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Higher Vote Thresholds for Incumbents, Effort and Selection

Hans Gersbach (ETH Zurich)

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Hans Gersbach

CER-ETH - Center of Economic Research
at ETH Zurich and CEPR
Zürichbergstrasse 18
8092 Zurich, Switzerland
hgersbach@ethz.ch

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The election mechanism has difficulties in selecting the most able candidates and deselecting less able ones. In a simple model we show that the power of elections as a selection and incentive device can be improved by requiring higher vote thresholds than 50% for incumbents. A higher vote threshold makes it impossible for office-holders of low ability to pool with more able office-holders in order to be reelected. As a consequence, the average ability of reelected politicians and the average effort level tends to increase. The socially optimal threshold can be set by the public. Alternatively, one could allow candidates to compete with individual vote thresholds.

Keywords: elections, political contracts, vote-share thresholds, incumbents, selection, effort

JEL Classification: D7, D82, H4

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1 Introduction

Once in office, politicians in parliament enjoy an incumbency advantage; incumbent reelection rates top the 90 percent mark. One hypothesis is that the power of the election mechanism in selecting the most competent candidates for office is too limited.

To improve the selection of candidates in a democracy we suggest using higher vote thresholds for incumbents. Incumbents competing for reelection would then need to reach a vote-share threshold above one-half in order to be reelected. If the incumbent does not obtain enough votes to reach the vote-share threshold, either his challenger is elected, or a run-off ballot between two new candidates takes place.

We illustrate the workings of vote-share thresholds in a model in which a polity selects an office-holder from a pool of candidates with different but privately observed abilities. Office-holders undertake public projects whose output depends on his effort and his ability. Policy-makers also choose ideological policies encountering conflicting interests among voters. In such environments, elected candidates of low ability can try to mimic high-ability office-holders by exerting higher efforts in order to secure reelection. As a consequence, all office-holders or at least the vast majority may be reelected, as their expected ability is no worse than the expected ability of a challenger. Moreover, the efforts needed for low-ability office-holders to mimic high-ability ones may be low.

Imposing higher vote thresholds for incumbents essentially eliminates the worst possible equilibria and thus on balance improves the average ability of reelected incumbents and tends to improve expected efforts exerted by office-holders. This is socially desirable. As an illustration, consider the polar case where all policy-makers pool and produce the same output of public projects. Low-ability policy-makers exert more effort than high-ability ones to produce the same output, but the average level of effort tends to be low compared to equilibria in which low-ability policy-makers and high-ability ones choose different output levels and only the latter group is reelected. In such an equilibrium, in which all policy-makers pool, they will merely obtain 50% of the votes and get reelected. From the perspective of the voters, the expected abilities of the incumbent and of a challenger are the same. Imposing a higher vote threshold than 50% for incumbents essentially eliminates such bad equilibria. In order to reach higher vote shares, higher-ability office-holders must necessarily distinguish themselves from the low-ability ones. As a consequence, only those equilibria obtain which, on balance, yield higher welfare.

Technically, we characterize in this paper the set of equilibria with standard elections and those where higher vote thresholds for incumbents are imposed. We show how higher vote thresholds than 50% eliminate the worst equilibria and on balance improve welfare. We also establish the existence of a socially optimal vote threshold.

The paper is organized as follows: In the next section we look at the related literature. In section 3 we introduce the model. Section 4 discusses the benchmark case where there are only standard elections. In section 5 we investigate how higher thresholds for incumbents affect effort and ability of office-holders and overall welfare. In section 6 we discuss various extensions, applications, and generalizations of the model. Section 7 concludes.

2 Relation to the Literature

Our paper is closely related to the large body of literature dealing with the fact that incumbents are extraordinarily successful when they seek reelection.

Several explanations have been advanced for the existence of incumbency advantages. First, the incumbent may be perceived as a safer bet than his challengers (Bernhardt and Ingberman (1985), Anderson and Glomm (1992)). For example, the incumbent may have gained a communication advantage over his challengers. Second, incumbents may have, on average, higher qualities than challengers. Either candidates who have won in the past are of higher quality[1] or challengers may be deterred from running against them (Jacobson and Kernell (1983), Cox and Katz (1996), Stone, Maisel and Maestas (2004), and Gordon, Huber and Landa (2007)).

Third, incumbents try to increase their reelection prospects by the provision of constituency service (Cain, Ferejohn and Fiorina (1987)) or (socially) costly actions like higher government expenditures or wars (Rogoff and Sibert (1988), Alesina and Cukierman (1990), Hess and Orphanides (1995, 2001), and Cukierman and Tommasi (1998)).[2]

[1]See Samuelson (1984), Londregan and Romer (1993), Banks and Sundaram (1998), Zaller (1998), Ashworth (2005), and Diermeier, Keane, and Merlo (2005).

[2]Other explanations of the incumbency advantage are based on the incumbents' voting behavior and face-recognition (Ansolabehere, Snyder an Stewart (2000) and Prior (2006)). Finally, challengers may have less access to campaign funds (Gerber (1988)). Whether these explanations can themselves be explained by a quality-based incumbency advantage is addressed in Ashworth and Bueno de Mesquita (2007). Given the existence of major incumbency advantages, Buchler (2007) challenges the assumption that competitive elections are a priori socially desirable.

