From Malthus to Solow: How Did the Malthusian Economy Really Evolve?

Nicholas Crafts

(University of Warwick)

and

Terence C. Mills

(Loughborough University)

January 2007

We are very grateful to Greg Clark for making his wage data available to us and to Esteban Nicolini for letting us see his paper prior to publication. The usual disclaimer applies.

1. Introduction

The advent of unified growth theory has awakened renewed interest in the idea of a Malthusian economy. In the well-known paper of Galor and Weil (2000) there is a three-stage model in which the onset of modern economic growth is endogenous and is triggered by increases in the size of the population. The defining characteristic of a Malthusian phase in economic development is that there is no long run trend growth in real wages. Improvements in productive potential are swallowed up by population growth. At the same time population growth is regulated by a preventive check on fertility and a positive check on mortality such that a homeostatic equilibrium is maintained.

The basic idea of the Malthusian model is captured in Figures 1 and 2. In Figure 1 the preventive check is reflected in the upward-sloping curve, which shows a positive relationship between the birth rate and the real wage, and the positive check is reflected in the downward-sloping curve which depicts a negative relationship between mortality and the real wage. At the intersection of these two curves fertility and mortality are equal, population growth is zero and wages are at the Malthusian equilibrium level. The population level that is compatible with this real wage is shown by the production function relationship in the lower panel, which embodies diminishing returns to labour. Figure 2 displays a Malthusian model with "technological progress"[1] that increases the demand for labour over time such that a larger population can be sustained at a constant real wage. Here, equilibrium is at the real wage that implies an excess of fertility over mortality such that population growth is at the rate, *r*, which is compatible with constant real wages over time.

The pre-industrial English economy was one in which population grew over time as Figure 3 shows. By 1800 population was 8.6 million compared with 2.8 million in 1541 (Wrigley and Schofield, 1981). Estimates of real wages were made many years ago by Phelps-Brown and Hopkins (1956) and a much-improved series has recently been published by Clark (2005): these are graphed in Figure 4. Both series exhibit considerable volatility but do not appear to demonstrate trend growth over the period from the 13th to the 18th century. Figure 5 confirms that, from the mid-16th century to the end of the eighteenth century, a tripling of the size of the population was absorbed without any deterioration in the real wage rate. Thus, to a first approximation this experience is similar to the Malthusian model of Figure 2 rather than that of Figure 1.

[1] The literature has generally referred to the outward shift of the labour demand schedule as "technological progress" and we maintain that usage in this paper. Strictly speaking, the accumulation of other factors of production may also be partly responsible and estimates of the shift parameter should not be taken literally to measure TFP growth as in growth accounting.

The Malthusian configuration of Figure 2 is similar to the one envisaged by Galor and Weil (2000) as the first phase of their unified growth model. They also suppose that increases in population size promote faster technological progress and that eventually, as the Malthusian phase comes to an end, faster technological progress induces parents to invest in the education of their children. A virtuous circle ensues in which there is a positive feedback between technological progress and the accumulation of human capital in a post-Malthusian phase that culminates in the demographic transition to low fertility and high human capital and a modern growth regime.

The classic account of population growth in England before the industrial revolution is that of Wrigley and Schofield (1981). They painted a picture of a Malthusian model with weak homeostasis, i.e., with large shocks and a slow return to an equilibrium real wage which remained constant until 1800, in which population growth could be sustained at about 0.5 per cent per year. They argued that there was clear evidence of the operation of a preventive check which dominated the system as a whole but that the positive check had disappeared by the 17th century.

Wrigley and Schofield (1981) relied on informal inference using graphs, but their view of the workings of the pre-industrial English Malthusian economy was largely confirmed by Lee and Anderson (2002) using Kalman-filter econometric methods to analyze the real wage series of Phelps-Brown and Hopkins (1956) and the demographic series of Wrigley and Schofield (1981). The results obtained by Lee and Anderson (2002) were that from 1600 to 1795 population growth at 0.47 per cent per year, driven by technological progress at a similar rate, was consistent with a constant level of real wages, that fertility responded positively and mortality negatively to wages with elasticities of 0.12 and –0.08, respectively, that the intercepts in these fertility and mortality equations varied greatly over time such that most of the changes in vital rates were unrelated to changes in wages, while the system only returned to the Malthusian equilibrium slowly, the half-life of a shock being 107 years.

Nicolini (2007) analyzed the same data with a focus on the evolution of the preventive and positive check relationships within the Malthusian system using vector autoregressive (VAR) methods to compare three hundred-year intervals, 1541-1640, 1641-1740 and 1741-1840. He found that there were both positive and preventive checks in the first period, a preventive but no positive check in the second period, and that there was neither a preventive nor a positive check in the third period. In other words, the relationships which bring the Malthusian system back to equilibrium had disappeared by the mid-18th century.

Clark (2005) reviewed the implications of his new real wage series, although without using modern time-series econometric methods. He suggested that, although the pattern of fluctuations was different from that in the earlier Phelps-Brown and Hopkins series, the same story of a Malthusian equilibrium at constant real wages prevailing until 1800 still obtains. But he also detected the beginnings of an escape from the Malthusian stagnation – what Galor and Weil (2000) would see as the later phase of the Malthusian epoch – in the 1640s when the wage to population (production function) relationship started to move out from the origin and the economy went from Figure 1 to Figure 2.

The publication of the Clark real wage series, which is a major advance on its predecessor, allows us to revisit the quantification of the Malthusian system of pre-industrial England. We re-trace the steps of Lee and Anderson (2002) and Nicolini (2007) using the new wage data, in each case introducing some methodological refinements. We also examine the evolution of economic-demographic interactions from the standpoint of the implications for the unified growth model of Galor and Weil (2000). In particular, we address the following questions:

1) When did wages cease to be Malthusian?

2) When did fertility and mortality stop exhibiting Malthusian responses to changes in real wages?

3) Was the economy characterized by weak homeostasis?

4) Why was population so much bigger in 1800 than in 1550?

5) What can be inferred about the rate of technological progress prior to the industrial revolution?

2. Data

Annual demographic estimates for England (crude birth rate, crude death rate and population size) are available for 1541 to 1871 in Wrigley and Schofield (1981, pp. 531-535). In common with all the earlier papers cited above, these are the basis for our analysis. Those earlier papers also used the annual real wage series for 1500-1912 that Wrigley and Schofield generated from the original Phelps-Brown and Hopkins work, where missing observations were interpolated as explained in Wrigley and Schofield (1981, pp. 639-640). In what follows this is labelled the WS wage series.

We prefer the real wage series recently published by Clark. This is based on a better sample of wage and price information, a better procedure to calculate national averages and superior index number methods. Clark (2005, pp. 1321-1338) provides a detailed explanation and comparison with the earlier Phelps-Brown and Hopkins estimates. Clark has supplied us with the annual series underlying the estimates reported in his paper. Our econometric models are estimated using both the Clark and the WS series so that it can be seen how much difference the revised real wage estimates make.

The Clark real wage series begins in 1263. This permits a longer-run assessment of trends in real wages than is possible for modelling economic-demographic interactions. It also allows an alternative way to fill in the missing observations in the Phelps-Brown and Hopkins series, by assuming that changes were parallel to those in the Clark series. The resultant real wage series is labelled PBH and starts in 1264. This is graphed together with the Clark series in Figure 4.

3. Univariate Modelling of Real Wages

We begin by examining the properties of the Clark and PBH real wages series from their start dates to 1913. Figures 6 and 7 show the logarithms of the series, with simple linear breaking trend functions fitted to them. The break points of the trends were identified using the search procedure and tests of Sayginsoy and Vogelsang (2004), which are robust to the presence of highly persistent serial correlation, including unit root errors.

The fitted trend models are of the form

$$x_t = \beta_0 + \beta_1 DT_t + \beta_2 DU_t + u_t \qquad (1)$$

where x_t is the logarithm of real wages and

$$DT_t = \begin{cases} 0 & t < 1800 \\ t - 1800 & t \geq 1800 \end{cases} \qquad DU_t = \begin{cases} 0 & t < 1800 \\ 1 & t \geq 1800 \end{cases}$$

Table 1 reports the results of fitting equation (1) and also includes estimates for the WS real wage series for 1899 to 1912. *DU* allows a discontinuity ('crash') at 1800 and is significantly negative in each case, although the magnitude of the crash is much smaller for the Clark series than the PBH and WS series. The trend level of the series before 1800 is given by β_0 and trend growth after 1800 by β_1. OLS estimation was used under the condition that the errors u_t follow Assumption 1 of Sayginsoy and Vogelsang (2004) and

HAC standard errors are reported. Inclusion of a time trend as an additional regressor to allow trend growth to be non-zero pre-1800 led to insignificant slope coefficients in each regression.

In each series a break was found at 1800: before that date the trend function was horizontal, so that trend growth was zero; after that date the trend function implies trend growth of 1% per annum for the PBH and WS series, but 1.2% for the Clark series. As can be seen from the figures, models which allow further breaks could be fitted, but these would only identify segments corresponding to the long swings in real wages prior to 1800: our focus here is to identify a break in real wages at which the generating process switched from stationary to nonstationary.

All series show long swings around the constant trend level pre-1800. Figures 8 and 9 show the pre-1800 PBH and Clark series with 'long cycles' superimposed. These are calculated as local cubic polynomials and should be regarded as purely descriptive devices. The PBH long cycle has peaks at 1469 and 1745 and troughs at 1291 and 1631, while the Clark long cycle has peaks at 1474 and 1750 and troughs at 1306 and 1615 (ignoring the further trough near the end of the 18^{th} century which is probably a consequence of end-point effects on the local cubic calculation). The average amplitude (distance between successive turning points) is greater for PBH than for Clark (0.67 compared to 0.49), while the variability about swing is also higher for PBH than for Clark (0.14 compared to 0.08). Pre-1800, the PBH series thus has a higher mean, greater swing amplitude and higher volatility about the swing than the Clark series.

Table 2 provides a breakdown of the Malthusian period of constant real wages to highlight the extent of deviations from this equilibrium. Over the whole period the PBH series was more than 6 per cent away from equilibrium more than 40 per cent of the time. The Clark series has lower variability but was still more than 6 per cent away from equilibrium more than 25 per cent of the time.

Two points emerge from this initial investigation. First, the conventional wisdom about real wage growth is confirmed. There is a clear break of trend at 1800, before which the real wage was stationary over the long-run as predicted by the Malthusian model. The Clark series is no different from the PBH or WS series in this respect. Second, prima facie, the picture is one of weak homeostasis as has also generally been supposed in the literature. The Clarke series shows lower deviations from equilibrium on average than the PBH series and the amplitude of its 'long cycles' is smaller but, nevertheless, in neither case does it appear that the economy quickly returns to the equilibrium real wage after a shock.

4. VAR modelling

Given that all the alternative real wage series demonstrate a structural shift in 1800, we now focus attention on the dynamic interactions between real wages and demographic variables, i.e., birth and death rates, for the pre-1800 period. Similar to Nicolini (2007), we consider a three-dimensional VAR containing the logarithm of real wages, x_t, and crude birth and death rates taken from Wrigley and Schofield (1981), denoted b_t and d_t respectively. These demographic variables are available from 1541, and the series are shown in Figure 10. Both series are stationary around constant levels of 33.2 and 27.6, respectively, with ADF test statistics of -4.53 and -8.59 clearly rejecting unit roots. Similarly, both the Clark and WS real wage series are found to be stationary around constant levels for the same sample period, with ADF test statistics of -2.96 and -2.86, which reject unit roots at the 4% and 5% levels respectively. These findings imply that we can consider a VAR in the levels of $Y_t = (x_t, b_t, d_t)^\top$, i.e.,

$$Y_t = \mathbf{\Pi}_0 + \sum_{i=1}^{p} \mathbf{\Pi}_i Y_{t-i} + U_t \qquad (2)$$

where $U_t = (u_t^x, u_t^b, u_t^d)^\top$ with $E(U_t) = 0$ and

$$E(U_t U_s) = \begin{cases} \mathbf{\Omega}, & t = s \\ 0, & t \neq s \end{cases}$$

Using a combination of information criteria and examination of the autocorrelation properties of the individual equation residuals, the order *p* of the VAR was selected to be 5 when using the Clark real wage series and 4 when using the WS series.

