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Institutions, Investing, and Fighting: A Game-Free Result About the Odds of War and Other Costly Outside Options

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Institutions, Investing, and Fighting: A game-free result about the odds of war and other costly outside options*

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Work in Progress: Preliminary and Incomplete

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1 Introduction

A large class of problems can be understood using models of bargaining with a disagreement point. Countries that cannot reach a settlement can go to war. Firms that fail to reach an agreement can use the courts, bureaucracy, or a legislature to try and resolve the dispute. In each of these contexts the parties can influence (usually at a cost) their odds of success in a war, court trial, agency hearing, or lobbying battle. It is well known that the institution governing the interaction can influence the probability of disagreement as well as the likely settlements. When players posses asymmetric information about the disagreement point the particular form of this uncertainty can also influence the odds of bargaining failure and lotteries over settlements. In most contexts, however, the utility of disagreement depends, at least partially, on actions that the parties take prior to disagreement. Countries can invest in military capacity, firms can experience and document losses, collect evidence, or establish relationships with corrupt government officials. Accordingly, it makes sense to think about disagreement payoffs as endogenous– the result of investment decisions that are made prior to negotiating.

It is not surprising that expectations about the negotiating process can influence the investment decision. For example, Plott (1987) considers a model in which two parties invest in legal fees and then a contest function determines the trial winner. He shows that the choice of legal rules (in particular who pays the legal fees after a trial) can influence the quantity of legal expenses that the parties absorb and points out that the English rule may be inefficient. The possibility that institutions governing how conflicts are resolved can have systematic upstream effects on investment decisions suggests that an understanding of institutions, and especially institution design, requires that we take a step back and model how the choice of the institution might influence actions that influence disagreement payoffs. This paper uses the mechanism design approach, allows players to make investments that are hidden actions in anticipation of a bargaining game, and proves an equivalence result. We find the probability of bargaining failure conditional on a level of investment is the same in every institution. This result illustrates that in a certain sense variation in the institution is entirely offset by changes in equilibrium investment strategies.

A small number of papers have studied the decision to invest strategically before playing games that are similar to the bargaining games central to our interests. Segal and Whinston (2002) study hold-up problems in which two parties each invest prior to the play of a contracting game. The investments influence the valuations of the disagreement option. In Segal and Whinston, however, the investment are not hidden actions and so the connections to the current paper are limited. Plott (1987) and Coughlan and Plott (1997) are also similar, and treats legal fees as an investment in the probability of winning a law suit, but he dose not include the option of settling after the investments are made. Plott and Coughlan illustrate that the institutional choice (who pays legal fees) influences the investment decisions. Jackson and Morelli (2008) study investment and war fighting. Here, again the investment decisions are public and the relationship between institutions and investments cannot be studied as only one particular institution is analyzed. Meirowitz and Sartori (2008) consider models in which investment decisions are hidden actions and players bargain after making their investment decisions. They find a strong condition that is necessary and sufficient for the existence of equilibria in which disagreement/war is avoided. Moreover, the possibility of disagreement in equilibrium and the presence of investment strategies that involve randomizing are shown to be equivalent. That paper, however, is limited as it focuses only on unmediated bargaining games leaving open questions about the more general problem of designing institutions to influence arming, negotiating and warfighting. Moreover, Meirowitz and Sartori do not provide tight characterizations of equilibrium play, they instead focus on whether or not war can be avoided. As a consequence the paper does not tell us much about the relationship between institutional choice and equilibrium behavior.

Another literature that is relevant to this study involves the use of mechanism design to establish "game free" results. One of the most influential applications of this approach is Myerson and Satterthwaite (1983) in which it is shown that agreement cannot be guaranteed in problems of bilateral trade. In the study of negotiations and war fighting Banks (1990) shows that in problems with one sided assymetric information, the equilibrium settlements and probability of fighting must be monotone in the unobserved capactify of the privately informed nation in any equilibrium to any bargaining game. More recently, Fey and Ramsay (2008) consider problems in which both players posses private information and investigate when it is possible to construct institions possessing equilibria in which the probability of war is 0. In these papers a nation or both players has private information, but the realization of these types is exogenous. In this paper we take a step back and consider the mechanism design problem when players select their type, but these investments are privately observed.

