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Coarse Matching with Incomplete Information

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Coarse Matching with Incomplete Information

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Abstract

We study two-sided markets with heterogeneous, privately informed agents who gain from being matched with better partners from the other side. Our main results quantify the relative attractiveness of a coarse matching scheme consisting of two classes of agents on each side, in terms of matching surplus (output), an intermediary's revenue, and the agents' welfare (defined by the total surplus minus payments to the intermediary). Following Chao and Wilson (1987) and McAfee (2002), our philosophy is that, if the worst-case scenario under coarse matching is not too bad relative to what is achievable by more complex, finer schemes, a coarse matching scheme will turn out to be preferable once the various transaction costs associated with fine schemes are taken into account. Similarly, coarse matching schemes can be significantly better than completely random matching, requiring only a minimal amount of information.

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1 Introduction

Achieving allocative efficiency when faced with a large diversity of alternatives and preferences requires complex price and allocation schedules. The studies by Chao and Wilson (1987), Wilson (1989) and McAfee (2002) have, however, uncovered instances in which rather simple schemes suffice to obtain most of the efficiency gains from the optimal schemes. Chao and Wilson (1987) and Wilson (1989) consider the case of a monopolist seller faced with a continuum of customers with different valuations for quality and a continuum of feasible qualities. They show that the efficiency loss due to the usage of a limited number of quality classes, n, converges to zero at a rate proportional to $1/n^2$, as n tends to infinity. This elegant result is, however, not very informative about the relative performance of schemes involving a small number of classes.

McAfee (2002) addresses this crucial issue in a matching model with a continuum of types on both sides. In his model, efficiency requires assortative matching, pairing the best agent from one side with the best agent from the other side, the second-best with the second-best, and so forth, i.e., the finest possible matching. This matching is contrasted with a scheme involving only two classes, high and low: high types pay common price for being randomly matched within the high class; low types get randomly matched within the low class. McAfee's remarkable result is that, for a broad class of distribution of types, the two-class scheme achieves *at least* as much surplus as the average of efficient assortative matching and completely random matching. In other words, the surplus loss associated with this very coarse scheme may in fact be rather modest.

The purpose of this paper is to extend the idea of McAfee (2002) to incomplete information settings. If the agents' types are private information, intermediated exchange consisting of price and matching schedules may induce agents to reveal their private information or parts of it, thus allowing the implementation of allocations that satisfy some optimality criterion with respect to the underlying preferences. Thus, even if coarse matching schemes may perform relatively well in terms of total surplus, it does not follow that it is similarly attractive to an designer who is not always committed to efficiency but rather seeks to maximize revenue and/or the agents' welfare (defined by the total surplus minus payments to the intermediary).[1] What is needed is an evaluation of the relative performance of coarse matching in terms of the revenue obtainable by an intermediary as well as the agents' net welfare. This is precisely the task of the present paper. The paper's

[1] Examples are employment and recruiting agencies, dating and marriage matchmakers, real estate brokers, technology and business brokers. Also, multi-product firms act as intermediaries by matching customers with heterogeneous tastes to products of different qualities. For other examples, see Spulber's (2005) recent survey of the literature on two-sided markets.

results will quantify, within worst-case scenarios, the revenue and net welfare gains for large classes of distributions of attributes - thus introducing the idea of coarse mechanism design.

In a nutshell, our philosophy is that, if the worst-case scenario is not too bad in terms of total surplus, revenue, and/or the agents' welfare relative to what is achievable by more complex, finer schemes, simple coarse matching scheme will turn out to be preferable once the transaction costs associated with fine schemes are taken into account. These transaction cost may take the form of communication, complexity (or menu), and evaluation costs for the intermediary (who needs more detailed information about the environment in order to implement a fine scheme) and for the agents (who need precise information about their own and others' attributes in order to optimally respond to a fine scheme), or higher production costs for firms offering different qualities. In addition, coarse matching schemes have the advantage that agents reveal only partial information about themselves, thus avoiding, at least to some extent, exploitation in the future. Finally, coarse matching schemes using a minimal amount of information may also generate significant welfare and revenue increases with respect to random matching.

McAfee's analysis of the surplus gains associated with coarse matching is based on Chebyshev's inequality. This approach is, however, not applicable to our analysis of total revenue and net welfare. New techniques are required and identified in our paper. Roughly speaking, we use several results from mathematical statistics that offer bounds on the variability and on the values of distribution functions at certain points in their range for large, non-parametric classes of functions. These bounds enable us to obtain lower bounds on the total revenue and net welfare obtainable through the two-class matching scheme for large classes of distributions of types. Moreover, applying these techniques to the analysis of total surplus also enables us to generalize the result obtained by McAfee.

The models of Chao and Wilson (1987) and Wilson (1989) assume privately informed customers, similar to the one-sided version of our model.[2] But in contrast to our analysis, the authors restrict attention to the efficiency effects of coarse allocation schemes. Accordingly, only those pricing schemes leading to full information revelation and maximum efficiency are characterized. In a model with a continuum of types and incomplete information on each side, Damiano and Li (2007) provide a characterization of incentive-compatible pricing schedules leading to coarse matching or, in the limit, to assortative matching. They employ the standard mechanism design approach in order to derive conditions under which the intermediary achieves a higher revenue by using the efficient scheme (assortative matching) rather than some other coarser scheme. The loss in terms

[2] Wilson's analysis is extended to the case of multi-unit demands in Spulber (1992).

of total surplus, revenue or/and net welfare associated with a limited number of classes is not addressed in their paper. Related is also Rayo (2003) who looks at a monopolist selling a menu of conspicuous items whose consumption is used as a signal about the agents' hidden characteristics (e.g., luxury goods). By restricting the variety of signals and forcing some subsets of consumers to pool (which corresponds to coarse matching), the monopolist can sometimes extract additional informational rents.[3]

The papers by Blumrosen and Feldman (2008) and Blumrosen and Holenstein (2008) who study mechanism design with restricted action spaces are closely related to ours. The results of Blumrosen and Feldman, however, focus on total surplus in a one-sided incomplete information setting, similar to Chao and Wilson (1987), Wilson (1989), and McAfee (2002). Comparing the optimal mechanism with unrestricted action space to mechanisms that employ only k actions, they find that the latter incurs an efficiency loss of $O\left(1/k^2\right)$ in their setting. Furthermore, they show that under certain conditions allocative efficiency may be achieved in environments with two players and two alternatives. For single-item auctions, Blumrosen and Holenstein compare the optimal auction to that of posted-price auctions in terms of the seller's revenue. It is shown that posted-price auctions with discriminatory (i.e., personalized) prices can be asymptotically equivalent to optimal full revelation auctions as the number of bidders increases.

The present paper is organized as follows: In Section 2 we describe the two-sided matching model with heterogeneous and privately informed agents on both sides of the market. As in McAfee (and most of the related matching literature), we assume that the value of a match is the product of the types of the agents (or the product of the agent's type and the good's quality in one-sided models). We further assume that matched agents share the surplus equally. Our analysis easily extends to other fixed sharing rules, as we illustrate in Section 5 for a one-sided model. In Section 3, we consider the incentive compatible price schedules that lead to assortative matching and coarse matching (Subsections 3.1. and 3.2, respectively). For assortative matching, we also establish a connection between those schedules and the unique stable payoff vector in the core of the market.

Section 4 contains our main results. Subsection 4.1 presents several definitions and results from mathematical statistics which we subsequently apply in our analysis of the relative performance of coarse matching. In Subsection 4.2, we first reconsider McAfee's surplus analysis. Using our new tools, we find that the surplus loss from coarse matching is less than one-quarter of the total surplus from assortative matching for the class of distributions considered by McAfee. Furthermore,

[3] This is related to the so-called "ironing conditions" emphasized by Mussa and Rosen (1978) and Maskin and Riley (1984).

we are able to extend McAfee's result to other classes of distributions. Our findings suggest that coarse matching performs relatively well in terms of total surplus if the distribution functions are logconcave and have a low coefficient of covariation - this offers some intuition for the rather "magic" analysis performed by McAfee.