The focus of this analysis, like that of Nicolini (2007), is to provide quantitative evidence on the workings of a Malthusian regime: in particular, of attempting to identify a period in which such a regime may actually have operated. Within the context of the above VAR, a Malthusian regime may be characterised by two key dynamic responses: (i) an increase in real wages should produce a decline in the death rate (the positive check); and (ii) an increase in real wages should produce an increase in the birth rate (the preventive check). Examining these responses in a multivariate setting is important because, for example, there should also be a positive response of real wages to an increase in the death rate, since higher mortality leads to a smaller population which, in turn, induces higher wages.

An important issue in computing dynamic responses (i.e., impulse response functions) is to be able to identify the shocks that impact upon the variables as structural or 'fundamental'. If the shocks U_t are contemporaneously correlated (i.e., Ω is not diagonal, so that the shocks are not orthogonal), then this cannot be done without imposing some identifying restrictions (e.g., by imposing an ordering on the variables that induces a recursive structure) or by defining responses to be invariant to such orderings. We follow the latter strategy and compute the generalised impulse responses of Pesaran and Shin (1997).

Since we are seeking to establish a period when Malthusian effects were in operation, we do not attempt to estimate a break date at which the VAR altered structure by using statistical tests alone. Rather, we estimate VARs for a variety of break dates throughout the period up to 1800 and calculate the implied preventive and positive checks pre- and post-break. These are estimated as the accumulated 25 year generalised impulse responses and are shown in Table 3 for WS real wages and Table 4 for Clark real wages. Examination of these sets of impulse responses suggests that Malthusian effects appear in samples that contain data up until the mid-1640s, but begin to disappear in samples that include data after the mid-1640s. Figure 11 plots the accumulated 25 year preventive and positive checks for the sample 1541–1645, while Figure 12 presents similar plots for the sample 1646–1799. Pre-1645, shocks to both real wage series produce similar preventive checks, with much of the positive response of birth rates being completed after twelve years. Both produce correctly signed positive checks but, whereas the response of the death rate to a WS real wage shock remains negative in the long-run, the response to a shock in Clark real wages dissipates towards zero over time. Post-1645, the preventive checks are stronger and more long lasting, but the positive checks are wrongly signed. (The rather wide confidence bands that accompany these accumulated responses are a typical feature of impulse response analysis in VARs and serve to temper any firm conclusions that might be drawn.)

However, looking at Table 3, there is no evidence to reject the null hypothesis that real wages had no effect on mortality, i.e., that there was no positive check, at any time in the period 1541-1800. Both Nicolini (2007) and Wrigley and Schofield (1981) thought that the positive check had disappeared by sometime early in the 17th century, which is understandable on the basis of the WS wage series, but with Clark real wages the positive check is nowhere to be seen with a 25-year window. The evidence for preventive checks is stronger, as earlier writers have supposed, and is clearer with the Clark than with the WS real wage series. Even so, it seems plausible using this lens that, by the late 17th century, the preventive check had also disappeared. Thus 18th century England seems to have been an economy in which the Malthusian control mechanisms no longer functioned: as Nicolini (2007) says, 'perhaps the world before Malthus was not so Malthusian'.

5. Structural Modelling

Lee and Anderson (2002) specify a structural model for the period 1540 to 1870 linking real wages, fertility and mortality via population, for which there is an identity linking the change in population to the crude birth and death rates plus net immigration, i.e., if p_t is the logarithm of population, then

$$p_t = p_{t-1} + b_t - d_t + e_t \qquad (3)$$

where e_t is defined so that the equation is true. Since $p_t - p_{t-1}$ is population growth, this must equal the crude birth rate minus the crude death rate plus the crude rate of net immigration, so that e_t is primarily net immigration.

The structural model is centred on the wage equation

$$x_t = \alpha_t - \beta p_t + s_t \qquad (4)$$

β is the elasticity of wages with respect to labour and the disturbance term s_t represents the impact of short-term shocks on wages, such as weather and harvest conditions, and is modelled as a stationary second order autoregression (AR(2)):

$$s_t = \gamma_1 s_{t-1} + \gamma_2 s_{t-2} + \varepsilon_t \qquad (5)$$

where ε_t is zero mean white noise. The time varying intercept α_t may be interpreted as the demand for labour, whose *change*, c_t, is assumed to follow a random walk:

$$\alpha_t = \alpha_{t-1} + c_t \qquad (6)$$

$$c_t = c_{t-1} + v_t \qquad (7)$$

where v_t is white noise. Why this particular specification? Over the period 1540 to 1870 the logarithm of population, p_t, is an $I(2)$ process, as argued by Bailey and Chambers (1993) and confirmed by unit root tests, while the logarithm of real wages, x_t, is $I(1)$. Thus equation (4) can only be 'balanced', and hence amenable to statistical analysis, if α_t is also $I(2)$ and cointegrated with p_t. The specification of equations (6) and (7) ensures that α_t is $I(2)$

unless the variance of v_t is zero, in which case α_t would collapse to a deterministic linear trend. Since α_t is unobservable, we can only assume that it is cointegrated with p_t, but indirect evidence for this will be provided on estimation of the AR coefficients in (5): if they imply stationarity then there is cointegration between α_t and p_t. In terms of its evolution, α_t is expected to increase over time: as industrialization becomes important this would accelerate its rate of increase through an upward drift or shift in c_t, which can be characterised as the rate of technological progress.

The crude death rate is modelled as a distributed lag of real wages plus a time varying intercept:

$$d_t = m_t + \sum_{i=0}^{k} \delta_i x_{t-i} + u_t \tag{8}$$

The positive check implies that $\sum \delta_i < 0$, while the disturbance term u_t represents short-term shocks to mortality, caused by environmental fluctuations such as weather and disease prevalence. Since these will tend to be correlated from year to year, u_t is modelled as a stationary AR(2) process

$$u_t = \lambda_1 u_{t-1} + \lambda_2 u_{t-2} + \rho_t \tag{9}$$

The intercept m_t reflects long-term exogenous influences on mortality, such as international exchange of diseases, changes in the virulence of disease, and cultural practices, and is assumed to follow the random walk

$$m_t = m_{t-1} + \phi_t \tag{10}$$

Both ρ_t and ϕ_t are assumed to be white noise. The crude birth rate is modelled in an identical fashion as

$$b_t = n_t + \sum_{i=0}^{k} \mu_i x_{t-i} + r_t \tag{11}$$

$$r_t = \kappa_1 r_{t-1} + \kappa_2 r_{t-2} + \psi_t \tag{12}$$

$$n_t = n_{t-1} + \xi_t \tag{13}$$

The preventive check implies $\sum \mu_i > 0$ and ψ_t and ξ_t are white noise. The AR(2) structure for r_t captures transient influences on fertility, such as variations in weather, health and morbidity, while the random walk structure of n_t models longer term effects such as occupational structure and changes in the rules and customs that regulate marriage.

Lee and Anderson (2002) set the lag lengths in equations (8) and (11) at $k = 4$ and estimate the structural model comprising equations (3) to (13) via the Kalman filter using the WS real wage series. Table 5 reports our estimates of this specification, obtained using the state space routine in EViews5. Since alternative state space routines can give different parameter estimates, it is important to compare the results in Table 4 with those reported by Lee and Anderson (2002, Table 1). The overall conclusion is that the main features of Lee and Anderson's results are preserved. The wage elasticity is significantly negative, although somewhat higher at -1.672 compared to -1.045. The variance of the innovation to the rate of technical progress is significantly positive and the autoregressive coefficients on the error process in the wage equation imply roots that are well below unity (being, in fact, $0.37 \pm 0.23i$). Thus the estimates are consistent with α_t and p_t being cointegrated. The sums of the lagged coefficients in the fertility and mortality equations are consistent with the preventive and positive checks found by Lee and Anderson and are of roughly similar magnitude. The variances of the innovations to the random walk intercept components in these equations are significantly positive, thus lending support to the time varying specification of the structural model. We are thus content that, within reasonable bounds, the estimates reported in Table 5a are consistent with those of Lee and Anderson (2002).

Table 6a reports estimates of the model when the Clark real wage series is used. Again, the general features found by Lee and Anderson are observed, but there are several subtle differences. The wage elasticity is rather smaller, -0.95, while the fertility response (the preventive check) is larger and more significant. Figure 13 shows the estimated demand for labour, $\hat{\alpha}_t$, and the rate of technological progress, $\hat{c}_t$, from the two alternative models (these, and all later, unobserved components are smoothed estimates). Both estimates of the demand for labour follow smooth paths up to 1800, with the average rate of technological progress averaging 0.75% per annum for WS real wages and 0.4% for Clark real wages. After 1800 there is a rapid acceleration in the demand for labour, with the rate of technological progress averaging 2.8% for WS real wages and 2.2% for Clark real wages (the different scales of the series shown in Figure 13 reflects both the different levels of the two series and the different estimates of β).

The 'rate of absorption', i.e., the rate at which population can grow consistent with maintaining constant real wages (cf. Figure 2), is defined in this model as c_t/β, so that its evolution through time is a scaled version of the rate of technological progress shown in Figure 13. For WS real wages, the rate of absorption thus averaged 0.45% per annum up until 1799 and 1.67% per annum after 1800. Using Clark real wages, these rates are 0.42%

and 2.32% respectively. These results for the WS series are also broadly similar to those of Lee and Anderson (2002), who obtained an absorption rate of 0.47% per annum for 1600 to 1795 and 2.01% for 1795 to 1870.
Figure 14 shows the estimates of the time varying fertility and mortality intercepts. These show similar patterns to those reported in Lee and Anderson (2002): the mortality intercept is almost identical for both real wage series, whereas the fertility intercept is smaller for Clark real wages than for WS real wages. Figure 14 implies that shifts of, rather than movements along, the birth and death rate schedules of Figures 1 and 2 were more important, so that real wages played relatively little part in the evolution of vital rates. Lee and Anderson (2002) found that the elasticities of fertility and of mortality with respect to real wages were 0.121 and -0.076, respectively. Our estimates with the WS series are very similar at 0.112 and -0.062, respectively. However, when estimation is based on Clark real wages, the elasticities are appreciably larger at 0.186 and -0.113, respectively, a result which reflects the lower volatility of Clark real wages.

Lee (1993) shows that the model will converge to equilibrium at a rate given by $\left(\sum\mu-\sum\delta\right)\beta$. For WS real wages, this takes the value $(0.00342+0.0197)\times 1.672$ $=0.0107$. The half-life of a an exogenous shock, *T*, is then obtained by solving $0.5=\exp(-0.0107T)$, which gives $T=65$ years. For Clark real wages, the rate and half-life are $(0.00594+0.00206)\times 0.948=0.0076$ and $T=91$ years. Thus, a picture of 'weak homeostasis' is confirmed.