The paper begins by defining the general bargaining problem and characterizing a coherent view of institutions in this setting. Our analysis proceeds by taking as given a fixed lottery over types and analyzes the induced problem of mechanism design with interdependent values. In an approach analogous to backward induction, we then use the results from this analysis to characterize equilibrium investment strategies and ultimately state and prove our main result.

2 The model

The first step is to define the set of situations for which our analysis applies. Consider the interaction between two players in anticipation of a negotiation. Each player, $i \in \{1, 2\}$ must first select a level of investment $a_i \in \mathbb{R}_+$ that will contribute to their disagreement payoff. In the case of international conflict, for example, this investment maybe spent on arms.[1] To allow investments to be in mixed strategies we write $F_i(\cdot)$ to capture the distribution function of a_i. We assume that the cost of investment a_i is given by $c_i(a_i)$ where $c_i(\cdot)$ is a strictly increasing and differentiable function. By $c_i'(a_i)$ we denote the first derivitive of the cost at a_i and by $c_i^{-1}(\cdot)$ we denote the inverse of the cost function. The investment choices are assumed to be hidden actions–player i knows its choice of a_i but it does not observe the choice by player $-i$. The players then negotiate over a resource or prize that is under dispute. Without loss of generality we assume the prize is of size 1. We will discuss how to think about the details of a negotiation process in the sequel, but the result of a succesful negotiation is a settlement (t_1, t_2). The payoffs t_i indicating what fraction of the prize goes to player i. Except when noted, we assume that there is no external subsidization and that players cannot be taxed, so that the settlements must be in the

[1] It is sometimes convenenient to refer to a player as i and to let $-i$ denote the other player.

unit simplex (each is nonnegative and they sum to atmost 1). Given a settlement t_1, t_2 and a pair of investments a_1, a_2 the payoffs are $t_1 - c_1(a_1)$ and $t_2 - c_2(a_2)$ respectively. If the players fail to reach an agreement they can invoke the costly outside option. If they do so, investment levels will influence the payoffs from disagreement. Rather than specifying a particular functional form for these payoffs, we just assume that the expected payoff to player i from a disagreement when its investment level is a_i, and the investment level of the other player is a_{-i}, is given by $p_i(a_i, a_{-i}) - c_i(a_i)$. We impose the natural assumption that $p_i(\cdot, \cdot)$ is strictly increasing in the first argument and strictly decreasing in the second argument. Throughout, we assume that these functions are twice continuously differentiable. To connect with the literature on inefficient bargaining failure, as in the case of war, we assume that $p_1(\mathbf{a}) + p_2(\mathbf{a}) < 1$ for all $\mathbf{a} \in \mathbb{R}_+$.

Our approach to the analysis proceeds without imposing or assuming any particular model of negotiation. We think of a negotiation procedure, protocol, game or "institution" as a sequence of interactions that must eventually either distribute settlements to players or result in the disagreement outcome. Important parts of this strategic environment will be considered exogenous and represent the game for that maps players action profiles into probability distributions over settlements and disagreement. We will then draw upon a key concept in the theory of mechanism design, *the revelation principle*. Our revelation result will allow us to discuss results in a vary large class of games or strategic interactions by focusing on a much smaller class of games called *direct revelation mechanisms*.

As a first order matter, a direct revelation mechanism–sometimes referred to as a direct mechanism–is an analytical device that allows us to characterize necessary consequences of Bayesian Nash equilibrium play in our strategic setting. Having discussed players utilities (payoffs) over outcomes and lotteries, the next important component of our analysis to to define a direct revelation mechanism.

A direct revelation mechanism is at its core a simple cheap talk game. This game is like any other, there exists an action space, where in a mechanism this is a set of messages M_i, and a mapping Γ taking profiles of player messages to probability distributions over outcomes. While, in general, mechanisms can consist of arbitrary message spaces and may or may not be "mediated", direct revelation mechanisms are simpler still. In a direct revelation mechanism M_i is simply the set of player i's types or possible hidden actions and all communication is to a disinterested mediator who can commit to producing specific probability distributions over outcomes given profiles of reports.