Subsection 4.3 analyzes the effects of coarse matching on the intermediary's revenue. We establish a counterpart to McAfee's surplus result, which requires, however, more restrictive conditions. Roughly speaking, these conditions ensure that the expected utility of agents in the upper class from being randomly matched with agents in the lower class is sufficiently small. As a consequence, agents in the upper class have a relatively high willingness to pay for being matched with partners in the upper class. For the two-class scheme, we also characterize the circumstances under which one side of the market pays more to the intermediary than the other. This question is intensely discussed in the literature on two-sided markets, which tends to focus on random matching among homogenous agents and on network externalities.[4] By contrast, our explanation is based on heterogeneity differentials.

In Subsection 4.4 we show that the two-class scheme may perform surprisingly well in terms of the agents' welfare when compared to both assortative and random matching. In particular, we identify conditions under which the agents' welfare under coarse matching even exceeds the one under assortative matching.[5]

In Section 5 we illustrate how the previous results can be applied to a model of price discrimination where a monopolist produces several levels of quality for a market with heterogeneous customers. Section 6 concludes. The Appendix contains several results about the concepts defined in Subsection 4.1 and most of our proofs.

2 The model

There are two groups of agents, "men" and "women". Each man is characterized by an attribute x, each woman by an attribute y. Agent i's attribute is private information to i. Attributes are distributed according to distributions F (men) and G (women) over the intervals $[0, \tau_F]$ and $[0, \tau_G]$, $\tau_F, \tau_G \leq \infty$, respectively. The distributions F and G are atomless and have continuous densities, $f > 0$ and $g > 0$, respectively. We assume here that the two groups have the same measure,

[4] See, e.g., Armstrong (2006) and Rochet and Tirole (2006) and the literature cited in these papers.

[5] This is related to a result by Hoppe, Moldovanu and Sela (2006) in a framework where partners use wasteful signals in order to match. They describe circumstances where agents may be better off under random matching (without any waste) than under assortative matching which needs to be sustained by wasteful signaling.

normalized to be one.

An intermediary who cannot observe the agent's types offers contracts that are characterized by a matching rule ϕ, a set-valued function that maps any $x \in [0, \tau_F]$ to a subset $\phi(x) \subseteq [0, \tau_G]$, and by price schedules, $p_m : [0, \tau_F] \to \mathbb{R}$ for men, and $p_w : [0, \tau_G] \to \mathbb{R}$ for women. If man x and woman y are matched, they generate an output (or matching surplus). To capture production complementarities in the simplest way, we assume that the surplus function takes the form $u(x, y) = xy$. We also assume that surplus is shared according to a fixed rule that does not react to market conditions: specifically, if matched, man x receives αxy and woman y receives $(1 - \alpha) xy$, where $\alpha \in [0, 1]$.[6] Thus, the utility of a man with attribute x that is matched to a woman with attribute y after paying p_m to the intermediary is given by $\alpha xy - p_m$ (and similarly for women).[7]

For any $A \subseteq [0, \tau_F]$ let $\nu_m(A)$ the measure of men announcing types in A. Similarly, define $\nu_w(A)$ for women. The matching rule ϕ is *feasible* if for any $A \subseteq [0, \tau_F]$, $\nu_m(A) = \nu_w(\phi(A))$. That is, each subset of men is matched to a subset of women of equal measure. As shown by Damiano and Li (2007), a feasible and incentive-compatible matching rule partitions the sets of men and women, respectively, into n corresponding subsets, matches the elements of this partition in a positively assortative way, and then, within each matched partition, matches agents randomly.

In this paper, we restrict attention to three special matching rules. First, we consider the limit case where the number of classes n goes to infinity - this is called *assortative matching*. Second, we consider the case of $n = 2$, i.e., *coarse matching* with two classes. Finally, we also consider the case $n = 1$, which yields completely *random matching*.

Throughout the paper we assume that those agents who are not served by the intermediary will be randomly matched with each other.

3 Matching and incentive compatible price schedules

In this section we derive formulas for total surplus and for the intermediary's revenue obtained when incentive compatible price schedules are used.

[6] Below we compare this setting to the benchmark where agents share output in a way that guarantees payoffs in the core of the matching market.

[7] The utility function is a straightforward generalization of the standard one-sided independent private value model considered in the auction literature. From that literature it is well-known that results beyond the case of risk neutrality are seldom analytically tractable. An advantage of this formulation (which is also used in most of the related matching literature, e.g., Chao and Wilson, 1987; McAfee, 2002; Damiano and Li, 2007) is that all our results can be stated solely in terms of properties of the distribution functions.

3.1 Assortative matching

Under *assortative matching* each man x is matched with a woman $\psi(x)$, where $\psi : [0, \tau_F] \to [0, \tau_G]$ is implicitly defined by

$$F(x) = G(\psi(x)) \tag{1}$$

If the matching surplus function $u(x, y)$ is supermodular, assortative matching yields the efficient outcome in terms of total output.[8] Given a matching rule ϕ, the sharing of the surplus among matched agents is called *stable* if:

$$\forall x, \ \delta(x) + \rho(\phi(x)) = x\phi(x) \tag{2}$$

$$\forall x, y, \ \ \delta(x) + \rho(y) \geq xy \tag{3}$$

As efficiency is easily shown to be a prerequisite for stability, the only feasible rule for which the surplus can be shared in a stable way is assortative matching ψ. Let $\varphi = \psi^{-1}$. It is well-known, and straightforward to show that the unique stable shares are given by:

$$\delta(x) = \int_0^x \psi(z)\,dz \tag{4}$$

$$\rho(y) = \int_0^y \varphi(z)\,dz \tag{5}$$

In order to implement the assortative matching rule ψ (given a fixed sharing rule α among the matched partners), the intermediary's price schedules p_m and p_w need to satisfy the following incentive-compatibility constraints:

$$\alpha x\psi(x) - p_m(x) \geq \alpha x\psi(\hat{x}) - p_m(\hat{x}) \tag{6}$$

$$(1-\alpha)\,\psi^{-1}(y)\,y - p_w(y) \geq (1-\alpha)\,\psi^{-1}(\hat{y})\,y - p_w(\hat{y}) \tag{7}$$

for all $x, \hat{x} \in [0, \tau]$, and $y, \hat{y} \in [0, \tau]$, respectively.

The following result establishes the relation between stable shares and incentive-compatible price schedules (see the Appendix for the proof).

Proposition 1 *1. The incentive compatible price schedules for the fixed sharing rule α satisfy*

$$p_m(x) = \alpha\rho(\psi(x)) \tag{8}$$

$$p_w(y) = (1-\alpha)\,\delta(\varphi(y)), \tag{9}$$

[8] A matching surplus is $u(x, y)$ is supermodular if $u_1 > 0, u_2 > 0$, and $u_{12} > 0$ where u_i, $i = 1, 2$ denotes the derivative with respect to the i'th coordinate, and u_{12} denotes the mixed derivative. It is not difficult to generalize our results about assortative matching (Proposition 1) to the case of a general supermodular matching surplus functions $u(x, y)$.

Consequently, the net utilities of any agents x *and* y *are* $\alpha\delta(x)$ *and* $(1-\alpha)\rho(y)$, *respectively.*

2. *The intermediary's revenue from each matched pair satisfies:*

$$\min(\alpha, 1-\alpha)\, x\psi(x) \leq p_m(x) + p_w(\psi(x)) \leq \max(\alpha, 1-\alpha)\, x\psi(x)$$

3. *In particular, the revenue from each matched pair is exactly half the matching surplus if* $\alpha = 1/2$.

The above result shows that the incentive compatible price schedules effectively correct for the induced distortion of incentives caused by the fact that the fixed sharing rule α does not respond to outside options: the net utilities of the agents form a stable sharing of the output that is left after payments to the intermediary were made.

Total surplus and the intermediary's revenue from price schedules (8) and (9) are, respectively, given by:

$$U^a = \int_0^{\tau_F} x\psi(x)\, dF(x) \tag{10}$$

$$R_\alpha^a = \alpha \int_0^{\tau_F} \psi(x) \left[x - \frac{1-F(x)}{f(x)}\right] dF(x) + (1-\alpha) \int_0^{\tau_F} x \left[\psi(x) - \psi'(x)\frac{1-F(x)}{f(x)}\right] dF(x) \tag{11}$$

Until the application of Section 5, we will henceforth assume that surplus is shared equally, i.e., $\alpha = 1/2$, and we will therefore omit this index.