Two variants of this model were investigated. First, insignificant lags in the mortality and fertility equations were excluded, along with any insignificant autoregressive terms (see Tables 5b and 6b). This had the affect of making inferences on the sums of lag coefficients (the preventive and positive checks) sharper, as the standard errors of these sums are reduced (note that in Tables 5a and 6a, no sum is significantly different from zero at conventional levels). The preventive check, the positive response of fertility to real wages, is then found to be significant (*t*-ratios exceed 2), but the positive check, the negative response of mortality, is confirmed to be insignificant. Second, the wage elasticity β in equation (4) was allowed to time vary via a random walk specification. Although the innovation to the random walk was significantly positive, the induced coefficient variation was extremely small: using the Clark real wage, β varied between 0.924 and 0.926, while for WS real wages, this elasticity varied between 1.718 and 1.730.

To this point our results resemble quite closely those of Lee and Anderson (2002). Using the Clark real wage series does modify the details quite a bit but, nevertheless, the fundamental workings of the Malthusian model that these authors had in mind is still there. The picture remains one of weak homeostasis based on a relatively strong preventive check in a world

where shocks to fertility and mortality play a large part. The economy is still found to be capable of absorbing a little under 0.5 per cent per year population growth with real wages remaining constant.
Our earlier VAR modelling exercise suggests that it may be useful to explore in more detail how the Malthusian model evolved over time. To this end, the structural model was also fitted to the sub-periods 1540-1645, 1646-1799 and 1800-1870, in line with the breaks obtained from our earlier analyses, with crucial parameter estimates being shown in Table 7. The strongest results are found for the earliest sub-period, with large and significant wage elasticities, stronger and significant fertility effects, and half-lives of less than 20 years. For the later periods, results break down quickly, so that for the last period from 1800 all crucial parameters are insignificant and the system is unstable, presumably confirming that Malthusian effects had completely disappeared by then.

In fact, these results for the pre-1800 period have strong similarities with those that were obtained from the VAR modelling. There is no evidence of a positive check and after 1645 the real wage effect on mortality has the wrong sign (though insignificant) with both real wage series. Again the preventive check is stronger with the Clark real wage series but, even in that case, real wages do not have a significant effect on fertility after 1645. The conclusion that the post-1645 period was not so Malthusian after all emerges even more strongly as the half-life of a shock is now 431 years and the Malthusian controls had effectively broken down already.

Dividing the period in this way also facilitates an assessment of the claim by Clark that his real wage series implies that the break from the Malthusian era of little advance in efficiency in England began circa 1640 (2005, p. 1308). The structural modelling approach offers no support for this argument, as is apparent from a glance at Figure 13 which shows no increase in the rate of technological progress in the 1640s. The estimates reported in Table 8 show a fall in technological progress after 1645 when the absorption rate is underpinned by a fall in the estimated value of β, the wage elasticity.

The equilibrium properties of the model may be demonstrated in the following way. Given the wage equation (4), along with (6) and (7), then

$$\Delta x_t = \Delta\alpha_t - \beta\Delta p_t + \Delta s_t = c_t - \beta\Delta p_t + \Delta s_t$$

In equilibrium, $\Delta x_t = \Delta s_t = 0$, so that

$$0 = c_t - \beta\Delta p_t$$

thus implying that

$$\Delta p_t^e = \frac{c_t}{\beta}$$

is the 'equilibrium population growth consistent with equilibrium real wage growth of zero'. Using the average absorption rates calculated from the model fitted to the 1540-1645 and 1646-1799 periods shown in Table 8, we obtain the equilibrium population series

$$p_t^e = p_{t-1}^e + (c_t/\beta) = p_0^e + \sum_{j=1}^{t} (c_j/\beta)$$

with $p_0^e = p_0$. Plots of c_t/β against Δp_t and p_t^e against p_t are shown in Figure 15. Equilibrium population is higher than actual population because typically c_t/β is larger than Δp_t during the period.

Thus, in the two and a half centuries before 1800 population growth was underpinned by an absorption rate of a little under 0.5% per annum on average. Actual population growth deviated from this rate because there were significant shocks to both fertility and mortality and equilibriating mechanisms were very weak. This implied that population growth could be quite rapid once the Malthusian controls had broken down and by the late 18th century the natural increase in the population was well over 1% per annum. But from the mid-17th- to the mid-18th century shocks depressed population growth below the absorption rate so that the population in 1800 of 8.6 million was still under the equilibrium level of 9.2 million.

6. Implications for Unified Growth Theory

Our empirical results are clearly consistent with some of the key stylized facts of the Galor and Weil (2000) model. The Malthusian phase of economic growth, in which there is a constant equilibrium real wage, persists up to the Industrial Revolution and a post-Malthusian phase, in which there is quite rapid technological progress, is established in the early 19th century. The late-Malthusian period in which there is trend population growth is characteristic of the entire period from the mid-16th to the late-18th century such that population in 1800 is about 3 times the level of 1550.

In other respects, however, the results are less obviously compatible with the Galor and Weil (2000) model. In important ways the economy had ceased to be Malthusian from the mid-17th century, given that both preventive and positive checks were no longer apparent and the economy could move far away from the Malthusian steady state for prolonged periods. Moreover, the positive feedback from population growth to technological progress which eventually brings the Malthusian phase to an end in the unified growth model does not show up in our results in the sense that there is no general tendency for technological progress to accelerate as population increases.

This is not necessarily a decisive objection. It might be argued that there is a ‘peso problem’, i.e., that the probability of stronger technological progress was enhanced a bit by demographic growth but without the favourable outcome actually materializing in the Malthusian period. This may be more persuasive if it is recognized that, in order to have a perceptible macroeconomic impact, inventions have to occur in sectors in which demand conditions would allow them to be a substantial fraction of GDP. Breakthroughs in textiles would be massive compared with earlier technological progress in books, gunpowder, glassware, etc. (Clark, 2004).

The route to a more persuasive model of positive feedbacks that generate an exit from the Malthusian era may require some greater complexity, however. A promising approach may be to focus on the implications of the growth of urban population rather than total population. Allen (2000) estimated that the period from 1500 to 1800 saw substantial urbanization of England, with urban population rising from 0.18 million (7.2%) to 2.61 million (28.7%). In the circumstances of the late-Malthusian economy, with its virtually complete move to capitalist farming, an open economy, and sharply diminishing marginal productivity of labour in agriculture, population growth was a strong impetus to the economy becoming more industrial and more urban (Crafts and Harley, 2004).

In turn, the disappearance of the preventive check in the later-Malthusian English economy may be related to the growth of industrial employment and increasing urbanization was conducive to more innovation. Goldstone (1986) stressed that emerging industrialisation provoked changes in nuptiality during the 18th century which underwrote the substantial acceleration of population growth after 1750, while Bairoch (1991) showed that innovation was disproportionately urban in 18th- and 19th-century Europe.

That said, it may be inappropriate to focus on positive feedbacks between population growth and technological progress as the key to escape from the Malthusian era rather than considering alternative models of technological change that draw on ideas from endogenous growth theory in the large rather than unified growth theory in particular. For example, Acemoglu et al. (2005) have pressed the point that the seeds of the industrial revolution were laid by Atlantic trade and the impetus that it gave to the development of capitalist institutions. Alternatively, Mokyr (2002) has stressed the role of a ‘knowledge revolution’, associated both with the Enlightenment and the development of factory-based production, which reduced the access costs of knowledge. In the context of endogenous innovation models, these arguments would amount, respectively, to increases in the appropriability of returns and to increases in the productivity of inputs to innovation and thus to faster growth in either case. Future attempts by growth economists to model the transition to modern economic growth should perhaps pay more explicit attention to improvements in capabilities and in incentive structures that increased the probability of technological advance.

7. Conclusions

In the introduction we posed five questions, the answers to which we now summarize. Since we have argued that the Clark real wage series is to be preferred to the earlier Wrigley and Schofield/Phelps-Brown and Hopkins series, we discuss only the results that have been obtained using the Clark series, but we note where these differ from earlier estimates based on the older real wage series. Our key findings are as follows.

1) Wages ceased to be Malthusian at the end of the 18th century, after which they exhibited strong trend growth (Figure 7 and Table 2). This confirms earlier findings.

2) Our analysis was unable to find evidence of a positive check at any time during the period 1541 to 1800 and the preventive check cannot be found in samples drawn after the mid-17th century (Tables 4, 6 and 7). These results are much less supportive of the fundamental Malthusian relationships between fertility, mortality and real wages than earlier work. In this sense the Malthusian economy was not very Malthusian after all.

3) The economy was characterized by weak homeostasis up to the mid-17th century. Subsequently, equilibriating tendencies were extremely weak and the half-life of a shock is estimated at 431 years (Table 7).

4) Population in 1800 was about three times the mid-16th century level and this demographic growth was underpinned by an absorption rate based on an expanding demand for labour that averaged a little under 0.5% per annum (Table 8). This result is very similar to earlier work.

5) There is nothing to suggest that technological progress was accelerating long before the end of the Malthusian period and thus there is no explicit evidence of the positive feedback between population size and technological progress that is a key feature of unified growth models (Figure 13 and Table 8).

References

Acemoglu, D., Johnson, S. and Robinson, J., 2005. The rise of Europe: Atlantic trade, institutional change, and economic growth. *American Economic Review* 95, 546-579.

Allen, R. C., 2000. Economic structure and agricultural productivity in Europe, 1300-1800. *European Review of Economic History* 4, 1-25.

Bailey, R.E., Chambers, M.J., 1993. Long-term demographic interactions in pre-census England. *Journal of the Royal Statistical Society, Series A* 156, 339-362.

Bairoch, P., 1991. The city and technological innovation. In Higonet, P., Landes, D. and Rosovsky, H. (Eds.), *Favorites of Fortune.* Harvard University Press, Cambridge Mass., pp. 159-176.

Clark, G., 2004. The great escape: the industrial revolution in theory and history. Mimeo, UC Davis.

Clark, G., 2005. The condition of the working class in England, 1209 to 2004. *Journal of Political Economy* 113, 1307-1340.

Crafts, N. and Harley, C. K., 2004. Precocious British industrialisation: a general-equilibrium perspective. In Prados de la Escocura, L. (Ed.), *Exceptionalism and Industrialisation: Britain and Its European Rivals, 1688-1815*. Cambridge University Press, Cambridge, pp. 86-107.

Galor, O., Weil, D., 2000. Population, technology, and growth: from Malthusian stagnation to the demographic transition and beyond. *American Economic Review* 90, 804-828.

Goldstone, J., 1986. The demographic revolution in England: a re-examination. *Population Studies* 49, 5-33.

Lee, R., 1993. Accidental and systematic change in population history: Homeostatis in a stochastic setting, *Explorations in Economic History* 30, 1-30.

Lee, R., Anderson, M., 2002. Malthus in state space: macro economic-demographic relations in English history, 1540-1870. *Journal of Population Economics* 15, 195-220.

Mokyr, J., 2002. *The Gifts of Athena*. Princeton University Press, Princeton.

Nicolini, E. A., 2007. Was Malthus right?: a VAR analysis of economic and demographic interactions in pre-industrial England. *European Review of Economic History* 11, forthcoming

Pesaran, M.H., Shin, Y. 1997. Generalized impulse response analysis in linear multivariate models. *Economics Letters* 58, 17-29.

Phelps-Brown, E. H., Hopkins, S. V., 1956. Seven centuries of the prices of consumables compared with builders' wage rates. *Economica* 23, 296-314.

Sayginsoy, O., Vogelsang, T., 2004. Powerful tests of structural change that are robust to strong serial correlation, mimeo, CUNY.

Wrigley, E. A., Schofield, R. S., 1981. *The Population History of England, 1541-1871: a Reconstruction.* Arnold, London.