Formally we say a direct revelation mechanism is a pair of message spaces M_1, M_2 and a triple of mappings $t_1 : M_1 \times M_2 \to \mathbb{R}_+$, $t_2 : M_1 \times M_2 \to \mathbb{R}_+$, $q : M_1 \times M_2 \to [0, 1]$. So a direct revelation mechanism in our setting is then a cheap talk game where M_i is player i's possible hidden actions, q is the probability of disagreement, and t_i is i's report contingent transfer or payoff. We could assume that transfers are only positive when q is zero, but as t_i can depend on reports, our simpler description is without loss of generality. We also not that in our setting the outcome of a disagreement, i.e., its payoff to the players, depends on the level of investment that each player has made in the investment stage.

A few observations are worth making. We do not require, at the onset, that transfers satisfy budget balance. We do, however require that transfers are non-negative. This assumption can be viewed as one of limited liability. We focus on situations in which it is not possible to coerce a player to give up more than just its possible share of the stake. One reason for this assumption is that we are interested in circumstances where players participate voluntarily. For example, if an international institution could tax a state for expressing a claim on some territory then the state may be less willing to join the institution (or cede it the authority to adjudicate a claim). A second reason is that if the institution offered a negative settlment to player i then she would be is better off invoking the disagreement outcome than accepting the settlement.

3 Results

We begin with a fairly standard description of incentive compatible behavior in a direct mechanisms, treating the distribution functions $F_i(\cdot)$ as fixed. Let F_i be player i's mixed strategy equilibrium distribution over the hidden action. Recall our direct mechanism is a pair of functions $t_i(m_i, m_j) : R_+^2 \to [0, 1]$ that describes the report contingent transfer to i and a function $q(m_i, m_j) : R_+^2 \to [0, 1]$ that determines the probability of disagreement. Expected utility to i of making a report m_i in this direct mechanism, given investment a_i , can then be written as

$$U_i(m_i|a_i) = \int [t_i(m_i, m_j) + q(m_i, m_j)p(a_i, m_j)]dF_j(m_j).$$

It is convenient to define

$$T_i(m_i) = \int T_i(m_i, m_j)dF_j(m_j),$$
$$P_i(m_i|a_i) = \int q(m_i, m_j)p(a_i, m_j)dF_j(m_j),$$

where $T_i(m_i)$ is the expected transfers when player i reports m_i and $P_i(m_i|a_i)$ is the expected value of disagreement, including the chance it happens, conditional on a_i. This is only slightly more complicated than the expected value of disagreement times the expected probability, because the values and probabilities need not be independent. Thus we can describe a player's interim expected utility of a report m_i as

$$U_i(m_i|a_i) = T_i(m_i) + P_i(m_i|a_i)$$

In a slight abuse of notation, we let $U_i(a_i) = U_i(a_i|a_i)$.

Given this formal structure, we see this direct revelation mechanism framework is useful because it allows us to characterize incentives across game forms and various equilibria. To do this, we will invoke throughout our analysis the following revelation theorem.

Theorem 1 *If there exists a game with equilibrium investing decisions given by the mixtures F_1 and F_2 and the lottery $G(t_1, t_2, p_1, p_2)$ over transfers and disagreement payoffs, then there is a direct mechanism posessing an equilibrium in which investing strategies are given by F_1 and F_2 and the states report truthfully $m_i(a_i) = a_i$, which induces the same lottery over the outcomes.*

For fixed investment strategies the argument involves the standard composition strategy as found in Myerson (1979). Since investment decisions are privately observed and reports are unverifiable this first stage introduces no additional complications. Without loss of generality we will proceed by looking at equilibrium incentives in direct mechanisms and focus on Bayesian Nash equilibria to the induced games.

It is important to notice what a direct mechanism represents. The theorem states a triple $< t_1, t_2, q >$ represents *an equilibrium* to some game that induces a lottery over settlement payoffs and disagreement. This means two triples $< t_1, t_2, q >$ and $< t'_1, t'_2, q' >$ can represent equilibria from completely different game forms or two different equilibria selected from a given game form. Our results apply either way. The fact that these results are relevant for both constructions is important. This is because when we talk about triples as "institutions" we speak to two distinct visions of institutions in the literature. In the applied literature comparison of institutions is usually a statement about the equilibrium correspondence of two distinct extensive form games. For example, we may consider the difference of equilibrium payoffs and the probability of war across game forms where countries negotiate bilaterally and when they negotiate within a framework of a third party organization like the African Union. Similarly, in the law and economics literature one might compare litigation games under American and English rules for fees. In each case we can describe an equilibrium of each game form with its own mappings $< t_1, t_2, q >$.