We first show that low-ability incumbents may be successful in seeking reelection as they can mimic politicians with higher ability. Then we show that the inadequacy of the reelection mechanism in selecting office-holders of high ability can be improved by requiring higher vote thresholds than 50% for incumbents. Higher vote thresholds also tend to improve effort and the output of public projects.

In a companion paper (Gersbach (2007)), it is shown that higher vote share thresholds, also called vote share contracts, are even socially desirable when incumbents first choose effort before they realize their ability and thus no signalling can occur. We focus in this paper on the problem when a policy-maker wants to signal high ability to get reelected.

3 The Model

3.1 Agents

We consider elections in a two-period model. At the beginning of each of two periods, $t = 1$ and $t = 2$, voters must elect a politician. The same two candidates compete for office on both election dates. Candidates are denoted by k or $k' \in \{R, L\}$. Candidate R (L) is the right-wing (left-wing) candidate. The ability of a candidate is a random variable a_k distributed uniformly on $[-A, A]$, $A > 0$. Nature draws a_k at the beginning of period 1, which is the private information of candidate k. There is a continuum of voters. Each individual voter is indexed by $i \in [0, 1]$.

3.2 Policies

There are two types of policy problems the policy-maker faces.

- Public Project: P
  In each period the office-holder can undertake a public project. The result is determined by the effort invested by the policy-maker and by his ability. The amount of this public project in period t is given as

$$g_t = \gamma(e_{kt} + a_k), \gamma > 0, \tag{1}$$

where e_{kt} represents the effort exerted by the policy-maker in period t and a_k represents his ability. Voters will observe g_t. The citizens derive utility from the public project in accordance with the instantaneous utility function $U^P(g_t) = g_t$.

- Ideological (or Redistribution) Policy: I
  In each period the policy-maker decides on an ideological policy I that affects voters differently. The choice set for I is represented by a one-dimensional policy space $[0, 1]$. We assume that voters are ordered according to their ideal points regarding I. Voter i has preferences about I according to the instantaneous utility function
  $$U_i^I(i_{kt}) = -(i_{kt} - i)^2, \tag{2}$$
  where i_{kt} is the platform chosen by the policy-maker and i is the ideal point of voter i.

Some remarks are in order here. The only advantage we assume the incumbent may have from being in office is that he may be able to signal his ability to voters by choosing a particular output g. In the extension we consider the case where an incumbent obtains further advantages from office.

3.3 Utilities

In this section we describe the utilities of voters and candidates. The discount factor of voters and politicians is denoted by β with $0 < \beta \leq 1$.

The expected utility of voter i evaluated at the beginning of $t = 1$ is given by the discounted sum of the benefits from the public project and from the ideological policy. The lifetime utility of voter i if candidate k is in office in period t is given by

$$V_i = g_1 + U_i^I(i_{k1}) + \beta[g_2 + U_i^I(i_{k2})]. \tag{3}$$

The candidates derive utility from two sources.

- *Office-holding*
  A policy-maker derives private benefits b from holding office, including monetary and non-monetary benefits such as power and enhanced career prospects. He incurs costs amounting to $C(e_{kt}) = ce_{kt}^2$ $(c > 0)$ from the exertion of effort.

- *Benefits from policies*
  We assume that candidate L is a left-wing candidate, i.e. his most preferred point, denoted by μ_L with regard to policy I, satisfies $\mu_L < \frac{1}{2}$. Similarly, candidate R is a right-wing candidate with an ideal point $\mu_R > \frac{1}{2}$. To simplify the exposition we assume that $\frac{1}{2} - \mu_L = \mu_R - \frac{1}{2}$. Hence the candidates' ideal points are symmetrically distributed around the median's ideal point $\frac{1}{2}$. Moreover, the candidates derive the same benefits from public projects as voters.

To describe the overall utility of politicians we have to distinguish four cases. For example, politician R's lifetime utility, denoted by V_R, can be computed as follows:

(i) If R is in office over both periods:

$$V_R = b - (i_{R1} - \mu_R)^2 - ce_{R1}^2 + g_1 + \beta[b - (i_{R2} - \mu_R)^2 - ce_{R2}^2 + g_2].$$

(ii) If R is in office in $t = 1$ only:

$$V_R = b - (i_{R1} - \mu_R)^2 - ce_{R1}^2 + g_1 + \beta[-(i_{L2} - \mu_R)^2 + g_2].$$

(iii) If R is in office in $t = 2$ only:

$$V_R = -(i_{L1} - \mu_R)^2 + g_1 + \beta[b - (i_{R2} - \mu_R)^2 - ce_{R2}^2 + g_2].$$

(iv) If R never is in office:

$$V_R = -(i_{L1} - \mu_R)^2 + g_1 + \beta[-(i_{L2} - \mu_R)^2 + g_2].$$

3.4 Parameter assumptions

We assume that b is not too small. More particularly, we require that $b > \gamma A$. This assumption ensures that candidates of low ability are willing to exert greater effort to increase their reelection chances. To simplify the exposition we assume $\beta = 1$. The extension to $\beta < 1$ is straightforward.