% deviation from equilibrium	1264 - 1799		1264 - 1540		1541 - 1645		1646 – 1799	
	#	Σ	#	Σ	#	Σ	#	Σ
,-6)	81	15.1	35	12.6	42	40.0	4	2.6
[-6,-4)	58	25.9	27	22.4	22	60.9	9	8.4
[-4,-2)	51	35.4	32	29.6	15	75.2	16	18.8
[-2,-1)	42	43.3	10	33.2	10	84.8	22	33.1
[-1,0)	42	51.1	10	36.8	5	89.5	27	50.6
[0,1)	41	58.8	9	40.1	4	93.3	28	68.8
[1,2)	42	66.6	15	45.5	3	96.2	24	84.4
[2,4)	54	76.7	29	56.0	4	100.0	21	98.0
[4,6)	63	88.4	61	78.0	0	100.0	2	99.3
[6,	62	100.0	61	100.0	0	100.0	1	100.0

(a) Clark

% deviation from equilibrium	1264 - 1799		1264 - 1540		1541 - 1645		1646 – 1799	
	#	Σ	#	Σ	#	Σ	#	Σ
,-6)	108	20.1	22	7.9	56	53.3	30	19.5
[-6,-4)	53	30.0	20	15.2	8	61.0	25	35.7
[-4,-2)	71	43.3	29	25.6	12	72.4	30	55.2
[-2,-1)	34	49.6	7	28.2	15	86.7	12	63.0
[-1,0)	49	58.8	20	35.4	8	94.3	21	76.6
[0,1)	30	64.4	15	40.8	5	99.0	10	83.1
[1,2)	18	67.7	7	43.3	1	100.0	10	89.6
[2,4)	38	74.8	24	52.0	0	100.0	14	98.7
[4,6)	23	79.1	21	59.6	0	100.0	2	100.0
[6,	112	100.0	102	100.0	0	100.0	0	100.0

(b) PBH

Table 1 Variability of the real wage rate about its equilibrium level. # denotes the cell count; Σ the cumulative proportion.

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
PBH	4.39	0.010	-0.33
	(0.03)	(0.001)	(0.05)
Clark	3.99	0.012	-0.08
	(0.02)	(0.000)	(0.03)
WS	4.22	0.010	-0.22
	(0.03)	(0.001)	(0.04)

Table 2 Estimates of breaking trend models for real wages. Figures in parentheses are HAC standard errors; see text for details.

1541 - t		t	$t+1$ - 1799	
Positive check $\overset{(-)}{x \to d}$	Preventive check $\overset{(+)}{x \to b}$		Positive check $\overset{(-)}{x \to d}$	Preventive check $\overset{(+)}{x \to b}$
-14.02	-0.88	1600	3.09	5.14*
-5.58	0.63	10	2.74	5.74*
-3.48	2.07	20	1.61	5.78*
-3.22	2.55	30	1.64	4.90
-3.05	2.49	40	1.34	6.28*
-3.24	2.34	1645	1.11	5.58
-2.96	2.93	50	0.12	1.48
-1.99	2.49	60	-0.66	0.44
-1.58	2.29	70	0.33	-0.09
-1.21	2.13	80	1.23	-0.33
-0.86	1.72	90	2.22	-1.34
-0.45	1.48	1700	1.27	-2.35
-0.08	-0.11	10	2.70	-4.83
0.09	0.69	20	1.19	-3.56
2.21	1.71	30	1.16	-4.60
2.39	2.24	40	-0.04	-2.62

Table 3 Accumulated 25 year generalised impulse functions using WS real wages. * denotes significant at 10% level.

1541 - t		t	$t+1$ - 1799	
Positive check $\overset{(-)}{x \to d}$	Preventive check $\overset{(+)}{x \to b}$		Positive check $\overset{(-)}{x \to d}$	Preventive check $\overset{(+)}{x \to b}$
5.24	4.15	1600	2.30	6.18*
2.56	4.97*	10	2.20	6.55*
0.58	4.87*	20	1.79	6.67*
-0.19	5.06*	30	1.27	6.02
-0.31	4.74*	40	0.51	8.17*
-0.66	3.01	1645	1.02	7.44*
-0.34	3.41	50	-0.08	1.34
1.50	1.60	60	-0.87	0.34
3.61	1.60	70	-0.77	0.84
3.40	1.37	80	0.05	0.94
4.45	0.79	90	0.57	0.59
4.28	0.66	1700	-0.48	-0.62
4.69	-0.88	10	-0.59	-2.64
4.27	0.17	20	-1.24	-3.04
5.24*	1.09	30	1.15	-4.40
4.99*	1.71	40	-0.04	-3.17

Table 4 Accumulated 25 year generalised impulse functions using Clark real wages. * denotes significant at 10% level.

Parameter	Description	Estimate	Standard error	*t*-ratio
β	Elasticity of *x* wrt *p*	1.672	0.894	-1.9
Fertility-wage coefficients	Effect of *x* on *b*			
μ_0	at lag 0	10.531	1.107	9.5
μ_1	at lag 1	-6.174	1.073	-5.8
μ_2	at lag 2	1.689	0.910	1.9
μ_3	at lag 3	-0.730	1.171	-0.6
μ_4	at lag 4	-1.895	0.822	-2.3
$\sum \mu$	Total effect	3.421	2.119	1.6
Mortality wage coefficients	Effect of *x* on *d*			
δ_0	at lag 0	-0.531	1.856	-0.3
δ_1	at lag 1	-6.261	1.794	-3.5
δ_2	at lag 2	0.303	2.555	0.1
δ_3	at lag 3	3.624	2.037	1.8
δ_4	at lag 4	0.896	2.598	0.3
$\sum \delta$	Total effect	-1.969	4.090	0.5
Variances	of innovations in			
$\ln V(\varepsilon)$	wage errors	-4.892	0.074	-66.0
$\ln V(v)$	rate of tech progress	-12.020	0.496	-24.2
$\ln V(\psi)$	fertility errors	-13.073	0.103	-126.0
$\ln V(\xi)$	fertility intercept random walk	-15.016	0.445	-33.7
$\ln V(\rho)$	mortality errors	-11.396	0.072	-158.4
$\ln V(\phi)$	mortality intercept random walk	-15.707	0.936	-16.8
AR coefficients	for errors in			
γ_1	wage	0.736	0.058	12.8
γ_2	wage	-0.188	0.061	-3.1
κ_1	fertility	0.495	0.075	6.6
κ_2	fertility	0.000	0.000	0.6
λ_1	mortality	0.545	0.047	11.6
λ_2	mortality	-0.091	0.045	-2.0

Notes: μ and δ coefficients expressed in per 000 of population.

Table 5a Estimates of structural model for 1540 – 1870 using WS real wage series.

Parameter	Description	Estimate	Standard error	*t*-ratio
β	Elasticity of *x* wrt *p*	0.948	0.471	-2.0
Fertility-wage coefficients	Effect of *x* on *b*			
μ_0	at lag 0	15.583	1.895	8.2
μ_1	at lag 1	-2.158	2.132	-1.0
μ_2	at lag 2	-3.364	2.023	-1.7
μ_3	at lag 3	3.111	1.880	1.7
μ_4	at lag 4	-7.764	1.918	-4.0
$\sum \mu$	Total effect	5.940	3.233	1.8
Mortality wage coefficients	Effect of *x* on *d*			
δ_0	at lag 0	-4.709	3.834	-1.2
δ_1	at lag 1	-10.907	3.835	-2.8
δ_2	at lag 2	1.208	4.121	0.3
δ_3	at lag 3	3.372	4.099	0.8
δ_4	at lag 4	8.981	3.893	2.3
$\sum \delta$	Total effect	-2.062	6.112	0.3
Variances	of innovations in			
$\ln V(\varepsilon)$	wage errors	-6.046	0.084	-72.0
$\ln V(v)$	rate of tech progress	-13.199	0.561	-23.5
$\ln V(\psi)$	fertility errors	-13.134	0.094	-139.0
$\ln V(\xi)$	fertility intercept random walk	-14.720	0.303	-48.6
$\ln V(\rho)$	mortality errors	-11.418	0.062	-183.2
$\ln V(\phi)$	mortality intercept random walk	-15.745	1.029	-15.3
AR coefficients	for errors in			
γ_1	wage	0.799	0.058	13.8
γ_2	wage	-0.255	0.057	-4.5
κ_1	fertility	0.274	0.094	2.9
κ_2	fertility	-0.000	0.000	-0.0
λ_1	mortality	0.532	0.044	12.1
λ_2	mortality	-0.058	0.045	-1.3

Notes: μ and δ coefficients expressed in per 000 of population.

Table 6a Estimates of structural model for 1540 – 1870 using Clark real wage series.

Parameter	Description	Estimate	Standard error	*t*-ratio
β	Elasticity of *x* wrt *p*	1.674	0.807	-2.1
Fertility-wage coefficients	Effect of *x* on *b*			
μ_0	at lag 0	10.564	0.986	10.7
μ_1	at lag 1	-6.146	1.017	-6.0
μ_2	at lag 2	1.485	0.855	1.7
μ_3	at lag 3	-		
μ_4	at lag 4	-2.087	0.736	-2.8
$\sum \mu$	Total effect	3.816	1.700	2.2
Mortality wage coefficients	Effect of *x* on *d*			
δ_0	at lag 0	-		
δ_1	at lag 1	-6.158	1.747	-3.5
δ_2	at lag 2	-		
δ_3	at lag 3	4.491	1.824	2.5
δ_4	at lag 4	-		
$\sum \delta$	Total effect	-1.667	2.530	-0.7
Variances	of innovations in			
$\ln V(\varepsilon)$	wage errors	-4.893	0.071	-68.9
$\ln V(v)$	rate of tech progress	-12.018	0.494	-24.3
$\ln V(\psi)$	fertility errors	-13.061	0.097	-134.6
$\ln V(\xi)$	fertility intercept random walk	-15.041	0.450	-33.4
$\ln V(\rho)$	mortality errors	-11.354	0.063	-179.7
$\ln V(\phi)$	mortality intercept random walk	-15.702	0.882	-17.8
AR coefficients	for errors in			
γ_1	wage	0.736	0.055	13.4
γ_2	wage	-0.188	0.059	-3.2
κ_1	fertility	0.501	0.073	6.8
κ_2	fertility	-		
λ_1	mortality	0.499	0.010	49.3
λ_2	mortality	-		

Notes: μ and δ coefficients expressed in per 000 of population.

Table 5b Estimates of structural model for 1540 – 1870 using WS real wage series; zero restrictions imposed.

Parameter	Description	Estimate	Standard error	*t*-ratio
β	Elasticity of *x* wrt *p*	0.920	0.459	-2.0
Fertility-wage coefficients	Effect of *x* on *b*			
μ_0	at lag 0	14.820	1.610	9.2
μ_1	at lag 1	-		
μ_2	at lag 2	-4.313	1.702	-2.5
μ_3	at lag 3	3.630	1.730	2.1
μ_4	at lag 4	-7.797	1.886	-4.1
$\sum \mu$	Total effect	6.339	3.100	2.0
Mortality wage coefficients	Effect of *x* on *d*			
δ_0	at lag 0	-5.238	3.567	-1.5
δ_1	at lag 1	-9.427	3.374	-2.8
δ_2	at lag 2	-		
δ_3	at lag 3	-		
δ_4	at lag 4	11.622	3.694	3.1
$\sum \delta$	Total effect	-3.043	5.400	-0.6
Variances	of innovations in			
$\ln V(\varepsilon)$	wage errors	-6.043	0.085	-71.0
$\ln V(v)$	rate of tech progress	-13.229	0.559	-23.6
$\ln V(\psi)$	fertility errors	-13.147	0.096	-137.5
$\ln V(\xi)$	fertility intercept random walk	-14.661	0.290	-50.6
$\ln V(\rho)$	mortality errors	-11.385	0.054	-209.3
$\ln V(\phi)$	mortality intercept random walk	-15.696	0.961	-16.3
AR coefficients	for errors in			
γ_1	wage	0.796	0.055	14.4
γ_2	wage	-0.251	0.051	-4.9
κ_1	fertility	0.256	0.093	2.8
κ_2	fertility	-		
λ_1	mortality	0.491	0.035	13.8
λ_2	mortality	-		

Notes: μ and δ coefficients expressed in per 000 of population.