3.2 Coarse matching

We now turn our attention to a matching scheme that involves only two categories on each side of the market. Under coarse matching, the intermediary sets only two prices: p_m^c, p_w^c. Men that are willing (not willing) to pay p_m^c are randomly matched with women that are willing (not willing) to pay p_w^c. Our setting thus captures situations where agents who are not willing to pay still have the possibility to match randomly with each other. The case where excluded agents remain unmatched is discussed below in the context of a multiproduct monopolist (see Section 5).

Denote by $\hat{x}$ ($\hat{y}$) the lowest type of men (women) who is willing to pay p_m^c (p_w^c). By the assumptions on the match surplus and utility functions, these types are well defined. Such a pricing scheme is incentive compatible if and only if the following equations are satisfied:[9]

$$\alpha \int_0^{\hat{y}} \frac{\hat{x}y}{G(\hat{y})} dG(y) = \alpha \int_{\hat{y}}^{\tau_G} \frac{\hat{x}y}{1-G(\hat{y})} dG(y) - p_m^c \tag{12}$$

$$(1-\alpha) \int_0^{\hat{x}} \frac{x\hat{y}}{F(\hat{x})} dF(x) = (1-\alpha) \int_{\hat{x}}^{\tau_F} \frac{x\hat{y}}{1-F(\hat{x})} dF(x) - p_w^c \tag{13}$$

$$\hat{y} = \psi(\hat{x}) \tag{14}$$

The prices p_m^c p_w^c are such that the cutoff types $\hat{x}$ and $\psi(\hat{x})$ are indifferent between being randomly matched in the high class (while paying the price) and being randomly matched in the low class (for free). Note also that $\hat{y}$ needs to be $\hat{x}$'s partner in assortative matching in order to ensure feasibility.

In order to assess the relative performance of the coarse matching scheme, we derive, in the next section, lower bounds on the total surplus, revenue and net welfare obtainable through the two-class scheme. Hence, as in McAfee, it will suffice to consider only a crude form of coarse matching where the cutoff between the upper and lower classes is determined by the mean of one of the distributions. Clearly, the lower bounds for either criteria obtained in this paper remain lower bounds if the cutoffs were optimally chosen to maximize surplus, revenue or net welfare, respectively.

Let U^{EX} be the total surplus from coarse matching with two categories where the cutoff point $\hat{x} = EX$, the mean of F. Furthermore, let EX_L be the mean x-type of the low class, and EY_L the mean y-type of the low class when using the cutoffs $\hat{x} = EX$ and $\hat{y} = \psi(EX)$. We have:

$$EX_L = \frac{\int_0^{EX} xf(x)dx}{F(EX)} = EX - \frac{\int_0^{EX} F(x)dx}{F(EX)}$$

$$EY_L = \frac{\int_0^{\psi(EX)} yg(y)dy}{G(\psi(EX))} = \psi(EX) - \frac{\int_0^{\psi(EX)} G(x)dx}{G(\psi(EX))}$$

Using these definitions, we can write the total surplus as:

$$U^{EX} = \int_0^{EX} \int_0^{\psi(EX)} \frac{xy}{F(EX)} g(y) f(x)\, dydx + \int_{EX}^{\tau_F} \int_{\psi(EX)}^{\tau_G} \frac{xy}{1-F(EX)} g(y) f(x)\, dydx \tag{15}$$

$$= EXEY + \frac{F(EX)}{1-F(EX)} (EX - EX_L)(EY - EY_L) \tag{16}$$

Similarly, we denote the respective revenue obtained with cutoff $\hat{x} = EX$ by R^{EX}.

[9] See also Damiano and Li (2007).

For $\alpha = 1/2$, we obtain:

$$\begin{aligned} R^{EX} &= \frac{1}{2}EX\left[\int_{\psi(EX)}^{\tau_G} ydG(y) - \frac{1-G(\psi(EX))}{G(\psi(EX))}\int_0^{\psi(EX)} ydG(y)\right] \\ &+\frac{1}{2}\psi(EX)\left[\int_{EX}^{\tau_F} xdF(x) - \frac{1-F(EX)}{F(EX)}\int_0^{EX} xdF(x)\right] \\ &= \frac{1}{2}[EX(EY-EY_L)+\psi(EX)(EX-EX_L)] \end{aligned} \qquad (17)$$

4 The relative performance of coarse matching

This section contains our main results. How attractive is coarse matching relative to assortative and random matching? In the following, we establish worst-case scenarios from the point of view of total surplus, the intermediary's revenue, and the agents' welfare, while focusing on the case where output is shared equally among matched partners, i.e., $\alpha = 1/2$.[10]

4.1 Failure rates and covariance

We first introduce several definitions and results that will be used in our analysis below:

Definition 1 *1) A distribution function F is said to have an increasing failure rate - IFR (decreasing failure rate - DFR) if $f(t)/[1-F(t)]$ is increasing (decreasing) on $[0,\tau_F)$, $\tau_F \leq \infty$.*[11]

2) A distribution function F is said to have an increasing reversed failure rate - $IRFR$ (decreasing reversed failure rate - $DRFR$) if $f(t)/F(t)$ is increasing (decreasing) on $[0,\tau_F)$, $\tau_F \leq \infty$.

3) Let X, Y be non-negative random variables on $[0,\tau_F]$ and $[0,\tau_G]$, $\tau_F,\tau_G \leq \infty$, respectively. The coefficient of covariation of X and Y is given by[12]

$$CCV(X,Y)) \equiv \sqrt{Cov(X,Y)/EXEY}$$

In particular, note that $E(XY) = (1+CCV^2(X,Y))EXEY$ for any two non-negative random variables.

[10] In Section 5 we provide a discussion of the case of $\alpha = 1$ where one side receives the whole match surplus.

[11] For example, the exponential, uniform, normal, power (for $\alpha \geq 1$), Weibull (for $\alpha \geq 1$), gamma (for $\alpha \geq 1$) distributions are *IFR*. The exponential, Weibull (for $0 < \alpha \leq 1$), gamma (for $0 < \alpha \leq 1$) are *DFR*. See Barlow and Proschan (1975) who use the terminology in the context of reliability theory. Other authors refer to "hazard rates".

[12] By the integral form of Chebychevs's inequality (see Theorem 236 in Hardy et al., 1934), $Cov(X,h(X)) \geq 0$ for any increasing function h. Hence the coefficient of covariation is well defined for any X non-negative random variable and for any increasing function h. The coefficient of covariation reduces to the more common coefficient of variation $CV(X) \equiv \sqrt{Var(X)}/EX$ when $X = Y$. This is a dimensionless measure of variability relative to the mean.

Lemmas 3 and 4 in the Appendix establish various relations among the above defined concepts and exhibit several bounds on values of distributions with the above properties. Using these results, we obtain the following lemma, which provides the main working horse for our analysis. The lemma is proved in the Appendix.

Lemma 1

1. *$EX - EX_L \leq (\geq)\ \frac{1}{2}EX$ if F is convex (concave), and $\psi(EX) - EY_L \leq (\geq)\ \frac{1}{2}\psi(EX)$ if G is convex (concave).*
2. *$EX - EX_L \leq (\geq) \frac{EXe^{-1}}{F(EX)}$ if F is IFR (DFR).*
3. *$EY - EY_L \leq (\geq)\ \frac{1}{2}EY$ if G is convex (concave) and ψ is concave (convex).*
4. *$EY - EY_L \geq (\leq) EX - EX_L$ if ψ is convex (concave) and if $EX \geq (\leq) EY$.*

4.2 Total surplus - revisiting McAfee (2002)

In Chao and Wilson (1987) and Wilson (1989) it is shown that the total surplus loss due to the usage of coarse matching with n categories is of order $1/n^2$, that is, $U(n) \geq U^a - O\left(1/n^2\right)$, where $U(n)$ denotes the maximal total surplus from coarse matching with n classes. Of course, the order of magnitude in their analyses may still mean that the surplus from coarse matching with only a few categories is small relatively to that in assortative matching. The following elegant result is due to McAfee (2002):

Proposition 2 *(McAfee, 2002) Let the distributions F and G be both IFR and $DRFR$. Then*

$$U^{EX} \geq \frac{U^a + U^r}{2} \tag{18}$$

where U^r denotes the total surplus from random matching.

Thus, for a broad class of distributions, the scheme involving only two classes achieves at least as much as the average of assortative and random matching.