There is also another view, more common in political economy, which takes institutions to be an equilibrium to some game or super-game, where there may exist opportunities for renegotiation or focal equilibrium selection–for example see Calvert (1995). Here, when comparing two different equilibria to such a game, we also have two different sets of mappings. That is, the method works equally well for comparing across equilibria of a single game and comparing equilibria across different games. It is prudent to be careful in deciding which interpretation one wishes to invoke and making sure that interpretations of results are consistent and coherent within a given framework.

We can now proceed with our results. Our first lemma is implied directly by incentive compatibility and is important for theorems that follow. From this lemma we learn that even though the probability of disagreement is a complicated and endogenous equilibrium property, the expected probability of disagreement is monotonically increasing.

Lemma 1 *Take any bargaining game with a disagreement point as described and as-*

sume $\mathbf{s}^*$ *is an equilibrium of this game. Then the expected probability of disagreement is increasing in the hidden information investments* a_i.

Proof. Suppose $\mathbf{s}^*$ is an equilibrium to such a game. By Theorem 1 there is a direct mechanism (t, q) such that players truthfully report and the mechanism induces the same lottery over outcomes. Consider some player i's strategy. In this direct mechanism incentive compatibility implies for any $a_i > a_i'$

$$T_i(a_i) + P_i(a_i|a_i) \geq T_i(a_i') + P_i(a_i'|a_i)$$

and

$$T_i(a_i') + P_i(a_i'|a_i') \geq T_i(a_i) + P_i(a_i|a_i')$$

Subtracting the inequalities we get

$$P_i(a_i|a_i) - P_i(a_i|a_i') \geq P_i(a_i'|a_i) - P_i(a_i'|a_i')$$

$$\int q(a_i, a_j)\left[p_i(a_i, a_j) - p(a_i', a_j)\right] dF_j(a_j) \geq \int q(a_i', a_j)\left[p_i(a_i, a_j) - p(a_i', a_j)\right] dF_j(a_j)$$

$$\int (q(a_i, a_j) - q(a_i', a_j))(p_i(a_i, a_j) - p(a_i', a_j))dF_j(a_j) \geq 0$$

By the Mean Value Theorem for integrals and the fact that p_i is increasing we have for some $\hat{a}_j$

$$(p_i(a_i, \hat{a}_j) - p(a_i', \hat{a}_j)) \int (q(a_i, a_j) - q(a_i', a_j))dF_j(a_j) \geq 0$$

$$\int q(a_i, a_j)dF_j(a_j) \geq \int q(a_i', a_j)dF_j(a_j)$$

proving the lemma. ■

This lemma tells us that in equilibrium, it must be the case that as players are investing more in bettering their disagreement payoffs, they must also be inducing higher probabilities of bargaining failure. This observation goes a long way toward helping characterize the incentives to invest in anticipation of bargaining games with the option of disagreement. Our second lemma shows that, in equilibrium, the probability of disagreement is well behaved.

Lemma 2 *Take any bargaining game with a disagreement point as described and assume* $\mathbf{s}^*$ *is an equilibrium of this game. Then* $\int q(a_i, a_j)dF_j(a_j)$ *is continuous in* a_i *almost everywhere.*

Proof. Suppose $\mathbf{s}^*$ is an equilibrium to such a game. By Theorem 1 there is a direct mechanism (t, q) such that players truthfully report and the mechanism induces the same lottery over outcomes. Let $\bar{q}(a_i) = \int q(a_i, a_j)dF_j(a_j)$. From Lemma 1 we know $\bar{q}(a_i)$ is monotone and because it is the expected probability of disagreement given a_i it has a bounded range. Clearly, a necessary condition for the range of $\bar{q}$ to be bounded is that the accumulation of the upward discontinuities must be finite. We now prove that this means the the number of discontinuities is countable. To see why, suppose not. That is, suppose that $\bar{q}$ is increasing, the range is bounded, but there are uncountably many jumps. By the fact that $\bar{q}$ is increasing we know that all jumps, Δq_j -the difference between the left and right limits of $\bar{q}(a_i)$- are positive. Let J be an index set such that we can label each jump. As J is uncountable, we can write the generalized sum of these jumps using

$$\sum_{j \in J} q_j = \sup\{\sum_{j \in A} |A \text{ is finite and } A \subset J\}.$$

That is, define the sum over the index set to be the supremum over all sums of finite subsets of J. This agrees with the usual definition of sum in the standard setting and generalizes to uncountable index sets.