Table 6b Estimates of structural model for 1540 – 1870 using Clark real wage series; zero restrictions imposed.

	β	$\sum \mu$	$\sum \delta$	T
1540 – 1645	4.528 $se = 2.136$ $t = 2.1$	6.75 $se = 3.47$ $t = 1.9$	-2.77 $se = 15.4$ $t = 0.2$	16
1646 – 1799	1.297 $se = 0.827$ $t = 1.6$	3.46 $se = 2.90$ $t = 1.2$	1.71 $se = 5.72$ $t = 0.3$	306
1540 – 1799	1.478 $se = 0.288$ $t = 5.1$	5.45 $se = 2.40$ $t = 2.3$	0.73 $se = 5.80$ $t = 0.1$	99
1800 – 1870	-1.701 $se = 2.431$ $t = 0.7$	-1.10 $se = 5.74$ $t = 0.2$	-4.67 $se = 3.22$ $t = 1.4$	-

(a) WS real wages

	β	$\sum \mu$	$\sum \delta$	T
1540 – 1645	2.211 $se = 0.939$ $t = 2.4$	10.57 $se = 5.31$ $t = 2.0$	-6.26 $se = 22.03$ $t = 0.3$	19
1646 – 1799	0.679 $se = 0.246$ $t = 2.8$	7.07 $se = 4.80$ $t = 1.5$	4.70 $se = 10.37$ $t = 0.5$	431
1540 – 1799	1.047 $se = 0.136$ $t = 7.7$	7.23 $se = 3.57$ $t = 2.0$	7.27 $se = 7.08$ $t = 1.0$	-
1800 – 1870	0.620 $se = 1.346$ $t = 0.5$	-6.99 $se = 9.06$ $t = 0.8$	-5.36 $se = 10.90$ $t = 0.8$	-

(b) Clark real wages

Table 7 Structural model; sub-period estimates.

	c/β	c	β
1540 – 1645	0.50%	1.10%	2.211
1646 – 1799	0.44%	0.30%	0.679

Table 8 Average absorption rates using Clark real wages calculated using sub-sample c_t and $\hat{\beta}$ from Table 7.

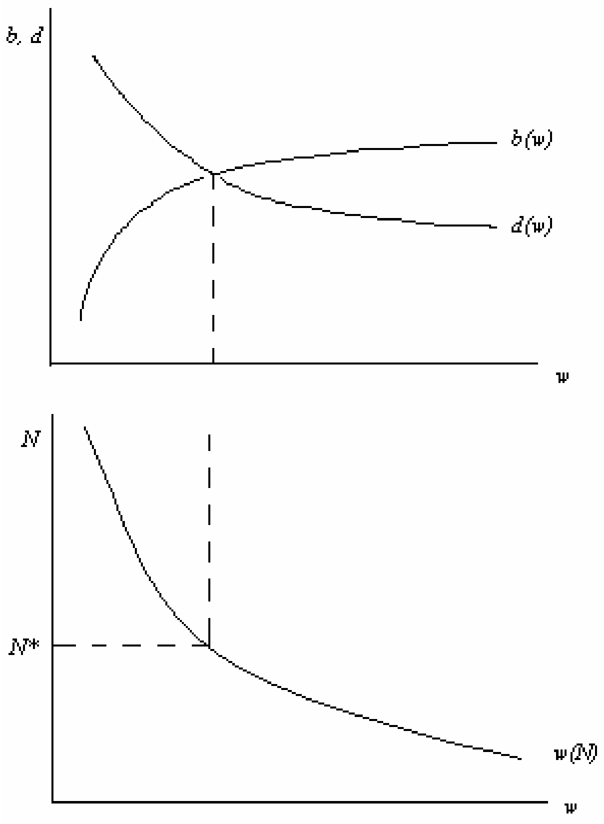


Figure 1 Economic-demographic equilibrium

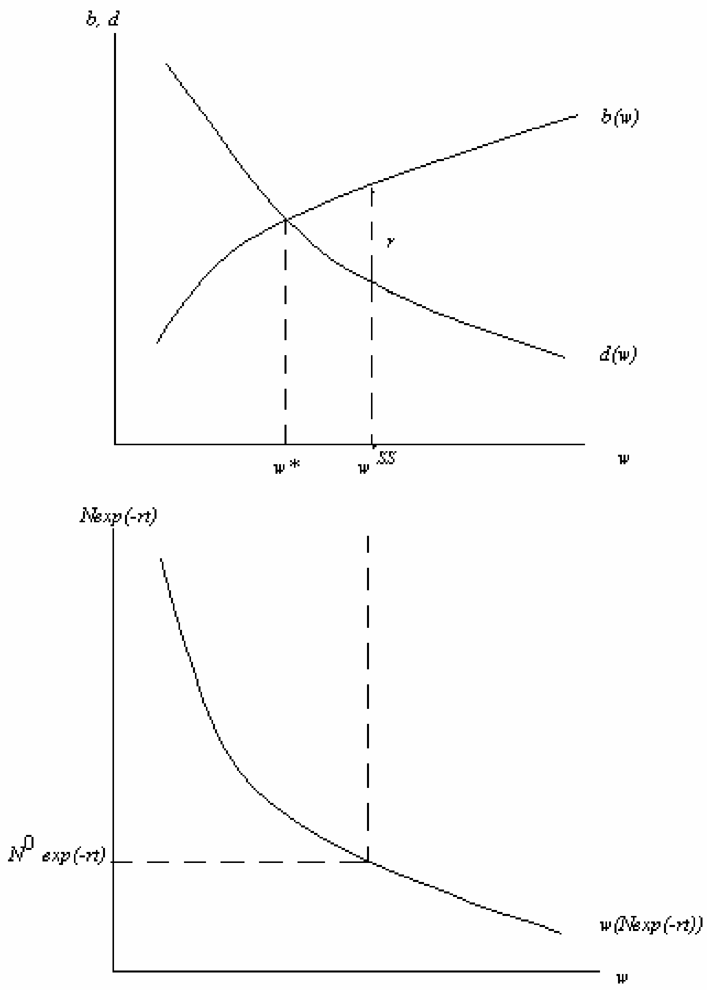


Figure 2 Labour demand increasing at a constant rate.

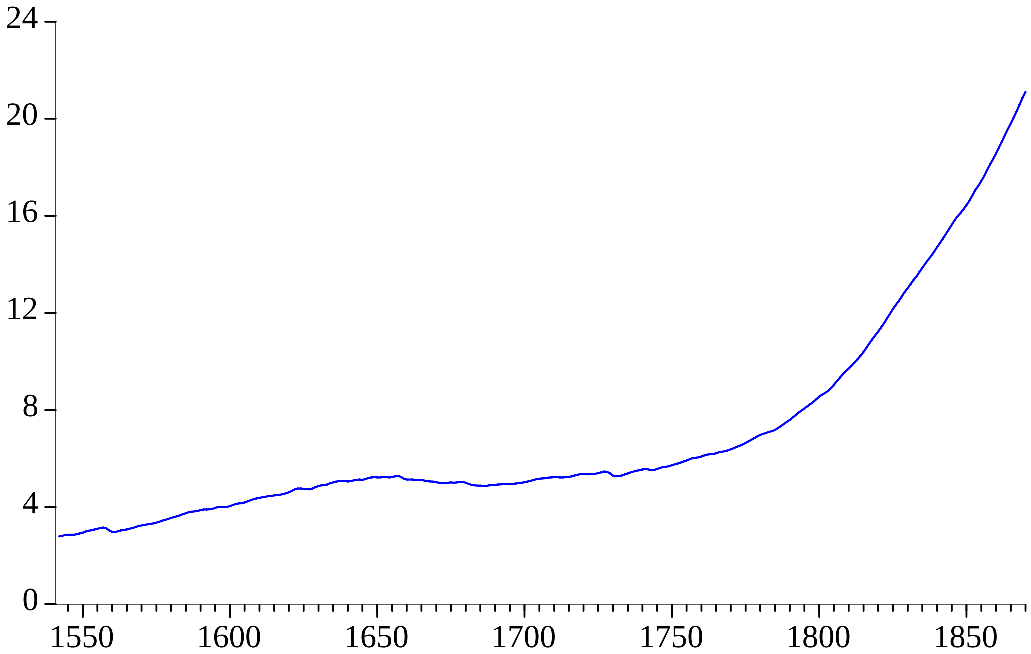


Figure 3 Population, 1541 – 1870 (millions).

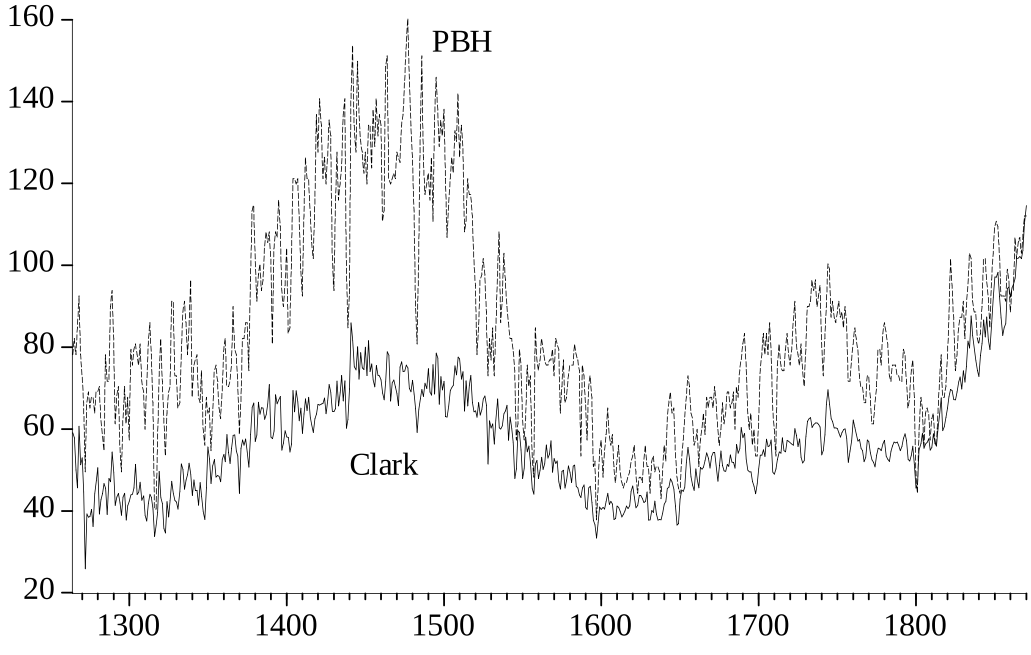


Figure 4 Real wages, 1264 – 1870.