Using Lemma 3-(9) (Appendix), it is straightforward to establish a tighter, explicit link between the surplus in assortative matching and the surplus in the two-class scheme for the class of distributions considered by McAfee (see the Appendix for the proof):

Corollary 1 *Under the conditions of Proposition 2, we have*

$$U^{EX} \geq \frac{3}{4}U^a. \tag{19}$$

Applying the results presented in Subsection 4.1, we are also able to provide new insights for the class of DFR distribution functions about which McAfee's result is silent. We first derive lower bounds on the total surplus from two-class coarse matching, expressed as a mark-up on the random output:[13]

Proposition 3 *1) Let F be DFR, and let ψ be convex. Then*

$$U^{EX} \geq \frac{3}{2} U^r \tag{20}$$

2) Let F be DFR, let ψ be convex, and assume that $EX \geq EY$. Then

$$U^{EX} \geq \frac{e}{e-1} U^r \simeq 1.582 U^r \tag{21}$$

3) Let ψ be concave, and switch the role of F and G . Then the above result holds for U^{EY}.

Proof. 1) Since F is DFR and ψ is convex, G must be DFR and hence concave. Thus, since ψ is convex, we know by Lemma 1-(3) that

$$EY - EY_L \geq \frac{1}{2} EY$$

Furthermore, since F is DFR, we have by Lemma 1-(2) that

$$EX - EX_L \geq \frac{EX}{eF(EX)}$$

Combining the above insights, we get:

$$\begin{aligned} U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX - EX_L)(EY - EY_L) \\ &\geq EXEY + \frac{F(EX)}{(1-F(EX)} \frac{EX}{eF(EX)} \frac{EY}{2} \\ &= EXEY + \frac{EXEY}{2e(1-F(EX))} \\ &\geq EXEY + \frac{eEXEY}{2e} = \frac{3}{2} EXEY = \frac{3}{2} U^r \end{aligned}$$

[13] Since $U^r = U^a / \left[1 + CCV^2(X, \psi(X)\right]$, this bound can be easily translated into a bound involving the coefficient of covariation and the surplus in assortative matching U^a.

2) By Lemma 1-(4) we know that $EY - EY_L \geq EX - EX_L$. Thus, together with Lemma 1-(2), we obtain the following chain of inequalities:

$$\begin{aligned} U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX - EX_L)(EY - EY_L) \\ &\geq EXEY + \frac{F(EX)}{1-F(EX)}(\frac{EX}{eF(EX)})^2 \\ &= EXEY + \frac{EXEY}{e^2F(EX)(1-F(EX)} \\ &\geq EXEY + \frac{e^2EXEY}{(e-1)e^2} = \frac{e}{e-1}U^r \end{aligned}$$

where the last inequality follows by Lemma 4-(7).

3) This is obvious by the above calculations. ■

It is instructive to compare the above result with McAfee's proposition for those distributions where both results apply. Consider $F = G = 1 - e^{-t}$ which is IFR and DFR. In this case, $CCV(X, X) = 1 \Leftrightarrow U^a = 2U^r$, and $U^{EX} = \frac{e}{e-1}U^r$. Thus, our estimate in Part 2 is tight. Note also that $\frac{e}{e-1}U^r = \frac{1}{2}U^r + \frac{2e-1}{2e-2}U^r = \frac{1}{2}U^r + \frac{2e-1}{4e-4}U^a > \frac{1}{2}U^r + \frac{1}{2}U^a$. Thus, by continuity, we obtain a better estimate than McAfee's for IFR distributions that are not too convex with respect to the exponential.

Proposition 3 is now used to extend McAfee's result beyond a subclass of IFR distributions (see the Appendix for the proof).

Proposition 4 *Let F be DFR, ψ be convex, $1 \leq CCV^2(X, \psi(X)) \leq \frac{2}{e-1}$, and $EX \geq EY$.*[14] *Then*

$$U^{EX} \geq \frac{U^a + U^r}{2}$$

Let ψ be concave, and switch the role of F and G . Then the above result holds for U^{EY}.

The following example illustrates the result:

Example 1 *Assume that F and G are Weibull: $F(x) = 1 - e^{-x^{\frac{19}{20}}}$, $G(y) = 1 - e^{-y^{\frac{19}{20}}}$ on $[0, \infty)$. Thus, F and G are not IFR, but the conditions of Proposition 4 are satisfied. In fact, $U^{EX} \simeq 1.712$, $[U^a + U^r]/2 \simeq 1.628$, and thus $U^{EX} > [U^a + U^r]/2$.*

To understand the intuitive reason behind these results, note that $DRFR$ and hence DFR distributions are logconcave (see Lemma 3 in the Appendix). Similarly to concavity, this property provides an upper bound on the survivor function of the distribution (see Sengupta and Nanda,

[14] Alternatively, one could assume that F is DFR, ψ is convex, $1 \leq CV^2(X) \leq \frac{2}{e-1}$, and $EX = EY$.

1999), implying an upper bound on the mass of agents with types above the mean. If the mass of these high-type agents is relatively small, the loss from matching them randomly instead of assortatively is relatively small. This tends to make coarse matching attractive. Note though that the surplus from matching agents randomly approaches the surplus under assortative matching as the coefficient of covariation goes to zero. Therefore, for the two-class scheme to perform sufficiently well, the property of logconcavity has to be combined with specific bounds on the coefficient of variation. For example, logconcavity of the survivor function, which is equivalent to IFR, ensures that the coefficient of variation is less than 1. Roughly speaking, if the distribution is IFR, then properly normalized differences between expected values of attributes in successive quantiles are decreasing. This implies an upper bound on the surplus gains from matching agents assortatively instead of randomly.

4.3 The intermediary's revenue

If the intermediary's goal is to maximize revenue, the decision to employ a coarse matching scheme will depend on how well it performs relatively to other schemes. Taking into account the fact that a larger number of classes will usually be associated with higher transaction costs (or production costs - see the application in Section 5), the two-class scheme may be superior to infinitely many classes if the simpler scheme yields a sufficiently high fraction of the revenue achievable from the more complex one. The crucial question of how well the two-class scheme fares against assortative matching in terms of revenue is addressed in the next proposition. In order to prove our result, we use the tools presented in Subsection 4.1.

Proposition 5 *Let F and G be both IFR and concave. Then*

$$R^{EX} \geq \frac{1}{2}R^a \tag{22}$$

Proof. By Lemma 1-(1), we know that

$$\begin{aligned} EX - EX_L &\geq \frac{1}{2}EX, \text{ and} \\ EY_L &\leq \frac{1}{2}\psi(EX), \text{ and hence } EY - EY_L \geq EY - \frac{1}{2}\psi(EX) \end{aligned}$$

This yields the following inequality chain:

$$\begin{aligned}
R^{EX} &= \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] \\
&\geq \frac{1}{2}[EX(EY - \frac{1}{2}\psi(EX)) + \psi(EX)\frac{1}{2}EX] \\
&= \frac{1}{2}[EXEY - \frac{1}{2}EX\psi(EX) + \frac{1}{2}\psi(EX)EX] \\
&= \frac{1}{2}U^r = \frac{1}{2}\left(\frac{E(XY)}{1 + CCV^2(X, \psi(X))}\right) \\
&= \frac{1}{1 + CCV^2(X, \psi(X))}R^a
\end{aligned}$$

By Lemma 3-(9) we know that $CCV^2(X, \psi(X)) \leq 1$ if F and G are IFR, which yields the result as stated. ■

Proposition 5 identifies conditions under which the revenue from two-class coarse matching is *at least* half the revenue from assortative matching. The proposition thus constitutes the analog of McAfee's surplus result in terms of revenue i.e., $R^{EX} \geq \frac{1}{2}R^a \Leftrightarrow R^{EX} \geq [R^a + R^r]/2$, where the revenue from matching agents randomly is zero. Interestingly, we find that the revenue relation holds for a subclass of the class of distribution functions identified by McAfee (Proposition 2). That is, while the surplus relation requires $DRFR$ (logconcavity), concavity is needed here, which is stronger (cf. Lemma 3 in the Appendix):

Example 2 *Assume that* $F = x^3$, $G = y^3$ *on* $[0, 1]$. F, G *are then* IFR *and* $DRFR$ *(but not concave). In fact,* $R^{EX} < \frac{1}{2}R^a$.