3.5 The overall game

We summarize the overall game in the following figure:

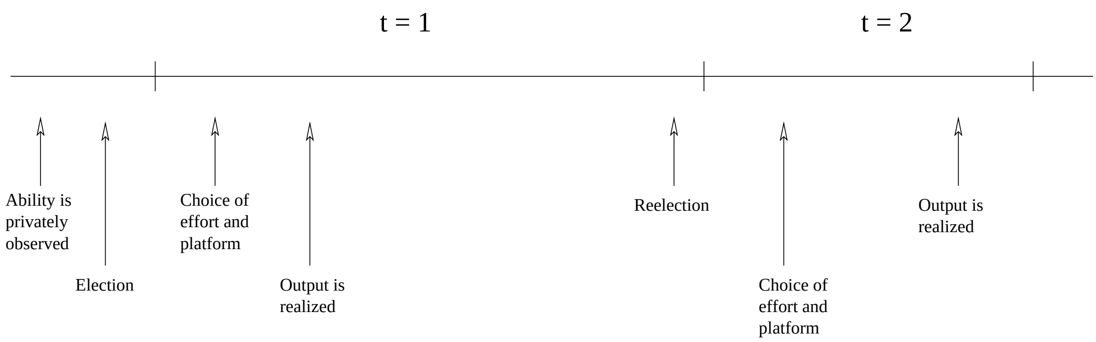

Figure 1: Time-line for standard elections

3.6 Assumptions and equilibrium concept

We assume that politicians cannot commit themselves to a policy platform. Voters observe the policy-maker's choice with regard to policy I. Moreover, we assume that voters observe output g_1 only and not its composition between effort and ability. Output g_1 is not contractable so it cannot be used to generate rewards for politicians beyond elections. Moreover, citizens are assumed to vote sincerely, i.e. they vote for the candidate from whom they expect higher utility.[3] To break ties we assume that voters reelect the incumbent if they are indifferent between him and the competitor. We are looking for perfect Bayesian Nash equilibria for the game under these assumptions.

[3]With a continuum of voters the individual voter has no influence on the outcome of an election. The optimality of sincere voting can be justified for a model variant with a large but finite number of voters or when the act of voting generates benefits.

4 Elections Alone

We first examine the standard case where elections are held. As a tie-breaking rule we assume that the probability of either candidate winning in the first period is 0.5 if they both have the same share of votes. In the second period the incumbent will be elected if he has 50% of the votes.

4.1 The second period

As candidates cannot commit to policy platforms, a policy-maker will choose his most preferred platform in the second period. In the Appendix we prove:

Proposition 1
Suppose candidate k is elected at date $t = 2$. Then

(i) he will choose $i_{k2} = \mu_k$ for policy I;

(ii) irrespective of whether k is in his first or second term, he will choose $e^*_{k2} = \frac{\gamma}{2c}$;

(iii) the expected utility of a policy maker at the beginning of period 2 is given by

$$V^*_{k2} = b + \frac{\gamma^2}{4c} + \gamma a_k$$

(iv) the expected utility of the politician $k' \neq k$ who has lost the second election is given by

$$V^D_{k'2} = \gamma\left(\frac{\gamma}{2c}\right) - (\mu_R - \mu_L)^2$$

4.2 The first period

We now look at the equilibria in the first period. As the candidates' ideal points are distributed symmetrically around the median voter's ideal point, the probability of either candidate winning is one half. Once in office, the candidate has to choose e_{k1} and i_{k1}. Without loss of generality we assume that candidate R has been elected. We first make a simple observation that will hold in every equilibrium with pure strategies.

Fact 1
Suppose candidate R is elected at date $t = 1$. Then he will choose $i_{R1} = \mu_R$.

This fact is obvious, as voters know that policy-makers will choose their bliss points in the last period. So politician R will not gain more votes in the second election by choosing a different platform than μ_R in period 1. We next derive the equilibrium effort choices made by the office-holder in the first period.

4.2.1 Semi-separating equilibria

We first look at equilibria that divide the pool of candidates into two groups. Such equilibria are called semi-separating. For this purpose a few preliminary steps are necessary. We first construct a separation of the pool of candidates into two groups as follows: The first group with ability equal to or higher than some critical threshold a^{cut}, $a^{cut} \in (-A, A)$, expect to be reelected with probability 1. A second group with ability smaller than a^{cut} expects to be deselected with probability 1. An office-holder with $a = a^{cut}$ is indifferent between being part of the first or the second group.

We next examine the conditions for the indifference of an office-holder with $a = a^{cut}$. Without loss of generality we assume that k is a right-wing politician. If he does not expect to be reelected, his utility is given by

$$V_R^{rejection} = b + \gamma(e_{R1} + a^{cut}) - ce_{R1}^2 + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2. \tag{4}$$

Given this expectation, the optimal choice of e_{R1} is given as $e_{R1} = \frac{\gamma}{2c}$, which yields

$$V_R^{rejection} = b + \frac{3\gamma^2}{4c} + \gamma a^{cut} - (\mu_R - \mu_L)^2. \tag{5}$$

If he expects to be reelected, his utility is

$$V_R^{reelection} = b + \gamma(e_{R1} + a^{cut}) - ce_{R1}^2 + b + \frac{\gamma^2}{4c} + \gamma a^{cut}. \tag{6}$$

The office-holder is indifferent between rejection and reelection if $V_R^{rejection} = V_R^{reelection}$, which yields

$$ce_{R1}^2 - \gamma e_{R1} + \frac{\gamma^2}{2c} - b - \gamma a^{cut} - (\mu_R - \mu_L)^2 = 0. \tag{7}$$

The solutions of this quadratic equation are given by

$$e_{R1} = \frac{\gamma \pm \sqrt{4c[b + \gamma a^{cut} + (\mu_R - \mu_L)^2] - \gamma^2}}{2c}. \tag{8}$$

The effort choice of an office-holder who will be rejected equals $\frac{\gamma}{2c}$. An incumbent who will be reelected will not choose a lower effort level than an incumbent who will be rejected. Hence the only viable solution is

$$\hat{e}_{R1} = \frac{1}{2c}\{\gamma + \sqrt{4c[b + \gamma a^{cut} + (\mu_R - \mu_L)^2] - \gamma^2}\}. \tag{9}$$

After these preparations we can now characterize the set of semi-separating equilibria. For that purpose we use $E_a[g]$ to denote the beliefs of voters regarding the expected ability of an office-holder if he produces output g.