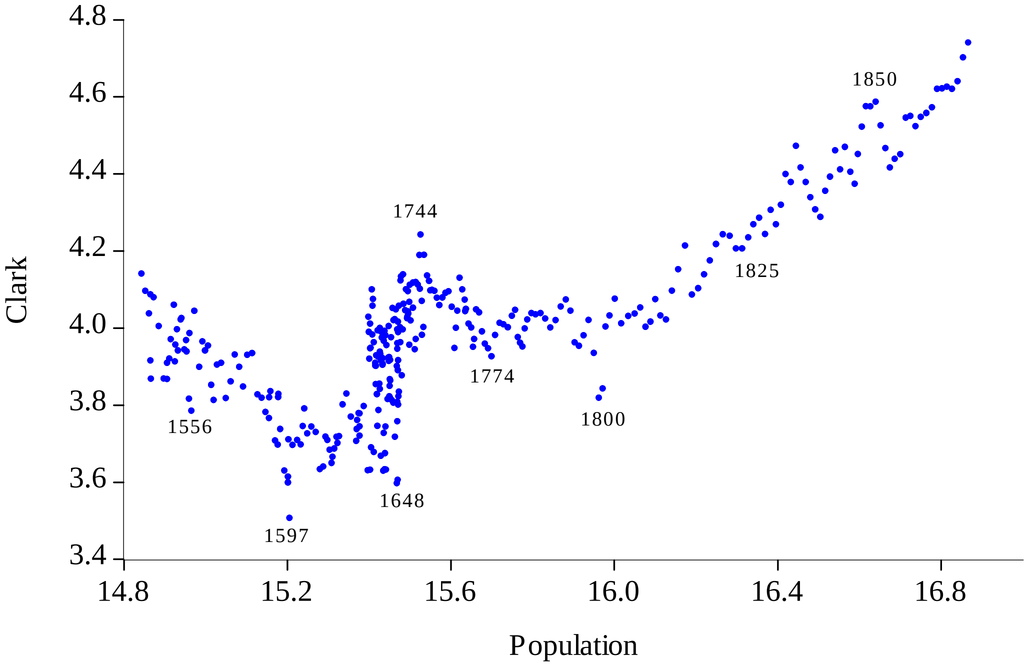


Figure 5 Scatterplot of Clark real wages and population; logarithms, 1541 – 1870.

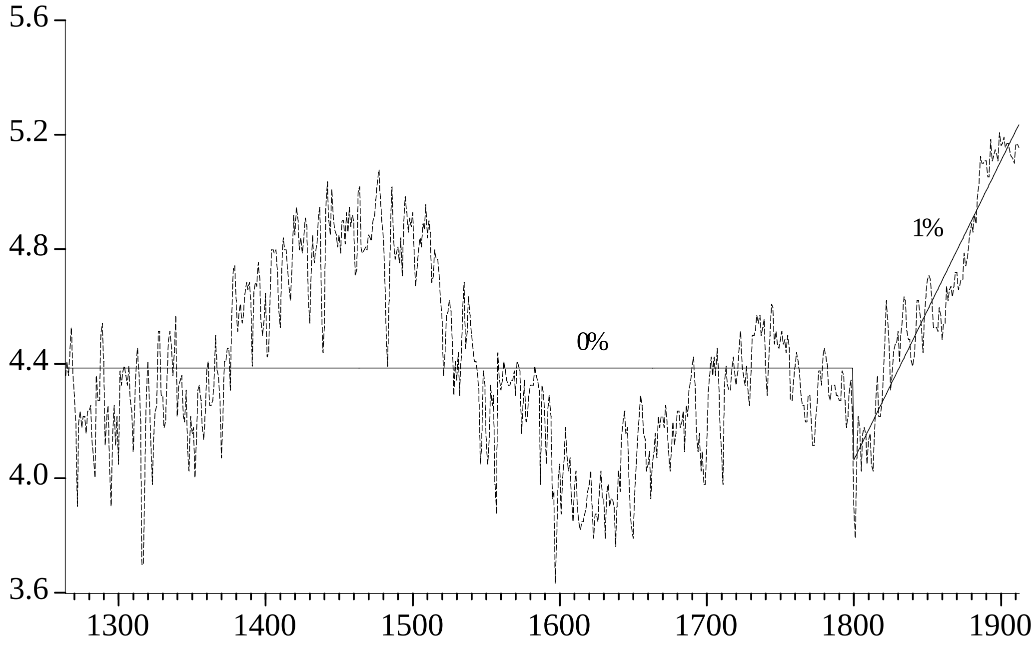


Figure 6 Logarithms of PBH real wages, 1264 – 1913, with breaking linear trend.

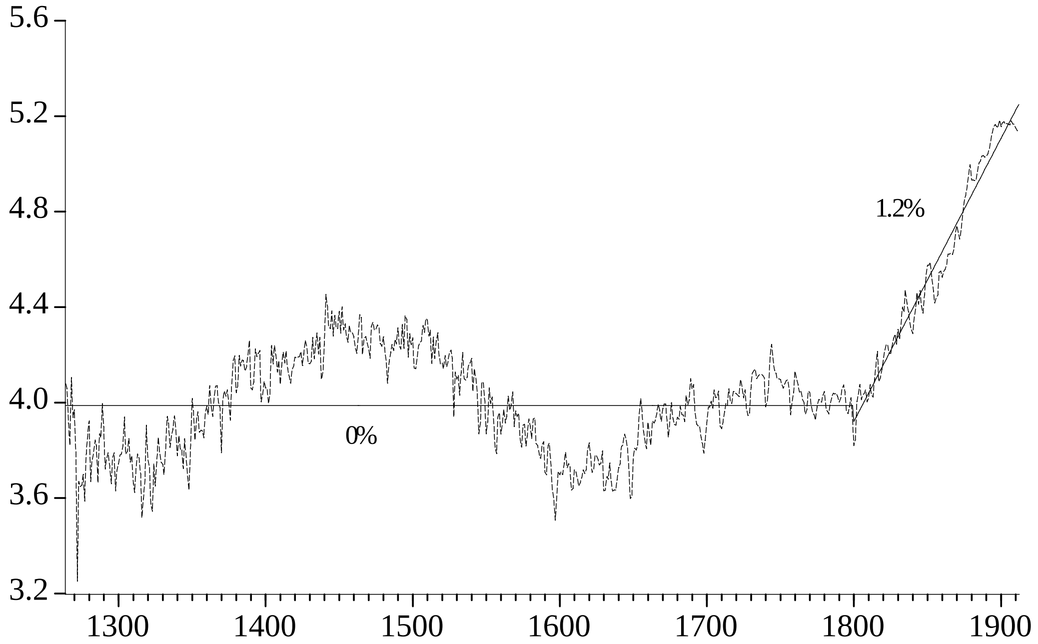


Figure 7 Logarithms of the Clark real wage series, 1263 – 1913, with breaking linear trend.

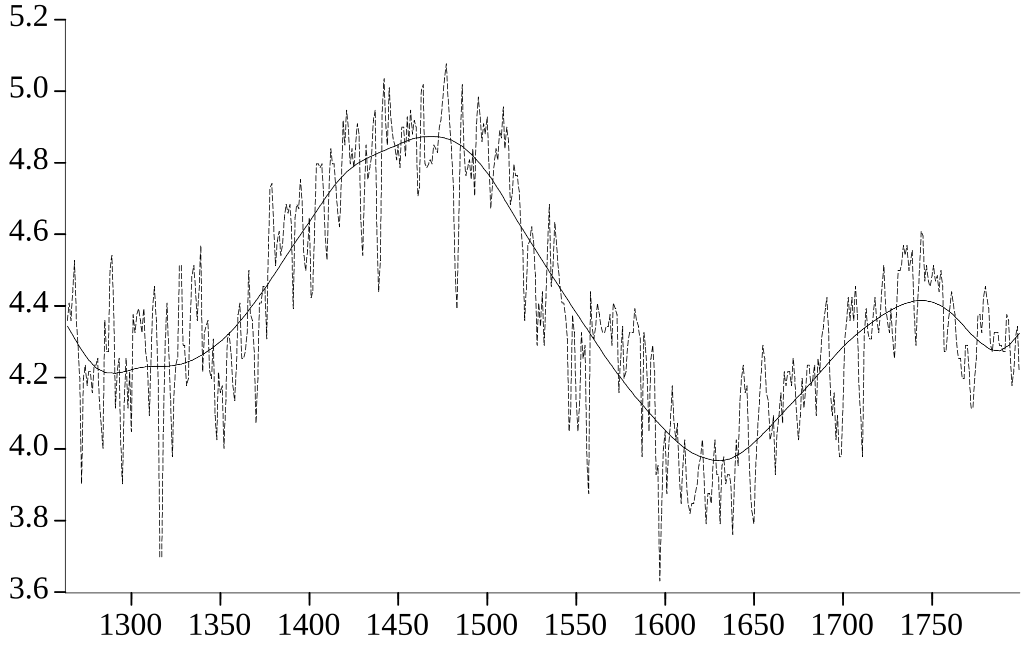


Figure 8 PBH real wages with fitted cycle, 1263 – 1799.

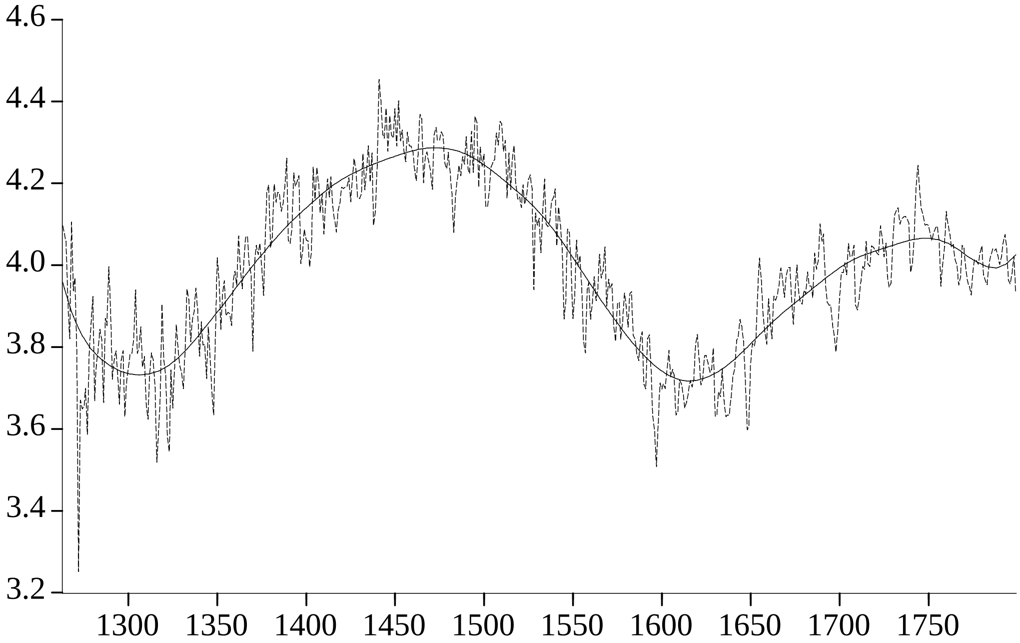
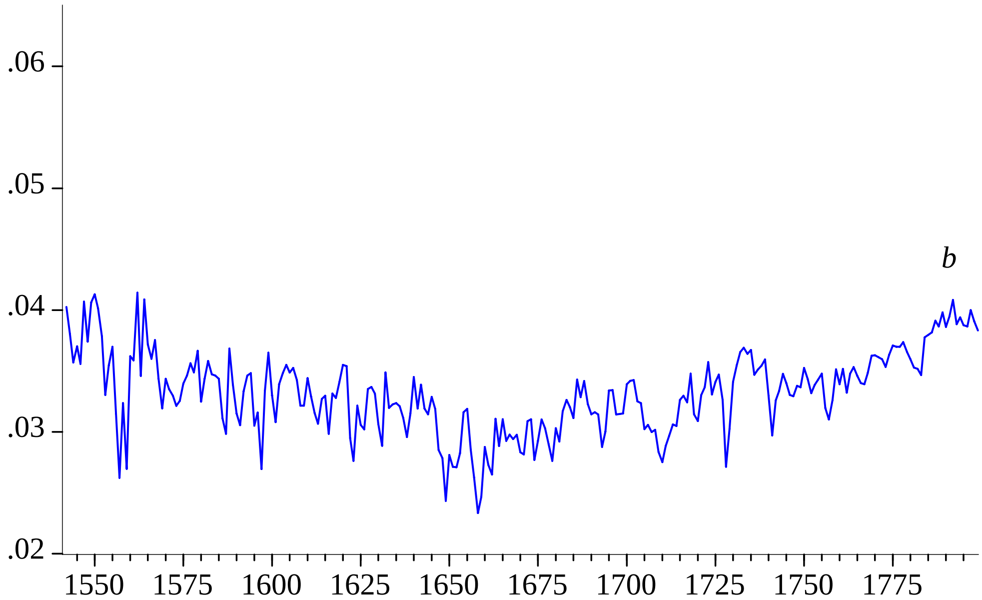


Figure 9 Clark real wages with fitted cycle, 1263 – 1799.