Now let $A_0 = \{j \in J | q_j \geq 1\}$ and $A_n = \{j \in J | 1/n > q_j \geq 1/(n+1)\}$. By standard convergence results, if any of these A_n are infinite then the sum will diverge on that set, violating the bounded range of $\bar{q}$. If all the the sets are finite, then their union is a countable set, contradicting that the set of positive jumps is uncountable.

Therefore, there are countably many jumps in $\bar{q}$ and it is absolutely continuous. ■

Our next result characterizes the value of playing such a game as a function of the pre-play investment choices.

Theorem 2 *Take any bargaining game with a disagreement point as described and assume* $\mathbf{s}^*$ *is an equilibrium of this game. Then for players* $i = 1, 2$

1. *for almost every* a_i

$$U_i'(a_i) = \int \left[\frac{\partial p_i(a_i, a_j)}{\partial a_i}\right] q(a_i, a_j) dF_j(a_j), \tag{1}$$

and

2. *the value of the game (net of costs) is given by*

$$U_i(\hat{a}_i) = U_i(0) + \int_0^{\hat{a}} \int \left[\frac{\partial p_i(a_i, a_j)}{\partial a_i}\right] q(a_i, a_j) dF_j(a_j) da_i. \tag{2}$$

Proof. Suppose $\mathbf{s}^*$ is an equilibrium to such a game. By Theorem 1 there is a direct mechanism (t, q) such that players truthfully report and the mechanism induces the same lottery over outcomes. Consider some player i's strategy. In this direct mechanism incentive compatibility implies for any $a_i > a_i'$

$$T_i(a_i) + P_i(a_i|a_i) \geq T_i(a_i') + P_i(a_i'|a_i)$$
and
$$T_i(a_i') + P_i(a_i'|a_i') \geq T_i(a_i) + P_i(a_i|a_i')$$

Multiplying all terms by $\frac{1}{|a_i - a_i'|}$ and taking limits for a sequence as a_i' goes to a_i allows us to conclude that

$$\lim_{a_i - a_i' \to 0} \int \frac{[p_i(a_i, a_j) - p(a_i', a_j)]}{|a_i - a_i'|} q(a_i, a_j) dF_j(a_j) \geq \lim_{a_i - a_i' \to 0} \frac{U_i(a_i) - U_i(a_i')}{|a_i - a_i'|}$$
$$\geq \lim_{a_i - a_i' \to 0} \int \frac{[p_i(a_i, a_j) - p(a_i', a_j)]}{|a_i - a_i'|} q(a_i', a_j) dF_j(a_j)$$

Since $p(\cdot, \cdot)$ is differnetiable and $q(a_i, a_j)[p_i(a_i, a_j) - p(a_i', a_j)]$ is bounded by an integrable function for all a_i, we obtain

$$\int \left[\frac{\partial p_i(a_i,a_j)}{\partial a_i}\right] \lim_{a_i - a_i' \to 0} q(a_i,a_j)dF_j(a_j)$$
$$\geq \lim_{a_i - a_i' \to 0} \frac{U_i(a_i) - U_i(a_i')}{|a_i - a_i'|}$$
$$\geq \int \left[\frac{\partial p_i(a_i,a_j)}{\partial a_i}\right] \lim_{a_i - a_i' \to 0} q(a_i',a_j)dF_j(a_j)$$

Now to get the equality we want from the taking of limits, we need to show that $\int \frac{\partial p}{\partial a_i}(a_i,a_j)q(a_i,a_j)dF_j(a_j)$ is a.e. continuous in equilibrium. Applying the Mean Value Theorem to this integral equation we get, for some $\tilde{a}_j$ the equation $\frac{\partial p}{\partial a_i}(a_i,\tilde{a}_j)\int q(a_i,a_j)dF_j(a_j) = \int \frac{\partial p}{\partial a_i}(a_i,a_j)q(a_i,a_j)dF_j(a_j)$ holds. By assumption $\frac{\partial p}{\partial a_i}(a_i,\tilde{a}_j)$ is continuous in a_i for all a_j and be Lemma 2, $\int q(a_i,a_j)dF_j(a_j)$ is almost everywhere continuous. Therefore, the product is continuous at each such point and for almost every a_i, we obtain from the limits the identity

$$U_i'(a_i) = \int \left[\frac{\partial p_i(a_i,a_j)}{\partial a_i}\right] q(a_i,a_j)dF_j(a_j)$$