Proposition 2

There exists a continuum of semi-separating equilibria parameterized by $a^{cut} \in (-A, +A)$. An equilibrium associated with a^{cut} is characterized as follows:

(i) Policy-makers with $a < a^{cut}$ choose

$$e_{R1} = \frac{\gamma}{2c}. \tag{10}$$

Voters perfectly infer their ability and deselect those policy-makers.

(ii) Policy-makers with $a \geq a^{cut}$ choose

$$e_{R1} = \hat{e}_{R1} + a^{cut} - a = \frac{1}{2c}\{\gamma + \sqrt{4c[b + \gamma a^{cut} + (\mu_R - \mu_L)^2] - \gamma^2}\} + a^{cut} - a. \tag{11}$$

They generate the same output given by

$$\gamma(\hat{e}_{R1} + a^{cut}) \tag{12}$$

and are reelected.

(iii) The beliefs of the voters are characterized by

α.) $E_a(\gamma(\hat{e}_{R1} + a^{cut})) = \frac{A+a^{cut}}{2}$
$E_a(\gamma(\frac{\gamma}{2c} + a)) = a \ \forall a \in [-A, a^{cut})$

β.) $E_a(g)$ *arbitrary if* $g > (\gamma(\hat{e}_{R1} + a^{cut})$
$E_a(g) < 0$ *if* $g < (\gamma(\hat{e}_{R1} + a^{cut})$ *and* $g \notin [\gamma(\frac{\gamma}{2c} - A), \gamma(\frac{\gamma}{2c} + a^{cut}))$.

The proof of Proposition 2 is given in the Appendix. The beliefs in (iii)α.) are on the equilibrium path, while (iii)β.) are conditions for out-of-equilibrium beliefs. Proposition

2 reveals that the selection power of the reelection mechanism may be severely limited. There are equilibria for which almost all incumbents are reelected. This occurs when a^{cut} is low. In such cases the output is low. We next consider pooling equilibria.

4.2.2 Pooling equilibria

In a pooling equilibrium all office-holders choose the same output levels. Such equilibria are characterized in the following Proposition:

Proposition 3
There exists a continuum of pooling equilibria characterized by output levels

$$g^p \in [g^p_{low}, g^p_{high}]$$

with

$$g^p_{low} = \frac{\gamma^2}{2c} + \gamma A \tag{13}$$

and

$$g^p_{high} = \frac{\gamma^2}{2c} - \gamma A + \frac{\gamma}{2c}\sqrt{4c[b - \gamma A + (\mu_R - \mu_L)^2] - \gamma^2}. \tag{14}$$

An equilibrium associated with g^p is characterized by

(i) Office-holders choose

$$e_{R1} = \frac{g^p}{\gamma} - a \tag{15}$$

and produce the same output g^p.

(ii) All office-holders are reelected.

(iii) Voters' beliefs are given by

α.) $E_a(g^p) = 0$

β.) $E_a(g)$ arbitrary for $g > g^p$
$E_a(g) < 0$ for $g < g^p$

The proof of Proposition 3 is given in the Appendix. Again, conditions in (iii)β.) restrict out-of-equilibrium beliefs.

Pooling equilibria can only exist if $g^p_{high} \geq g^p_{low}$. Otherwise the interval $[g^p_{low}, g^p_{high}]$ is empty, and no pooling equilibria exist. Comparing the expressions for g^p_{high} and g^p_{low} we observe that pooling equilibria exist if the utility from holding office or the disutility when the ideological platform of the opposing candidate is adopted are relatively high.

We would like to stress that one particular pooling equilibrium, characterized by g^p, requires that all voters associate the level of g^p with the equilibrium value, while all other values of g are interpreted as a deviation from this equilibrium value.[4]

We further note that pooling equilibria deter voters completely from gathering information regarding the ability of candidates. As a consequence, all incumbents are reelected. This represents an extreme case where the election mechanism has no power to select able candidates for public office.

4.2.3 Other equilibria

Finally we discuss whether further equilibria exist. For that purpose we introduce the following plausible refinement:

Output-Ability Monotonicity (OAM)

An equilibrium satisfies output-ability monotonicity (henceforth OAM) if voters believe that an office-holder who produces a higher output than another one has equal or higher ability. Formally, the belief function $E_a(g)$ in an equilibrium is non-decreasing in g.

The OAM refinement makes sense because higher ability for given effort levels translates into higher output.

We thus obtain

Proposition 4

There are no other equilibria in pure strategies than the semi-separating and pooling equilibria described in Propositions 2 and 3 and satisfying OAM.

The proof of Proposition 4 is given in the Appendix.

Proposition 4 demonstrates that no other equilibrium exists if we impose OAM. In the following we can reduce the set of equilibria by applying a plausible refinement.