The main message is that, for the class of distributions characterized in Proposition 5, coarse matching involving only two classes performs relatively well not only in terms of efficiency, but also in terms of revenue. Examples include the uniform distribution and the exponential distribution.

To understand why concavity plays an important role here, recall that payments are obtained only from agents in the high class. Their willingness to pay for being matched with a random partner from the high class on the other side of the market is determined by the value of the outside option, i.e. being randomly matched with a partner from the low class. As the distribution of types on the other market side becomes more concave, the mass of potential partners with very low types gets larger, reducing the value of the outside option. As a consequence, agents in the high class are willing to pay more, leading to a higher revenue. Recall, however, that the revenue from assortative matching is tightly linked to the assortative surplus (Proposition 1-(3)). Since the assortative surplus is linearly increasing in the coefficient of covariation, the property of concavity needs to be combined with a property ensuring that this coefficient is bounded from above. Note that the coefficient of variation is less than 1 for IFR distributions.

Which side pays more? In practice, price schedules observed in two-sided markets are often uneven, with one side paying more than the other.[15] Answers to the question of which side pays more tend to focus on network externalities.[16] A result in Hoppe, Moldovanu, Sela (2006) can be adapted to show that men's total payment is larger (smaller) than women's total payment if the intermediary uses assortative matching, and if the assortative matching function $\psi = G^{-1}F$ is convex (concave). In particular, agents are willing to pay more as the other side becomes more heterogeneous (since then the marginal gains in terms of winning a better matching partner are larger at the high end of the type range).

We inquire here whether this finding carries over to coarse matching. Let R_m^{EX} be the total payments obtained from men, and R_w^{EX} the total payments obtained from women under the two-class scheme.

Proposition 6 *1) Assume that either (a) F is convex and G concave, or (b) F is IFR and G is DFR. Then ψ is convex and $R_m^{EX} \geq R_w^{EX}$.*

2) Assume that $EX \geq EY$ and that ψ is convex. Then $R_m^{EX} \geq R_w^{EX}$.

Part 2 of Proposition 6 resembles the finding for assortative matching, but the intuition is slightly different here. Under coarse matching, the agents' willingness to pay is determined by the outside option of being randomly matched within the low class. If F and G have the same mean, but the distribution of women, G, has a higher variance, the chances for men who take the outside option to end up with a very low type partner are higher than for women. As a consequence, for types in the high class, men's willingness to pay is larger than women's.[17]

4.4 The agents' welfare

Another important criterion for measuring the relative performance of coarse matching is the agents' welfare, defined as total surplus minus the total payment to the intermediary. Let $W^a = U^a - R^a$ be the agents' welfare under assortative matching and $W^{EX} = U^{EX} - R^{EX}$ under coarse matching

[15] See, e.g., *The Economist*, "Matchmakers and trustbusters", p.84, Dec 10th, 2005.

[16] For an analysis of two-sided markets with network externalities, see, for instance, Rochet and Tirole (2003).

[17] In addition, there is a size effect (this is similar to assortative matching): if ψ is convex, the mass of men with types above a certain threshold is larger than the mass of women with types above that threshold.

with cutoff EX. Note that $W^r = U^r$ under random matching since the intermediary's revenue is zero.

The following two results exhibit the relations between the agents' welfare in these schemes. In order to prove the results, we need to find upper bounds on the revenue from coarse matching and combine them with lower bounds on the total surplus (see Appendix).

Proposition 7 *1) Let F and G be IFR and concave, and let ψ be convex. Then*

$$W^{EX} \geq \frac{3}{4} W^a \tag{23}$$

2) Let F and G be $DRFR$ and convex. Then

$$W^{EX} \geq W^a \tag{24}$$

The proposition indicates that coarse matching fares relatively well against assortative matching for subclasses of IFR distributions. Part 2 of Proposition 7 even identifies a class of distributions for which the agents' welfare exceeds the one obtainable under assortative matching! From our previous analysis, we know that for this class of distributions, coarse matching is relatively strong in terms of surplus (Proposition 2), but tends to be weak in terms of revenue (Proposition 5). Intuitively, the combination of these two features strengthens coarse matching from a net welfare point of view.

Example 3 *Assume that F, G are uniform on $[0, 1]$. Thus, the distributions are convex and $DRFR$. we get $W^{EX} = 3/16$, $W^a = 1/6$, and thus $W^{EX}/W^a = 9/8$.*

The next proposition shows that coarse matching, using a minimal amount of information, performs relatively well compared to random matching for the class of DFR distributions.

Proposition 8 *Let F and G be DFR and let ψ be convex. Then*

$$W^{EX} \geq \frac{3}{4} W^r \tag{25}$$

Recall that random matching generates zero revenue. In practice, many intermediaries must, however, respect a budget constraint. Our insights here together with those for revenue (Proposition 5) suggest that an intermediary who wishes to maximize the agents' welfare while collecting sufficient revenue to recover their costs of capital (analogous to the Ramsey-Boiteux problem in the theory of public finance) may find a coarse scheme to be quite attractive relative to both assortative and random matching.

5 An application to one-sided incomplete information models

Throughout the above analysis, we assumed that privately informed agents populate both market sides. Here we briefly illustrate how some of our previous insights can be modified for a context where there are privately informed agents only on one side who get matched to observable items (or partners) on the other side[18]. For instance, Wilson (1989) studies a multi-product seller in the electricity industry, seeking to match customers having heterogeneous valuations for power provision to different service qualities that represent different service probabilities. In this case, consumers naturally obtain the whole surplus from their purchase minus their payment to the seller. In other words the sharing rule corresponds to $\alpha = 1$, and we have to adjust our previous revenue and welfare results for this case. Total surplus is of course unaffected by the value of α.

For $\alpha = 1$, total revenue from assortative matching is given by

$$R_1^a = \int_0^{\tau_F} \psi(x) \left(x - \frac{1 - F(x)}{f(x)} \right) dF(x) \tag{26}$$

and the total revenue from coarse matching is given by

$$R_1^{EX} = EX(EY - EY_L) \tag{27}$$

Simple relations between revenues for $\alpha = 1$ and $\alpha = 1/2$ are:

Lemma 2 *If the assortative matching function ψ is convex (concave) then $R_1^a \geq (\leq)\ R_{1/2}^a$. If the assortative matching function ψ is convex (concave) and if $EX \geq (\leq)\ EY$ then $R_1^{EX} \geq (\leq)\ R_{1/2}^{EX}$.*

The next two results illustrate how our previous results can be adapted to this framework, and provide a comparison of the two matching rules in terms of revenue and agents' welfare:

Proposition 9 *Let F and G be IFR and concave, and ψ be convex. Then $R_1^{EX} \geq \frac{1}{4} R_1^a$.*

Proposition 10 *1) Let F and G be IFR and concave, and let ψ be convex. Then $W_1^{EX} \geq \frac{1}{2} W_1^a$. 2) Let F and G be convex and $DRFR$, and let ψ be convex. Then $W_1^{EX} \geq W_1^a$.*

In Wilson's model, the distributions of customers' valuations and the feasible distribution of service probabilities (and hence the assortative matching function ψ) are exogenously given.[19] Propositions 9 and 10 can therefore be directly used to identify lower bounds on the revenue and/or the agents' welfare associated with two service classes.

[18] The classical references are Mussa and Rosen (1978) , and Maskin and Riley (1984).

[19] See McAfee (2002) for a mapping of Wilson's set-up into the matching framework used here (albeit with complete information).

5.1 Price discrimination with quality costs

Here we briefly illustrate how Propositions 9 and 10 can be applied to the more realistic situation in which the monopolist takes production costs into account when determining the available qualities. In contrast to Wilson's model, the assortative matching function will be now endogenously determined.

We denote quality by q. There is a measure one of consumers, each demanding a unit of the good. Consumers are distributed over $[0,1]$ according to distribution F with $f = F' > 0$. The utility of type v from quality q is vq. The cost of producing y units of quality q is $c(q)\,y$ where $c(0) = c'(0) = 0$ and where $c'(q) > 0$ and $c''(q) > 0$ for $q > 0$.