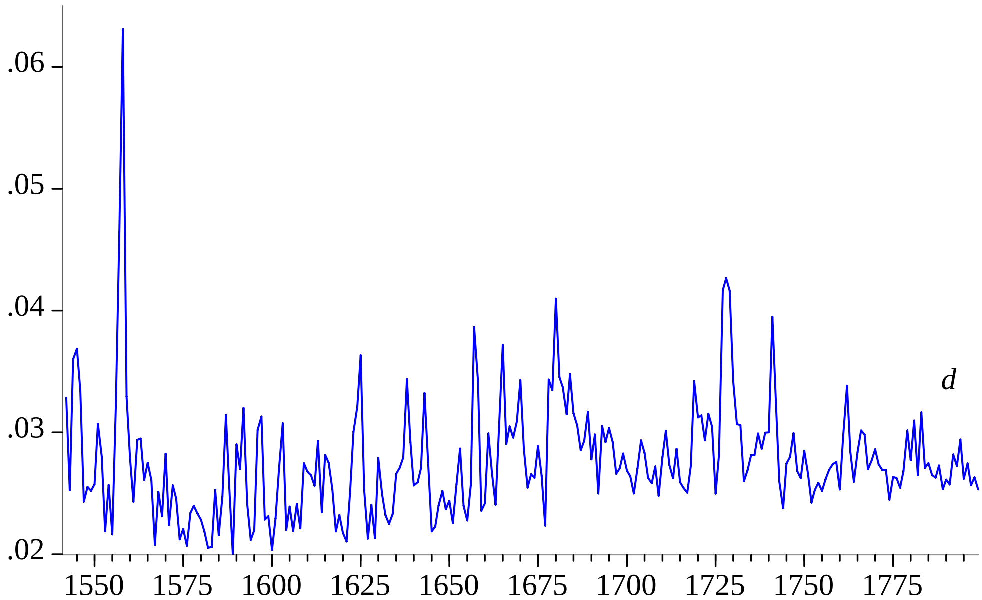
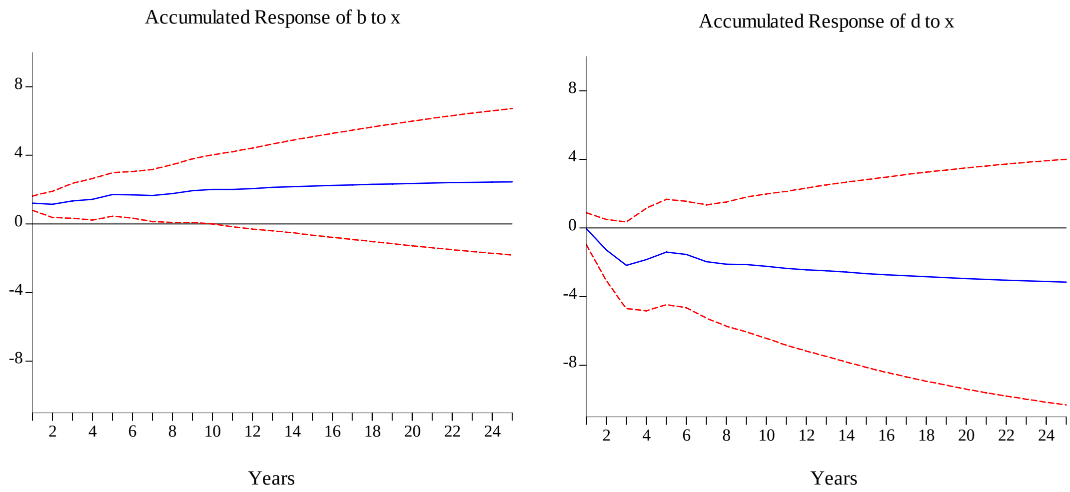
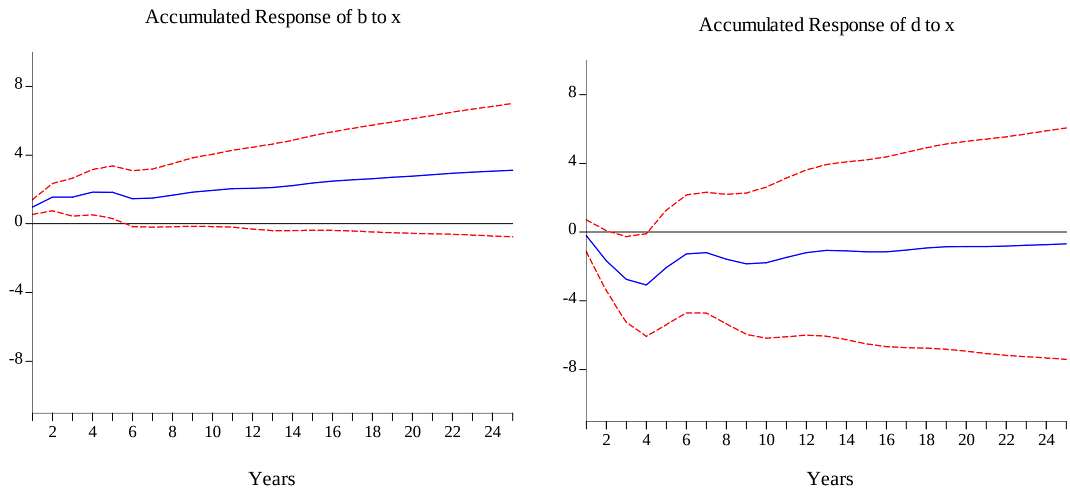


Figure 10 Crude birth (*b*) and death (*d*) rates; 1541 - 1799.

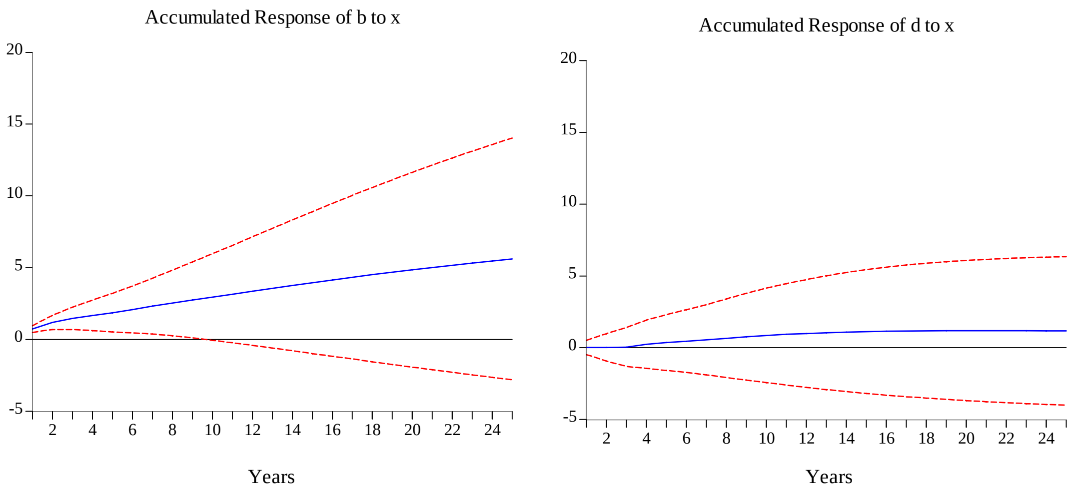


(a) WS responses

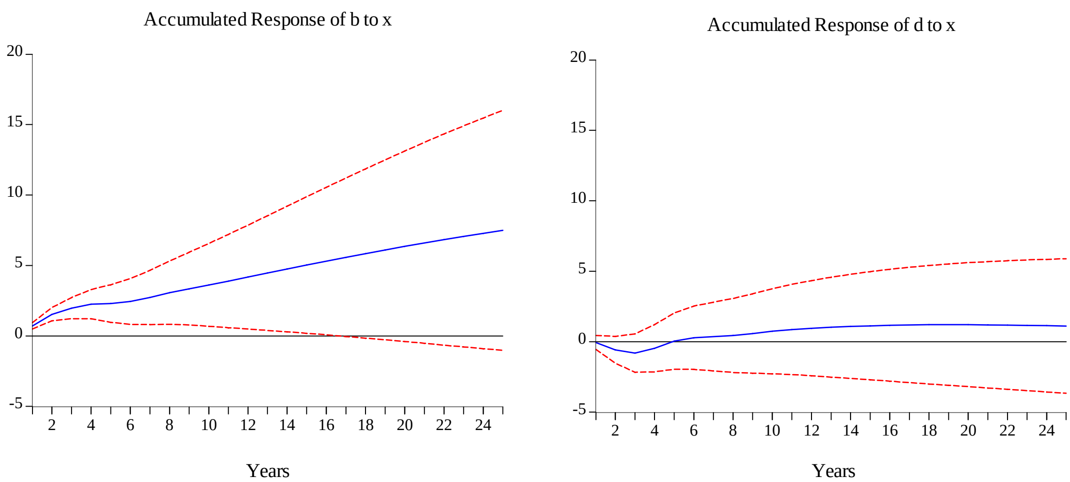


(b) Clark responses

Figure 11 Accumulated responses of the birth and death rates to a one standard deviation shock in (a) WS real wages, and (b) Clark real wages, using the sample 1542 – 1645 and with 2 standard error bands.