Integrating up both sides, the expected value of the mechanism to a player with investment a_i (net of costs) is then

$$U_i(\hat{a}_i) = U_i(0) + \int_0^{\hat{a}} \int \left[\frac{\partial p_i(a_i,a_j)}{\partial a_i}\right] q(a_i,a_j)dF_j(a_j)da_i.$$

■

With these results from the analsys of the problem while treating the distribution over investment levels as fixed we can now focus on the study of what types of investment strategies are actually possible in an equilibrium. What we find is that the equilibrium conditions from strategic investment pin down a number of characteristics of equilibrium. We begin with another lemma.

Lemma 3 *In any equilibrium to any game, if a_i is in the support of i's mixed strategy then*

$$c_i'(a_i) = \int \frac{\partial p_i(a_i,a_j)}{\partial a_i} q(a_i,a_j)dF_j(a_j)$$

Proof. Suppose $\mathbf{s}^*$ is an equilibrium to such a game. By Theorem 1 there is a direct mechanism (t, q) such that players truthfully report and the mechanism induces the same lottery over outcomes. For any mixed strategy, take any levels of arming chosen with positive probability (or density), a_i and a_i', then equilibrium requires that $U_i(a_i) - U_i(a_i') = c_i(a_i) - c_i(a_i')$. So for any two points in i's support we have, by methods like those above,

$$\int \frac{\partial p_i(a_i, a_j)}{\partial a_i} \lim_{\{a_i - a_i'\} \to 0} q(a_i, a_j) dF_j(a_j) \geq c_i'(a_i) \geq \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} \lim_{\{a_i - a_i'\} \to 0} q(a_i', a_j) dF_j(a_j).$$

This last conclusion implies that if $\int q(a_i, a_j) F(a_j)$ is continuous at a_i then

$$c_i'(a_i) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF_j(a_j)$$

We cannot conclude that $c_i'(a_i) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF(a_j)$ at every value of a_i because the right-hand side is known only to be a.e. continuous. Hence, it has at most countable discontinuities. To show that the equation holds for all a_i in the support of i's strategy, suppose not. That is, suppose $c_i'(a_i) < \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF(a_j)$. Then for a small enough value of $\varepsilon > 0$ a deviation to $a_i + \varepsilon$ will result in a larger increase $U_i(a_i + \varepsilon) - U_i(a_i)$ than the change in cost (since c_i is differentiable and thus continuous). Accordingly at any a_i in the support of an equilibrium strategy it must be the case that $c_i'(a_i) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF_j(a_j)$ and so this expression also has to be upper semi-continuous at any point in an equilibrium support. A similar argument holds for downward discontinuities, showing again upper semi-continuity. ■

This equilibrium condition illustrates that incentive compatibility and the indifference condition for investing require that the equilibrium investment strategies are closely related to $q(\cdot, \cdot)$ and the primitives, c and $p_i(\cdot, \cdot)$. An even starker representation is possible, however. Consider two distinct functions $q(\cdot, \cdot)$ and $q'(\cdot, \cdot)$ that are both attained in equilibrium with mixed strategies $F_j(\cdot)$ and $F_j'(\cdot)$ respectively. We then have

Theorem 3 *Fix the cost functions and disagreement payoff functions and consider two different equilibria to some incentive compatable mappings $q(\cdot, \cdot)$ and $q'(\cdot, \cdot)$. If investment level a_i is in the support of an equilibrium investment stratgy for player* *i in an equilibrium under each of these equilibria, then* $\Pr(disagreement|a_i)$ *is the same in both equilibria.*

Proof. Suppose $\mathbf{s}^*$ and $\hat{\mathbf{s}}^*$ are such equilibria. By Theorem 1 there are direct mechanisms (t, q) and $(t'q')$ such that players truthfully report and the mechanism induces the same lottery over outcomes.