Consider first a monopolist who uses standard non-linear pricing. In this case the monopolist chooses a menu of prices and qualities from which the consumers can pick their preferred combination. We make the standard assumption that the virtual valuation $v - (1 - F(v))/f(v)$ is increasing, which holds, for example, if F is IFR. Denote by $(p(v), q(v))$ the element that is chosen by type v. Using standard arguments, the monopolist's revenue and profit under incentive compatible price schedules and assortative matching are given by:

$$\begin{aligned} R_1^a &= \int_0^1 q(v)\left(v - \frac{1 - F(v)}{f(v)}\right) f(v)\,dv \\ \pi_1^a &= \int_0^1 \left[q(v)\left(v - \frac{1 - F(v)}{f(v)}\right) - c(q(v))\right] f(v)\,dv. \end{aligned}$$

Let r be such that $r - (1 - F(r))/f(r) = 0$. The function q that maximizes the above profit function is determined by

$$\begin{aligned} q(v) &= 0 \text{ if } v \leq r \\ c'(q(v)) &= v - \frac{1 - F(v)}{f(v)} \text{ if } v \geq r \end{aligned}$$

The profit maximizing menu contains a continuum of quality levels. Denote the distribution of quality levels by G, and note that $G(y) = F\left(q^{-1}(y)\right)$ for $y \in (0, q(1)]$. Thus, the function q plays here the role of the assortative matching function ψ.[20]

We first compare the revenue from the optimal menu derived above to the revenue that the monopolist can achieve by producing only two quality levels and charging two prices. Suppose

[20] Although $q^{-1}(0)$ is not defined here, q can be approximated by an everywhere strictly increasing function, allowing us to use previous results. In fact, since the monopolist does not get revenue from agents that are not served, the results from the approximation underestimate the ratio between revenue in coarse matching versus revenue in assortative matching, since only a fraction of the agents are assortatively matched with a positive quality.

that customers with valuations below $EV = \int_0^1 v dF(v)$ are matched with the quality level $Q^L = \int_0^{EV} q(z)\, dF(z) / F(EV)$ and customers with valuations above this cutoff are matched with the quality level $Q^H = \int_{EV}^1 q(z)\, dF(z) / (1 - F(EV)) = \frac{EQ - F(EV) Q^L}{1 - F(EV)}$.

Let p^L and p^H be the prices for low and high quality, respectively. By incentive compatibility these prices satisfy

$$p^H = \frac{EV \int_{EV}^1 q(x)\, dF(x)}{1 - F(EV)} - \frac{EV \int_0^{EV} q(x)\, dF(x)}{F(EV)} + p^L = EV \left(Q^H - Q^L\right) + p^L$$

The purpose of our analysis is to find a lower bound on the revenue associated with the two-quality scheme. For such a worst-case scenario, it suffices to consider the case of $p^L = 0$, since this case clearly underestimates the maximal revenue from coarse matching.[21] The revenue from the coarse provision of quality is then given by:

$$R_1^{EV} = EV \left(EQ - Q^L\right) \tag{28}$$

which is analogous to expression (27). Hence, Propositions 9 and 10 can be directly used to obtain revenue and welfare results for specific cost functions, as we illustrate in the following example.

Example 4 *Suppose that* $c(q) = q^2$ *and that* v *is uniformly distributed over* $[0, 1]$. *In this case,* $q(v) = 0$ *if* $v \leq \frac{1}{2}$ *and* $q(v) = \frac{2v-1}{2}$ *if* $v > \frac{1}{2}$. *Thus* $G(y) = \frac{1+2y}{2}$ *for* $y \in \left[0, \frac{1}{2}\right]$, *which is concave and* IFR. *Although* $G(0) > 0$, *we get* $CCV^2(X, \psi(X)) \leq 1$, *which is required for an application of Proposition 9. Straightforward algebra yields* $Q^H = \frac{1}{2}$, $R_1^a = \frac{1}{12}$, $R_1^{EV} \geq \frac{1}{16}$, *and thus* $R_1^{EV} \geq \frac{3}{4} R_1^a$. *Furthermore, considering total profit (that takes into account the production cost) we get* $\pi_1^a = \frac{1}{24}$, $\pi_1^{EV} \geq \frac{1}{32}$, *and* $\pi_1^{EV} \geq \frac{3}{4}\pi^a$. *Moreover, the conditions of Proposition 10-(1) are also satisfied, which yields* $W^{EV} \geq \frac{1}{2} W^a$.

Note that Proposition 9 deals only with revenue. What can be said about the profit comparison? Since costs are convex, the monopolist saves costs by producing the average quality levels Q^L and Q^H rather than producing the whole range of qualities. This does not immediately translate into a lower bound on profits from coarse provision of quality. Yet, if the cost function is sufficiently convex, the cost savings will be substantial and the bound on profits will match the bound on revenues (see the above example).

When the transaction costs of providing different qualities are also taken into account, our analysis suggests that the coarse provision of quality may well be the profit-maximizing choice of

[21] A price-discriminating monopolist can either set Q^L at zero or increase p^L to a positive value (thus excluding some customers) in order to increase revenue. Note that for maximizing the agents' net welfare, $p^L = 0$.

the monopolist for a broad range of customer type distributions and cost functions. Moreover, a public agency that is interested in the agents's welfare but has some revenue considerations (for budgetary reasons, etc.) will also have a strong incentive to offer coarse matching.

6 Conclusion

We have studied the performance of very coarse matching schemes and the associated price schedules in a two-sided market with heterogenous agents who are privately informed about their attributes. In a variety of settings we have shown that such schemes are very effective. The type of analysis performed in this paper is not standard in the Economics literature, which tends to concentrate on the zero-one dichotomy between optimality versus suboptimality: degrees of suboptimality are not quantified or compared. Several papers in mechanism design argue that only "simple" mechanisms are realistic.[22] But, a majority of these follow the above dichotomy, by completely specifying special settings where simple mechanisms are *fully* optimal.[23] Instead, we focus on a priori *suboptimal* mechanisms, while identifying settings where such mechanisms are very effective (and thus may become optimal once transaction costs associated with more complex mechanisms are taken into account).[24] The scarcity of "worst-case" studies (or of other quantifications of suboptimality) in the Economics literature should be contrasted to the wealth of papers following precisely this philosophy in the Operations Research/Computer Science literature.[25] We believe that both our understanding of existing trading institutions and our ability to design new effective institutions will profit from more studies in this vein.

[22] This is one argument in what is sometimes called the "Wilson doctrine" (see Wilson, 1987).

[23] E.g, Myerson (1981) shows that standard one-object auctions with a reserve price are revenue maximizing in the symmetric, independent private value environment. Holmstrom and Milgrom (1987) identify conditions where linear contracts are optimal for the provision of intertemporal incentives in a principal-agent relationship.

[24] Our analysis is similar in spirit to Neeman's (2003) study about the effectiveness of the English auction in environments where this auction is not revenue maximizing. See also Rogerson (2003) who describes an example where simple contracts achieve a substantial shares of the profit obtainable by full non-linear pricing in a cost-based procurement and regulation framework.

[25] Koutsoupias and Papadimitriou (1999) and Roughgarden and Tardos (2004) are excellent examples since they also combine game-theoretic reasoning. These authors study the "price of anarchy" in network routing games. This is defined as the ratio between the welfare in the worst Nash equilibrium and the welfare in the Pareto-optimal allocation.

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Appendix

The next lemma gathers several main relations among the concepts defined in Subsection 4.1. For the less obvious implications, see Barlow and Proschan (1975). We first need the following definition:

Definition 2 *A distribution function F is said to be new better than used in expectation - $NBUE$ (new worse than used in expectation - $NWUE$) if* $\int_t^{\tau}(1-F(x))dx \leq (\geq) EX(1-F(t)),\ t \geq 0$.