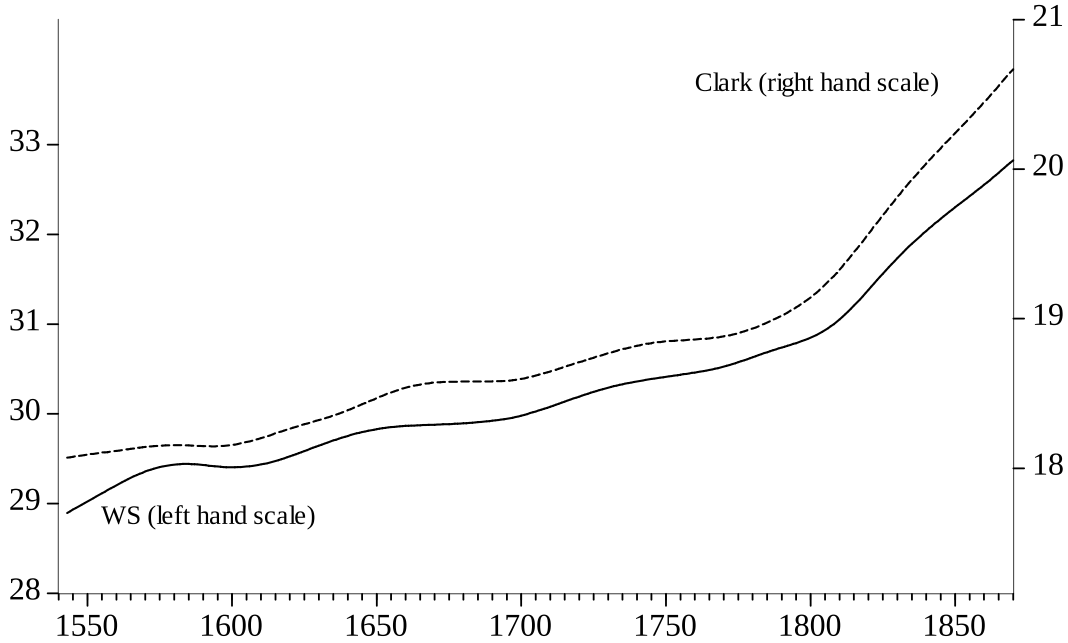


(a) WS responses

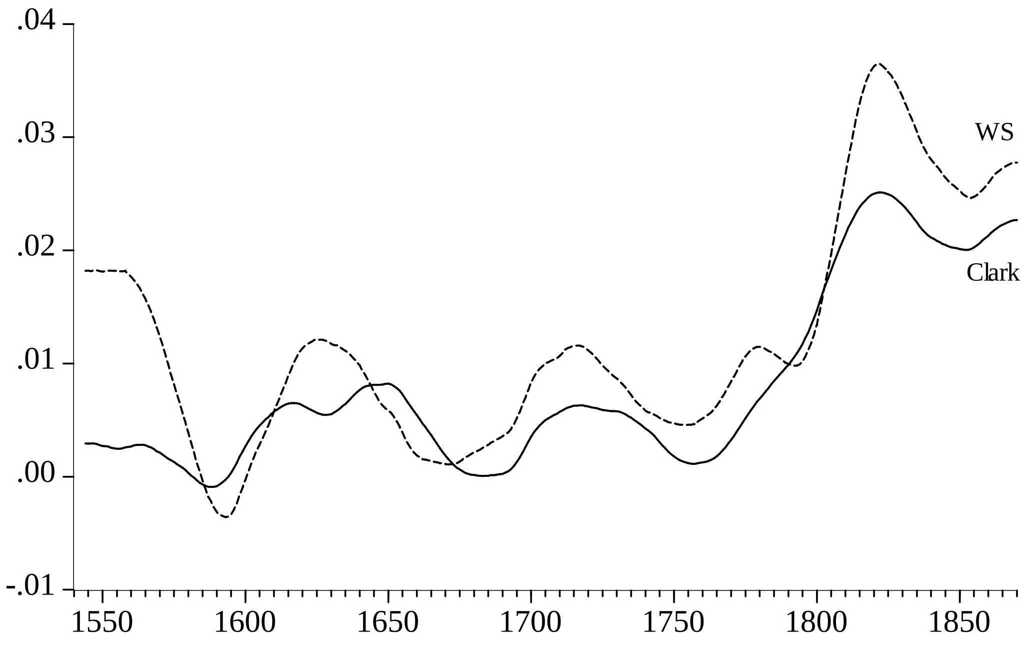


(b) Clark responses

Figure 12 Accumulated responses of the birth and death rates to a one standard deviation shock in (a) WS real wages, and (b) Clark real wages, using the sample 1646 – 1799 and with 2 standard error bands.

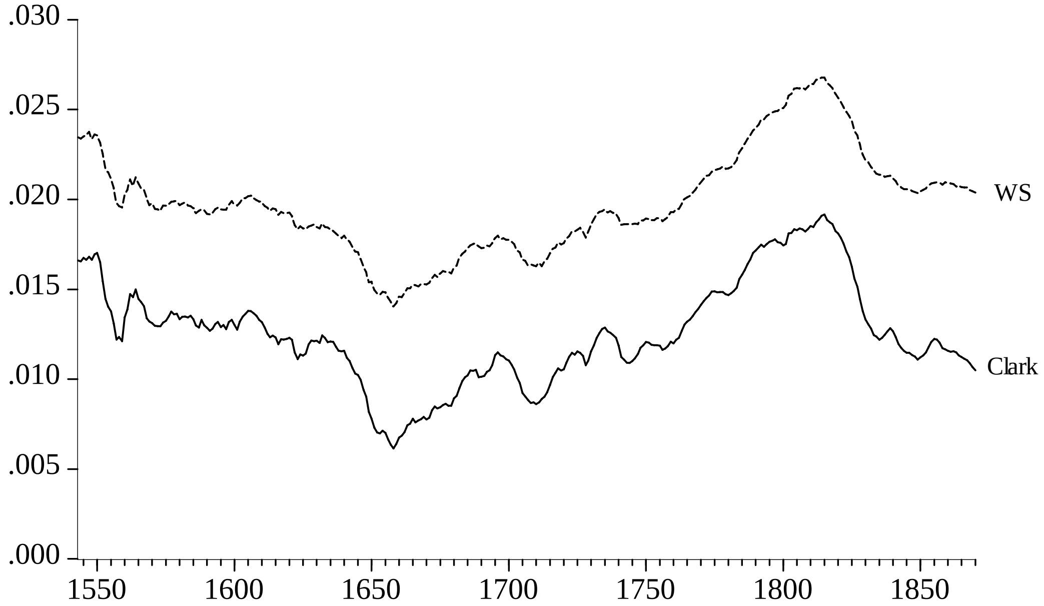


(a) Demand for labour

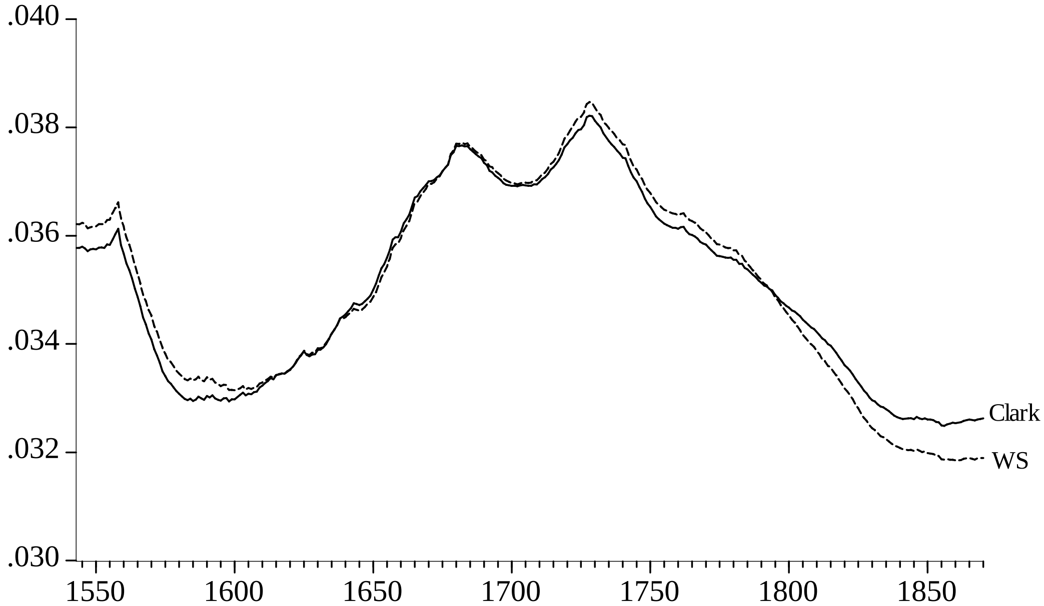


(b) Rate of technological progress

Figure 13 Estimates of the demand for labour, $\hat{\alpha}_t$, and the rate of technological progress, $\hat{c}_t$, for 1540 – 1870.



(a) Fertility intercept



(b) Mortality intercept

Figure 14 Estimated fertility, n_t, and mortality, m_t intercepts, 1540 – 1870.

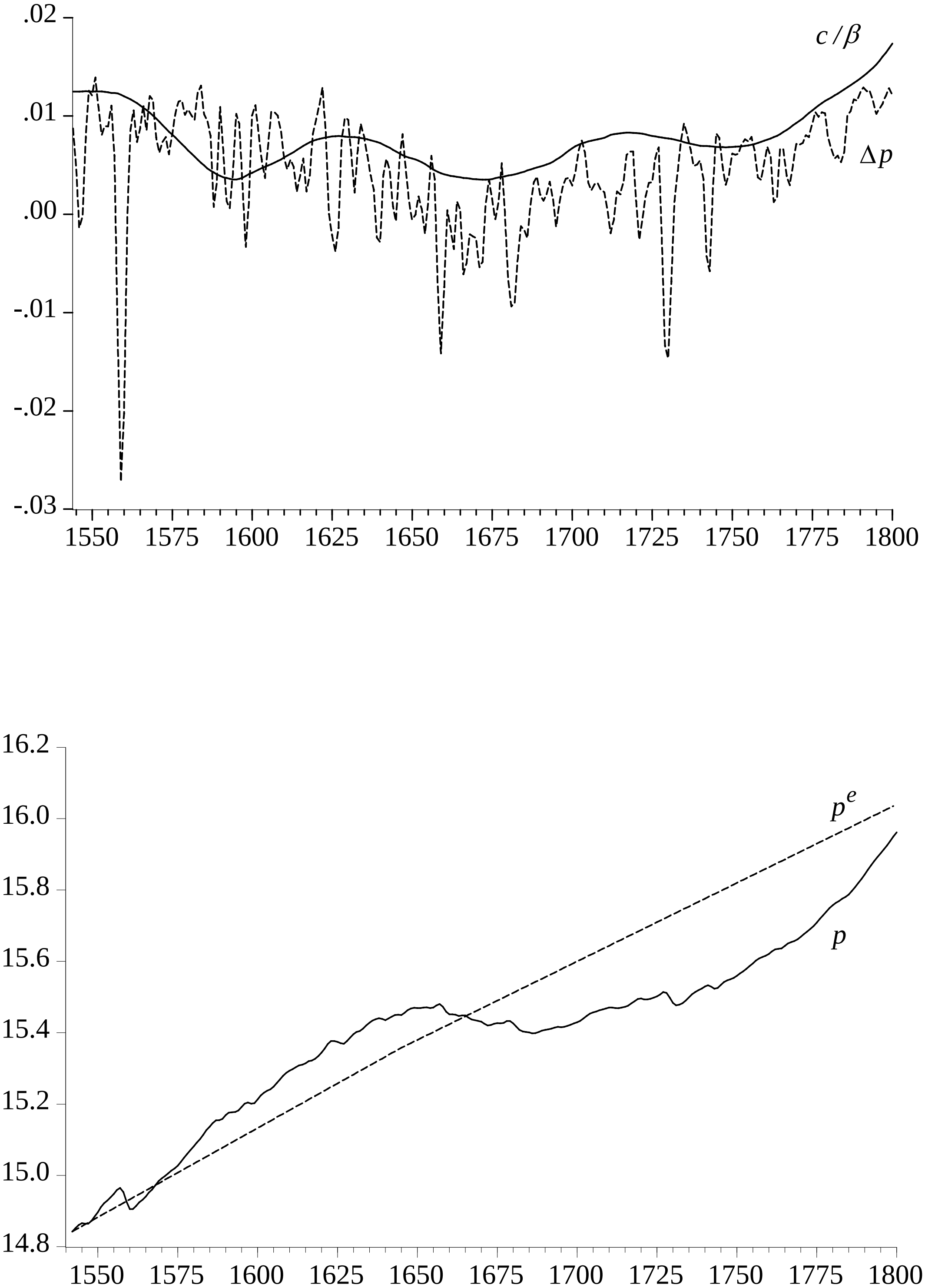


Figure 15 Top panel: Absorption rate, c_t/β, and population growth, Δp_t; Bottom panel: Actual, p_t, and equilibrium, p_t^e, population (logarithms), 1543 – 1800.