Holding fixed the cost and disagreement payoff function, we attain the following consequence of the equilibrium condition

$$\int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q'(a_i, a_j) dF_j'(a_j) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF_j(a_j).$$

Linearity of the intergral yields

$$\int \frac{\partial p_i(a_i, a_j)}{\partial a_i} \big(q'(a_i, a_j) dF_j'(a_j) - q(a_i, a_j) dF_j(a_j)\big) = 0$$

Using the Mean Value Theorem we know that there is some value a_j^* s.t.

$$\frac{\partial p_i(a_i, a_j^*)}{\partial a_i} \int \big(q'(a_i, a_j) dF_j'(a_j) - q(a_i, a_j) dF_j(a_j)\big) = 0$$

Since $\frac{\partial p_i(a_i, a_j^*)}{\partial a_i}$ is strictly positive for all values of a_j, we know that $\frac{\partial p_i(a_i, a_j^*)}{\partial a_i} > 0$. This and the previous equlity require that

$$\int \big(q'(a_i, a_j) dF_j'(a_j) - q(a_i, a_j) dF_j(a_j)\big) = 0.$$

We then conclude

$$\int q'(a_i, a_j) dF_j'(a_j) = \int q(a_i, a_j) dF_j(a_j),$$

proving the theorem. ■

We can, in fact go slightly farther and provide a sometimes useful characterization of the conditional probability of war. Recall that the equilibrium mixing condition requires that

$$c_i'(a_i) = \int \frac{\partial p_i(a_i, a_j)}{\partial a_i} q(a_i, a_j) dF_j(a_j)$$

Applying the Mean Value Theorem again implies

Corollary 4 *There exists some value a_j^{**} such that*

$$\Pr(disagreement \mid a_i) = \int q(a_i, a_j) dF_j(a_j) = \frac{c_i'(a_i)}{\frac{\partial p_i(a_i, a_j^{**})}{\partial a_i}}$$

at any investment level a_i in the support of an equilibrium investment strategy for some incentive compatable institution.

This result establishes that, as a function of i's investment, the probability of disagreement in any equilibrium is linear in the cost and decreasing in the marginal effect of investment on i's war payoff. While comparisons across games, or even across equilibria to the same game are complicated because the equlbrium mixtures (and supports might) vary, the result is quite powerful. In the spirit of a revenue equivalence theorem this result says that even though investment strategies vary, the overal probability of disagreement, conditional on a_i, is the same in every equilibrium to every game in which a_i is in i's support. To compare this with the well known revenue equivalence result, note that the Revenue Equivalence Theorem provides an equivalence on the function relating realizations of types into revenue given an assignment rule. But expected revenue in any equilibrium to any auction (when the theorem applies) requires integrating this function over the exogenous distribution. For our problem in which the lotteries over types are endogenous the result does not depend on the analogue to an allocation rule, for us it would be $q(\cdot, \cdot)$. Equilibrium investment decisions (lotteries) respond to incentive compatible q in a way to completely offset variation in the q function.

Additionally, we see the way that changing game forms or switching equilibria may affect the probability of disagreement. By iterated expectations, it is clear that only in those instances where the change of equilibrium changes investment strategies do we find that probability of disagreement changes. Interestingly this must be true for both players. That is, in an equilibrium if the change only influences one player's investment strategy that is not sufficient. This result is not about probabilities conditional on profiles of reports, but on individual reports.

4 Conclusion

In this paper we confront two problems that face the theoretical study of conflict. First, we take serious the idea that players choose their strengths and, to a great degree, make their choices with strategic foresight. In particular, decision-makers worry about how their choices of investment levels will effect their bargaining position in negotiations. Second, we are interested in both comparing results across bargaining institutions and procedures as well as highlighting common incentives that exist in a variety of bargaining situations. To make progress on this issue we pursue game free results, and consider how the interaction between the institutional form of a bargaining game and incentives to invest effect various equilibrium outcomes.

Even with minimal structure, we are able to learn a number of things about bargaining with "endogenous strength." We know from our utility theorem that the strategic arming incentives do not depend on the size of the settlement offers from the bargaining game and the probability of disagreement, given a level of investment, does not depend on the underlying institution. This is because any inducements toward settlement a bargaining institution might create, then create incentives for players to "game the system" in a way that the conditional probability of war is unchanged. The other implication here, is that we can effect the probability of disagreement with institutions that effect decisions to invest in winning in the case of outside options. That is, our results do not imply that institutions are irrelevant. They suggest that if we ignore the effect institutions have on strategic investment, we may be missing the big effect institutions have on conflict. So while strategic investment decisions offset institutional effects on the probability of settlement downstream when institutions are given, institutions can effect investment and incentives to reach agreements.

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