Lemma 3

1. *Any IFR distribution is $NBUE$.*
2. *A distribution F is IFR if and only if its survivor function $(1-F)$ is logconcave.*
3. *Any DFR distribution is $NWUE$.*
4. *Any convex distribution is IFR*
5. *Any DFR distribution is concave.*
6. *Any concave distribution is $DRFR$.*
7. *A distribution F is $DRFR$ if and only if F is logconcave.*
8. $CV^2(X) \leq (\geq)\ 1$ *if F, the distribution of X, is $NBUE$ $(NWUE)$.*
9. $CCV^2(X, G^{-1}F(X)) \leq (\geq)\ 1$ *if both F and G are IFR (DFR) distributions.*

Properties such as mentioned in the above lemma are of interest here since they describe large, non-parametric classes of distribution functions for which various upper or lower bounds on the values of distributions at various points in their respective range hold. Such bounds are exhibited in the next lemma:

Lemma 4

1. $F(EX)(1-F(EX)) \leq \frac{1}{4}$ *for all F.*
2. $F(EX) \leq (\geq) \frac{1}{2}$ if F *is convex (concave).*
3. $F(t) \leq (\geq) 1-e^{-\frac{t}{EX}}$ $, t < EX$ $(t \leq EX)$ if F *is IFR (DFR).*
4. $\int_0^{\tau}(1-F(x))dx = EX$ for all F
5. $\int_0^t F(x)dx \leq (\geq) t - EX(1-e^{-\frac{t}{EX}})$ *if F is $NBUE$ $(NWUE)$.*
6. $\int_0^{EX} F(x))dx \leq (\geq)\ EXe^{-1}$ if F *is $NBUE$ $(NWUE)$.*
7. $F(EX)(1-F(EX)) \leq \frac{e-1}{e^2} \approx 0.23254$ *if F is DFR.*

Proof. 1) This is immediate by observing that on the interval $[0,1]$ the function $x(1-x)$ has a maximum at $x=\frac{1}{2}$.

2) Note that $F(X)$ is a uniformly distributed random variable on the interval $[0,1]$, and hence $E[F(X)]=\frac{1}{2}$. The claim follows then by Jensen's inequality since $F(EX)\leq(\geq)\ E[F(X)]=\frac{1}{2}$ if F is convex (concave).

3) The assertions are contained in Theorems 4.4 and 4.7 in Barlow and Proschan (1965).

4) This is immediate from integration by parts.

5) This follows by 4) and by rearrangement of terms from the fact that $\int_t^{\tau}(1-F(x))dx\leq(\geq)\ EXe^{-\frac{t}{EX}}$ if F is $NBUE$ ($NWUE$) (see Barlow and Proschan (1975), page 187)

6) This is just the instance of 5) for $t=EX$.

7) By 3) for $t=EX$, we know that $F(EX)\geq 1-e^{-1}$ for F DFR. Since $1-e^{-1}\geq\frac{1}{2}$, we obtain that the function $x(1-x)$ is decreasing for $x\geq 1-e^{-1}$. Thus, $F(EX)(1-F(EX))\leq(1-e^{-1})(e^{-1})=\frac{e-1}{e^2}$. ■

Proof of Proposition 1. Using the envelope theorem and condition 6, we obtain

$$\alpha x\psi'(x)-\frac{dp_m(x)}{dx}=0.$$

Since the man with the lowest type will be matched for sure with the woman with the lowest type, the willingness to pay of this type is always zero, which yields the boundary condition $p_m(0)=0$. Hence, we obtain

$$p_m(x)=\int_0^x \alpha z\psi'(z)\,dz. \tag{29}$$

The incentive-compatible price schedule for women is analogously derived. Letting $\varphi=\psi^{-1}$, we have

$$p_w(y)=\int_0^y (1-\alpha)\,z\varphi'(z)\,dz. \tag{30}$$

To derive the net utilities of matched agents x and $\psi(x)$, note that

$$u(x,\psi(x))=\int_0^x \psi(z)\,dz+\int_0^x z\psi'(z)\,dz$$

Thus, the net utilities of agents x and $\psi(x)$ from contracting with the intermediary (and being matched with each other) can be written, respectively, as

$$\begin{aligned}\alpha x\psi(x)-p_m(x) &= \alpha\int_0^x \psi(z)\,dz=\alpha\delta(x)\\ (1-\alpha)\,x\psi(x)-p_w(\psi(x)) &= (1-\alpha)\int_0^y \varphi(z)\,dz=(1-\alpha)\rho(y)\end{aligned}$$

where $\delta(x)$, $\rho(y)$ are the stable shares, respectively. ■

Proof of Lemma 1. 1) Geometrically, the term $\left[\int_0^{EX} F(x)dx\right] / [EXF(EX)]$ is the ratio among the area below F up to the mean, and the area of the rectangle with sides of EX and $F(EX)$, respectively. Note that any chord to the graph of a continuous convex (concave) function lies entirely above (below) the graph. Thus $\int_0^{EX} F(x)dx$ covers less (more) than $\frac{1}{2}$ of the area of the rectangle when F is convex (concave). Observe that the equality $EX - EX_L = \frac{1}{2}EX$ is indeed obtained for the uniform distribution (on any interval), which is both convex and concave. By the same geometric argument, we get $\psi(EX) - EY_L = \left[\int_0^{\psi(EX)} G(x)dx\right] / [G(\psi(EX))] \leq (\geq)\frac{1}{2}\psi(EX)$ if G is convex (concave).

2) This follows immediately from Lemma 3-(1),(3) and Lemma 4-(6).

3) The result follows from by statement 1 and because $\psi(EX) \geq (\leq)E(\psi X) = EY$ if ψ is concave (convex) by Jensen's inequality.

4) Assume first that ψ is convex. If $F = G$, the result is obvious. Thus, assume $F \neq G$. By Theorem 6.2 in Barlow and Proschan (1981) $F(t) \leq G(t)$ for $t < EX$. In other words, the unique crossing of F and G (which must exists in this case) cannot occur below $t = EX$. Since $F \neq G$, we obtain $\psi(EX) < E(\psi X) = EY \leq EX$. Thus

$$F(t) \leq G(t) \text{ for } t \leq \psi(EX)$$

This yields the following chain:

$$\begin{aligned}
\frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} &\geq \frac{\int_0^{\psi(EX)} F(t)dt}{F(EX)} \Leftrightarrow \\
\frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} + \frac{\int_{\psi(EX)}^{EX} F(EX)dt}{G(\psi(EX))} &\geq \frac{\int_0^{\psi(EX)} F(t)dt}{F(EX)} + \frac{\int_{\psi(EX)}^{EX} F(EX)dt}{G(\psi(EX))} \Leftrightarrow \\
\frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} + \frac{\int_{\psi(EX)}^{EX} F(EX)dt}{G(\psi(EX))} &\geq \frac{\int_0^{\psi(EX)} F(t)dt}{F(EX)} + \frac{\int_{\psi(EX)}^{EX} F(t)dt}{G(\psi(EX))} \Leftrightarrow \\
EX - \psi(EX) + \frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} &\geq \frac{\int_0^{EX} F(t)dt}{F(EX)} \Leftrightarrow \\
EY - (\psi(EX) - \frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))}) &\geq \frac{\int_0^{EX} F(t)dt}{F(EX)} \Leftrightarrow \\
EY - EY_L &\geq EX - EX_L
\end{aligned}$$

The first inequality holds by the above observation and because $G(\psi(EX)) = F(EX)$. The third holds because F is increasing.

If ψ is concave, then ψ^{-1} is convex, and the argument holds with reversed roles. ■

Proof of Corollary 1. The result follows because $U^r = U^a / \left[1 + CCV^2(X, \psi(X)\right]$ and because $CCV^2(X, \psi(X)) \leq 1$ by Lemma 3-(9). ■

Proof of Proposition 4. Assume that F be DFR, ψ be convex, and that $EX \geq EY$. We need to show that $\left[U^{EX} - U^r\right] / \left[U^a - U^r\right] \geq \frac{1}{2}$. Note that $U^a - U^r = CCV^2(X, \psi(X))U^r$, and $U^{EX} - U^r \geq \frac{1}{e-1} U^r$ by Proposition 3-(2). Solving for $CCV^2(X, \psi(X))$ yields the result as stated. ■

Proof of Proposition 6. 1) a) By the concavity of G, we get that $R_m^{EX} = \frac{1}{2} EX\,(EY - EY_L) \geq \frac{1}{4} EXEY$. By the convexity of F and ψ, we get $R_w^{EX} = \frac{1}{2}\psi\,(EX)\,(EX - EX_L) \leq \frac{1}{2} EY\,(EX - EX_L) \leq \frac{1}{4} EXEY$. The result follows.

b) Because G is DFR and ψ is convex, using Lemma 4-(3), we obtain the following chain:

$$\begin{aligned} EY - EY_L &= EY - \psi\,(EX) + \frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi\,(EX))} \\ &\geq EY - \psi\,(EX) + \frac{\int_0^{\psi(EX)} (1 - e^{-\frac{t}{EY}})dt}{(G\psi\,(EX))} \\ &= EY - \psi\,(EX) + \frac{\psi\,(EX) - EY + EYe^{-\frac{\psi(EX)}{EY}}}{G(\psi\,(EX))} \\ &\geq \frac{(EY - \psi\,(EX))(G(\psi\,(EX)) + \psi\,(EX) - EY + EY(1 - G(\psi\,(EX)))}{G(\psi\,(EX)} \\ &= \frac{\psi\,(EX)\,(1 - G(\psi\,(EX))}{G(\psi\,(EX))} \end{aligned}$$

This yields the following chain:

$$\begin{aligned} R_m^{EX} &= \frac{1}{2} EX\,(EY - EY_L) \geq \frac{EX\psi\,(EX)\,(1 - G(\psi\,(EX))}{2G(\psi\,(EX))} \\ &= \frac{EX\psi\,(EX)\,(1 - F(EX)}{2F\,(EX))} \geq \frac{1}{2}\psi\,(EX)\,(EX - EX_L) = R_w^{EX} \end{aligned}$$

where the last inequality follows because F is IFR.