[4] This requires coordination among voters by some commonly-known criterion or norm. For instance, voters may coordinate on a norm that politicians tend to be lazy and thus believe that the equilibrium associated with g^p_{low} is played.

4.2.4 Refinement

We impose the widely-applied Intuitive Criterion (see e.g. Cho and Kreps (1987)) regarding the set of pooling and semi-separating equilibria identified in the last sections.[5] As we will see, this criterion rules out equilibria with high levels of a^{cut}. These are implausible because an equilibrium with a high value of a^{cut} means that only a small share of policy-makers with the highest ability are reelected. Policy-makers of high ability, but below a^{cut}, are deselected.

If such a high-ability policy-maker increases both his effort and the output in order to separate himself from lower-ability ones, then the out-of-equilibrium beliefs will cause voters to believe that he has below-average ability. As only high-ability policy-makers are willing to increase output by a sufficiently high amount, these out-of-equilibrium beliefs are implausible.

The pooling equilibria satisfy the Intuitive Criterion. For the semi-separating equilibria we obtain the following Proposition:

Proposition 5
Semi-separating equilibria fulfill the Intuitive Criterion if and only if $a^{cut} \leq 0$.

For the remainder we concentrate on equilibria that fulfill the Intuitive Criterion, i.e. on pooling equilibria and semi-separating equilibria with $a^{cut} \leq 0$.

5 Vote-Share Thresholds

In this section we assume that the public sets a reelection threshold for incumbents m with $\frac{1}{2} \leq m \leq 1$. Throughout the section we assume that $\frac{2\mu_R - 1}{2A\gamma} < \frac{1}{2}$, which ensures interior solutions.[6] The interpretation is as follows: If politician k takes office in $t = 1$, he must win a share of votes at least equal to m at the next election if he wants to retain office. Otherwise the challenger will take office.[7]

[5] The criterion says that if the information set following a message is off the equilibrium path and if this message is equilibrium-dominated for a certain type, then the receiver's belief should give this type zero probability.

[6] Corner solutions are an important variant of our model. If $\frac{2\mu_R - 1}{2A\gamma} > \frac{1}{2}$, the incumbent may have an incentive to forgo high effort, since reelection chances are too low or possibly zero when vote-share thresholds are high.

[7] Another practical solution is to allow for a runoff between two new candidates. Such a procedure ensures that all candidates elected to public office gain at least 50% of the votes.

The timing of the extended game is summarized in the following figure:

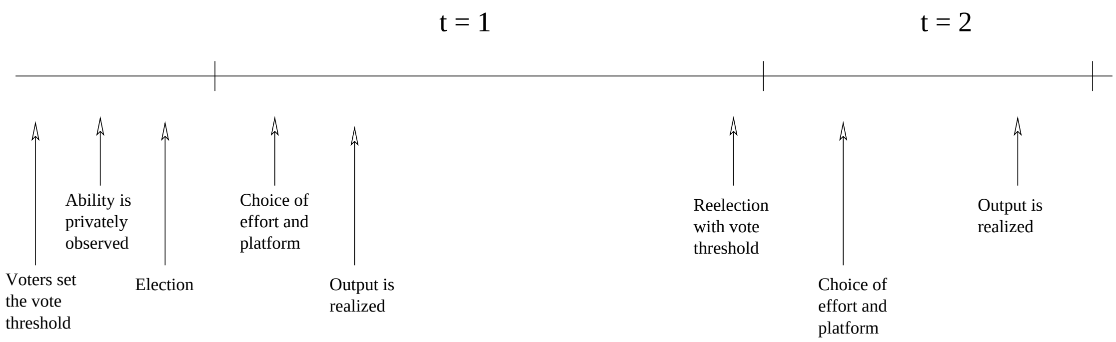

Figure 2: Time-line for elections and vote-share thresholds

For the following analysis we assume that a candidate k, say R, has been elected and that the vote-share threshold has been set at m with $m \geq \frac{1}{2}$.

In the second period the choice regarding P and I by R (if he remains in office) or by L (if he enters office) will remain the same as in Proposition 1. Hence we can concentrate on the first period.

5.1 The first period

For the first period we assume without loss of generality that candidate R has been elected. We obtain

Proposition 6

Suppose $m > \frac{1}{2}$. Then,

(i) the pooling equilibria do not exist.

(ii) semi-separating equilibria parameterized by a^{cut} exist if and only if $a^{cut} \geq a^{crit}(m)$, where the critical quality level $a^{crit}(m)$ is given by

$$a^{crit}(m) := -A + \frac{2}{\gamma}(2\mu_R - 1)(2m - 1). \tag{16}$$

The proof of Proposition 6 is given in the Appendix. We note that $a^{crit}(m)$ is larger than $-A$ and monotonically increasing in m. For $m = \frac{1}{2}$ we obtain $a^{crit} = -A$, and all semi-separating equilibria exist.

Proposition 6 shows that higher thresholds for incumbents destroy pooling equilibria and eliminate semi-separating equilibria where the average ability of reelected incumbents is low. The reason is that an incumbent can only gain a vote share that exceeds 50% marginally if his perceived average ability exceeds 0 marginally. In the next section we discuss the welfare implications of these results.

5.2 Welfare

To prepare the ground for welfare implications we first calculate the welfare associated with a particular equilibrium. As aggregate utility from the ideological policy is independent of the type of politician, we use W to denote the expected welfare voters derive from public project P in a particular equilibrium. Specifically, $W^{pool}(g^p)$ is the expected welfare in a pooling equilibrium associated with output $g^p \in [g^p_{low}, g^p_{high}]$. $W^{sep}(a^{cut})$ is the expected welfare in a semi-separating equilibrium with cut-off ability a^{cut}.