2) From Lemma 1-(4) we know that $EY - EY_L \geq EX - EX_L$. Since $EX = EY \geq \psi(EX)$, we obtain $R_m^{EX} = EX(EY - EY_L) \geq \psi(EX)(EX - EX_L) = R_w^{EX}$. ■

Proof of Proposition 7. To prove the proposition we first need to derive upper bounds on the revenue from coarse matching:

Lemma 5 *1) Let F be concave (convex), ψ be convex (concave) Then*

$$R^{EX} \leq (\geq) \frac{1}{2}(U^{EX} - EX_L EY_L) \tag{31}$$

2) Let F and G be convex. Then

$$R^{EX} < \frac{1}{2} U^{EX} \tag{32}$$

Proof. For the first part, consider the following chain that holds for ψ convex and F concave (the other direction is analogous):

$$\begin{aligned}
U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX - EX_L)(EY - EY_L) \\
&\geq EXEY + (EX - EX_L)(EY - EY_L) \\
&= 2EXEY - EXEY_L - EYEX_L + EX_L EY_L \\
&= 2[\frac{1}{2}(EX(EY - EY_L) + EY(EX - EX_L))] + EX_L EY_L \\
&\geq 2[\frac{1}{2}(EX(EY - EY_L) + \psi(EX)(EX - EX_L))] + EX_L EY_L \\
&= 2R^{EX} + EX_L EY_L
\end{aligned}$$

The first inequality follows from Lemma 4-(1) and the second inequality holds since $EY \geq \psi(EX)$ for ψ convex. The last equality uses formula (17).

For the second case where F, G are both convex, we use (17) to obtain:

$$\begin{aligned}
R^{EX} &= \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] \\
&\leq \frac{1}{2}[EX(EY - \frac{1}{2}\psi(EX)) + \psi(EX)\frac{1}{2}EX] \\
&= \frac{1}{2}[EXEY - \frac{1}{2}EX\psi(EX) + \frac{1}{2}\psi(EX)EX] \\
&= \frac{1}{2}U^r < \frac{1}{2}U^{EX}
\end{aligned}$$

where the first inequality follows from Lemma 1. ■

1) By Lemma 5-(1), we know that $R^{EX} < \frac{1}{2}U^{EX.}$. Hence,

$$\begin{aligned}
W^{EX} &= U^{EX} - R^{EX} > \frac{1}{2}U^{EX.} \geq \frac{1}{4}(U^a + U^r) \\
&\geq \frac{1}{4}(U^a + \frac{1}{2}U^a) = \frac{3}{8}U^a = \frac{3}{4}W^a
\end{aligned}$$

The first inequality follows from $R^{EX} < \frac{1}{2}U^{EX.}$. The second follows by McAfee's result (recall that any concave distribution is $DRFR$). The last equality follows by Proposition 1.

2) We know from the proof of Lemma 5 that $R^{EX} \leq \frac{1}{2}U^r$ for F and G convex. This gives:

$$W^{EX} = U^{EX} - R^{EX} \geq \frac{1}{2}(U^a + U^r) - \frac{1}{2}U^r = \frac{1}{2}U^a = W^a$$

where the first inequality follows from $R^{EX} \leq \frac{1}{2}U^r$ together with McAfee's result (recall that convex distributions are IFR), and the last equality follows by Proposition 1. ■

Proof of Proposition 8. We have the chain:

$$W^{EX} = U^{EX} - R^{EX} \geq U^{EX} - \frac{1}{2}(U^{EX} - EX_L EY_L) = \frac{1}{2}U^{EX} \geq \frac{3}{4}U^r = \frac{3}{4}W^r$$

The first inequality follows from Lemma 5-(1), and the second from Proposition 3-(1). ■

Proof of Lemma 2. The claim for $\alpha = 1$ follows by observing that $dR^a/d\alpha = \int_0^1 \left(x\psi'(x) - \psi(z)\right)(1 - F(z))\,dz > (<)\,0$ if ψ is convex (concave). The claim for $\alpha = 1/2$ follows by noting that $R_1^{EX} = EX(EY - EY_L) \geq \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] = R_{1/2}^{EX}$ follows by the same argument as the one used in the proof Proposition 6.

Proof of Proposition 9. By Lemma 1, we know that $EY - EY_L \geq \frac{1}{2}EY$ if G is concave and ψ is convex. This yields:

$$\begin{aligned}
R_1^{EX} &= EX(EY - EY_L) \\
&\geq \frac{1}{2}EXEY \\
&= \frac{1}{2}U^r = \frac{1}{2}\left(\frac{E(XY)}{1 + CCV^2(X, \psi(X))}\right) \\
&= \frac{1}{2}\frac{1}{1 + CCV^2(X, \psi(X))}U^a \\
&\geq \frac{1}{2}\frac{1}{1 + CCV^2(X, \psi(X))}R_1^a
\end{aligned}$$

The result follows then by noting that $CCV^2(X, \psi(X)) \leq 1$ if F and G are both IFR. ■

Proof of Proposition 10. **1)** We have the following chain:

$$\begin{aligned}
U^{EX} &= EXEY + \frac{F(EX)}{1 - F(EX)}(EX - EX_L)(EY - EY_L) \\
&\geq EXEY + (EX - EX_L)(EY - EY_L) \\
&= EXEY + R_1^{EX} - EX_L EY + EY_L EX_L \\
&= R_1^{EX} + EY(EX - EX_L) + EY_L EX_L \\
&\geq R_1^{EX} + \frac{1}{2}EXEY + EY_L EX_L \\
&= R_1^{EX} + \frac{1}{2}EXEY - \frac{1}{2}EXEY_L + \frac{1}{2}EXEY_L + EY_L EX_L \\
&= \frac{3}{2}R_1^{EX} + \frac{1}{2}EXEY_L + EY_L EX_L
\end{aligned}$$

The second line follows from Lemma 4-(2) since F is concave, the third line is due to $R_1^{EX} = EX(EY - EY_L)$, the fifth line follows from Lemma 1-(1). This implies that

$$\begin{aligned} W_1^{EX} &= U^{EX} - R_1^{EX} \geq \frac{1}{3}U^{EX.} \geq \frac{1}{6}(U^a + U^r) \\ &\geq \frac{1}{6}(U^a + \frac{1}{2}U^a) = \frac{1}{4}U^a \geq \frac{1}{2}W_{1/2}^a \end{aligned}$$

where the second inequality follows from McAfee's result (recall the concave distributions are $DRFR$), the third inequality is due to the fact that $U^r \geq \frac{1}{2}U^a$ if F and G are both IFR, and the last inequality is due to the fact that $U^a < 2R_1^a$ if ψ is convex.

2) If F, G are both convex, we use Lemma 1. to obtain:

$$R_1^{EX} = EX(EY - EY_L) \leq \frac{1}{2}U^r < \frac{1}{2}U^{EX}$$

This implies that

$$W_1^{EX} = U^{EX} - R_1^{EX} \geq \frac{1}{2}(U^a + U^r) - \frac{1}{2}U^r = \frac{1}{2}U^a \geq W_1^a$$

where the second inequality follows from McAfee's result since convex distributions are IFR. The last inequality follows from the same argument as the one at the end of part 1. ■