Using Propositions 1, 2, and 3 yields

$$\begin{aligned} W^{pool}(g^p) &= \int_{-A}^{+A} \gamma(\frac{g^p}{\gamma} - a + a)da + \int_{-A}^{+A} \gamma(\frac{\gamma}{2c} + a)da \\ &= [g^p a + \frac{\gamma^2}{2c}a + \frac{1}{2}\gamma a^2]^{+A}_{-A} \\ &= g^p A + \frac{\gamma^2}{2c}A + \frac{1}{2}\gamma A^2 + g^p A + \frac{\gamma^2}{2c}A - \frac{1}{2}\gamma A^2 \\ &= \frac{\gamma^2 A}{c} + 2g^p A \end{aligned} \tag{17}$$

and

$$\begin{aligned} W^{sep}(a^{cut}) &= \int_{-A}^{a^{cut}} (\frac{\gamma^2}{2c} + \gamma a + \frac{\gamma^2}{2c})da + \int_{a^{cut}}^{+A} (\gamma \hat{e}_{R1} + \gamma a^{cut} + \frac{\gamma^2}{2c} + \gamma a)da \\ &= [\frac{\gamma^2}{c}a + \frac{1}{2}\gamma a^2]^{a^{cut}}_{-A} + [\gamma \hat{e}_{R1} a + \gamma a^{cut} a + \frac{\gamma^2}{2c}a + \frac{1}{2}\gamma a^2]^{+A}_{a^{cut}} \\ &= \frac{\gamma}{2c}[\gamma(3A + a^{cut}) + 2c(\hat{e}_{R1} + a^{cut})(A - a^{cut})]. \end{aligned} \tag{18}$$

We obtain

Proposition 7

(i)

$$\lim_{a^{cut}\to -A} W^{sep}(a^{cut}) = W^{pool}(g^p_{high}) = \frac{2\gamma^2 A}{c} - 2A^2\gamma + \frac{A\gamma}{c}\sqrt{4c[b-\gamma A+(\mu_R-\mu_L)^2]-\gamma^2}$$

(ii) There exists a^{cut*} for which the semi-separating equilibrium yields the highest possible welfare.

The proof of Proposition 7 is given in the Appendix. Note that the semi-separating equilibrium with $a^{cut} = -A$ may not be the worst among the class of semi-separating equilibria. This tends to be the case when b is very large. In such a case it could be optimal to set $m^* = \frac{1}{2}$.[8]

In the next Proposition we state a sufficient condition for $a^{cut} = -A$ to constitute the worst semi-separating equilibrium.

Proposition 8

- *If the following condition holds:*

$$\frac{3}{4}\gamma^2 + 3c\gamma A > 3c[b + (\mu_R - \mu_L)^2]\,, \tag{19}$$

  then $W^{sep}(-A) < W^{sep}(a^{cut})$ for all $a^{cut} \in (-A, 0]$.

- *The corresponding upper boundary for benefits b is given by*

$$b < \frac{\gamma^2}{4c} + \gamma A - (\mu_R - \mu_L)^2.$$

The proof of Proposition 8 is given in the Appendix. The upper boundary for b confirms the intuition stated above: If b is too large, $a^{cut} = -A$ will not yield the worst aggregate welfare among the semi-separating equilibria. Furthermore, note that equation (19) is more likely to be fulfilled for large A, i.e. when there is a wide range of ability levels, and for large γ, i.e. when effort and ability translate into higher public-project output. The impact of cost parameter c is ambiguous.

[8] This depends on the welfare levels in the pooling equilibria lower than $\lim_{a^{cut}\to -A} W^{sep}(a^{cut})$.

In the following we assume that the semi-separating equilibrium with $a^{cut} = -A$ is the worst among the semi-separating equilibria. A sufficient condition for this case is given in Proposition 8. Here Proposition 7 reveals that there are two types of equilibria associated with low welfare. Pooling equilibria are the worst. Semi-separating equilibria with a^{cut} close to $-A$ produce lower welfare than semi-separating equilibria with intermediate values for a^{cut}. Combining Proposition 6 and Proposition 7 yields our main result.

Theorem 1

(i) Higher vote thresholds than $\frac{1}{2}$ eliminate the worst equilibria (pooling equilibria).

(ii) Vote thresholds $m = \frac{1}{2} + \Delta$ for Δ sufficiently small eliminate semi-separating equilibria with a^{cut} close to $-A$.

Theorem 1 is the main rationale for advocating higher vote thresholds than $\frac{1}{2}$. To determine a socially optimal vote threshold one has to make an assumption regarding the likelihood with which an equilibrium will be played. We perform an exercise assuming that each equilibrium has the same chance of being realized. Then we obtain

Proposition 9

There exists a welfare optimal vote threshold m^* with $m^* > \frac{1}{2}$.

The proof is given in the Appendix.

Two remarks are in order here. First, it is important to stress that slightly higher vote thresholds than 50% can never be harmful for welfare as the vote-share of reelected incumbents in semi-separating equilibria with high welfare is well above 50%. Such equilibria will not be eliminated.

Second, the utility of politicians in office is negligible in our model, as we have a continuum of voters. Here their utility does not affect welfare considerations. In a finite version of our model the utility of the politician and the cost of exerting effort will affect the welfare-optimizing vote-share threshold. As a result the welfare-optimal vote-share in a finite version of our model tends to be slightly lower.

6 Extensions, Applications and Generalizations

We have illustrated how higher vote thresholds for incumbents can improve the selection power of elections. Numerous extensions can and should be pursued to test the robustness and validity of the argument.

6.1 Further incumbency advantages

It is useful to allow for further sources of incumbency advantage discussed in the literature. Suppose a candidate can gain advantages in office that make him more attractive than a challenger, even if his ability is the same. Such advantages may be name recognition or lower uncertainty (variance) about ideological positions for incumbents than for challengers. Such advantages generated by holding office further weaken the selection power of elections, as office-holders with expected ability slightly below average will be reelected. In such circumstances higher vote thresholds for incumbents are even more advantageous, as setting them sufficiently high ensures that only office-holders with above-average ability will be reelected.

6.2 Learning by doing

Another fruitful extension is learning by doing. Suppose the politician in office experiences learning effects during the first term in office. Then his marginal effort costs may decline for the second term. In such circumstances it is socially desirable to reelect incumbents with expected ability slightly below average. As a consequence, higher vote thresholds for incumbents than for the election of a newcomer are still welfare-improving, but they have to be set at a lower level than in the model variant without learning-by-doing effects.

6.3 Alternative election procedures

Two alternative election procedures involving vote-share thresholds for incumbents can be considered. First, if the incumbent fails to reach the threshold, a separate election between a new right-wing candidate and candidate L will take place. Such a procedure ensures that politicians are only elected if they receive at least 50% of the votes.

Second, instead of the public, candidates may propose vote thresholds for themselves which become effective if they take office. Both variants of the model yield the same (latter version) or qualitatively similar results (former version). The result is obvious for the latter version. Competing newcomers will tend to offer the socially optimal vote threshold, as the voters in the center will prefer those candidates who are closest to the socially optimal threshold.

6.4 Repeated competition with vote thresholds

A useful extension of the model is to consider a larger time horizon or a version of the model with an infinite horizon. In a finitely repeated version of our model there is no a priori welfare reason why vote thresholds for incumbents should increase further at the end of their second or third term. However, if the incumbency advantages connected with the office of the kinds discussed in subsection 6.1 increase with the number of terms in office, this could be a justification for vote thresholds increasing in the number of terms an incumbent has been in office.

6.5 Generalizations of the model

Finally, it is useful to study a general version of the present model that does not rely on specific functional forms. A generalization of the model can be stated as follows:

- Ability is distributed on $[-A, +A]$ according to some density function $f(A)$. The expected ability is normalized to zero.
- The amount of the public project in period t is given by $g_t = h(e_{kt}, a_k)$ with
  - $\lim_{e_{kt} \to \infty} h(e_{kt}, a_k) = \infty$ for all $a_k \in [-A, +A]$
  - $\frac{\partial h(\cdot,\cdot)}{\partial e_{kt}} > 0$, $\frac{\partial^2 h(\cdot,\cdot)}{\partial e_{kt}^2} \leq 0$, $\frac{\partial h(\cdot,\cdot)}{\partial a_k} > 0$, $\frac{\partial^2 h(\cdot,\cdot)}{\partial a_k^2} \leq 0$
- $U_i^I(i_{kt}) = -k(|i_{kt} - i|)$ with $k'(\cdot) > 0$ and $k''(\cdot) > 0$.
- $C(e_{kt})$ satisfies $C(0) = 0$, $C'(0) = 0$, $C'(e_{kt}) > 0$ for $e_{kt} > 0$, $\lim_{e_{kt} \to \infty} C'(e_{kt}) = \infty$ and $C''(\cdot) > 0$.

The assumptions on $h(e_{kt}, a_k)$ ensure that it is always possible for agents with low ability to mimic the output of high-ability office-holders by exerting a sufficiently high level of effort. The same analysis as in this paper can also be performed in this more general setting. While no explicit solutions can be derived for the general case and the set of equilibria may vary, the qualitative considerations remain unchanged.[9]

7 Conclusion

The main insight of this paper is that higher vote thresholds increase the selection power of elections, which is socially desirable. As it is easy to implement in practice, it will be useful to experiment with this new institution.

[9] Conditions for the existence of the equilibria and for socially optimal thresholds above $\frac{1}{2}$ are available on request.

Appendix

Proof of Proposition 1

The first point is obvious. The optimization problem of the office-holder regarding his effort choice is given by

$$\max_{e_{k2}}\{\gamma(e_{k2}+a_k)-ce_{k2}^2\},$$

which yields $e_{k2}^*=\frac{\gamma}{2c}$. The expected utility of a policy-maker at the beginning of period 2 is given by

$$b+\gamma\left(\frac{\gamma}{2c}+a_k\right)-c\left(\frac{\gamma}{2c}\right)^2=b+\frac{\gamma^2}{4c}+\gamma a_k.$$

■

Proof of Proposition 2

Step 1

Office-holders with $a<a^{cut}$ could mimic the behavior of incumbents with $a\geq a^{cut}$ in order to get reelected. Mimicking requires an effort level

$$e_{R1}=\hat{e}_{R1}+a^{cut}-a, \tag{20}$$

which would yield a utility

$$V_{R1}^{dev}=b+\gamma(\hat{e}_{R1}+a^{cut}-a+a)-ce_{R1}^2+b+\frac{\gamma^2}{4c}+\gamma a. \tag{21}$$

This will be smaller than their equilibrium utility

$$V_{R1}=b+\frac{3\gamma^2}{4c}+\gamma a-(\mu_R-\mu_L)^2 \tag{22}$$

if and only if the following condition holds:

$$b+\frac{3\gamma^2}{4c}+\gamma a-(\mu_R-\mu_L)^2>b+\gamma(\hat{e}_{R1}+a^{cut}-a+a)-ce_{R1}^2+b+\frac{\gamma^2}{4c}+\gamma a. \tag{23}$$

Rearranging terms yields

$$ce_{R1}^2-\gamma\hat{e}_{R1}+\frac{\gamma^2}{2c}-b-\gamma a^{cut}-(\mu_R-\mu_L)^2>0. \tag{24}$$

As $a < a^{cut}$, we have $\hat{e}_{R1} < e_{R1}$. As (24) holds by construction as an equality for $e_{R1} = \hat{e}_{R1}$, the deviation is not profitable.

Step 2

Candidates with $a \geq a^{cut}$ could choose to lower their effort, thereby risking a deselection. The equilibrium utility is given by

$$V_{R1} = b + \gamma(\hat{e}_{R1} + a^{cut} - a + a) - ce_{R1}^2 + b + \frac{\gamma^2}{4c} + \gamma a. \tag{25}$$

Deviating with $e_{R1} = \frac{\gamma}{2c}$ yields

$$V_{R1}^{dev} = b + \frac{3\gamma^2}{4c} + \gamma a^{cut} - (\mu_R - \mu_L)^2. \tag{26}$$

Deviation is not profitable if

$$ce_{R1}^2 - \gamma\hat{e}_{R1} + \frac{\gamma^2}{2c} - b - \gamma a - (\mu_R - \mu_L)^2 \leq 0. \tag{27}$$

As $a \geq a^{cut}$, we have $\hat{e}_{R1} \geq e_{R1}$. By construction, inequality (27) holds as an equality for $e_{R1} = \hat{e}_{R1}$. Hence the deviation is not profitable.

Step 3

Voters' equilibrium beliefs about utility and voting decisions are given as follows:

- If output is $\gamma(\hat{e}_{R1}+a^{cut})$, expected ability is given by $E_a(\gamma(\hat{e}_{R1}+a^{cut})) = \frac{A+a^{cut}}{2} > 0$ and office-holders producing this output are reelected.
- If output is $\gamma(\frac{\gamma}{2c}+a)$ with $-A \leq a < a^{cut} \leq 0$, voters will believe that the candidate has ability a and he will be deselected because his ability is below-average.
- If output is below $\gamma(\hat{e}_{R1} + a^{cut})$ and out of the equilibrium, voters will believe that candidates' ability is below zero.
- If output is above $\gamma(\hat{e}_{R1} + a^{cut})$, then the belief of voters is arbitrary.

■

Proof of Proposition 3

If he plays the equilibrium strategy, the utility of a politician with ability a is given by

$$V_{R1}^{pool} = b + \gamma(\frac{g^p}{\gamma} - a + a) - c(\frac{g^p}{\gamma} - a)^2 + b + \frac{\gamma^2}{4c} + \gamma a. \qquad (28)$$

If he deviates to a slightly higher effort $e_{R1} = \frac{g^P}{\gamma} - a + \epsilon$, his utility would amount to

$$V_{R1}^{hdev} := b + \gamma(\frac{g^p}{\gamma} - a + \epsilon + a) - c(\frac{g^p}{\gamma} - a + \epsilon)^2 + b + \frac{\gamma^2}{4c} + \gamma a \qquad (29)$$

with ϵ being a small positive number. Such a deviation is not attractive if $V_{R1}^{pool} \geq V_{R1}^{hdev}$, which yields

$$g^p \geq \frac{\gamma^2}{2c} + a\gamma - \frac{\gamma}{2}\epsilon. \qquad (30)$$

Condition (30) has to hold for all $\epsilon > 0$ and for all $a \in [-A, +A]$. For type $a = A$ not to deviate,

$$g^p_{low} = \frac{\gamma^2}{2c} + A\gamma. \qquad (31)$$

Deviation to a lower effort than in the pooling equilibrium will result in deselection and would yield

$$V_{R1}^{ldev} = b + \frac{\gamma^2}{4c} + \gamma a + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2. \qquad (32)$$

There will be no downward deviation if $V_{R1}^{pool} \geq V_{R1}^{ldev}$, which yields

$$\begin{aligned} g^p \in [\ & \tfrac{\gamma^2}{2c} + a\gamma - \tfrac{\gamma}{2c}\sqrt{4c[b + \gamma a + (\mu_R - \mu_L)^2] - \gamma^2}, \\ & \tfrac{\gamma^2}{2c} + a\gamma + \tfrac{\gamma}{2c}\sqrt{4c[b + \gamma a + (\mu_R - \mu_L)^2] - \gamma^2}\ ]. \end{aligned} \qquad (33)$$

The condition has to hold for all $a \in [-A, +A]$. The worst type $a = -A$ will not want to lower his effort if

$$g^p \leq \frac{\gamma^2}{2c} - A\gamma + \frac{\gamma}{2c}\sqrt{4c[b - \gamma A + (\mu_R - \mu_L)^2] - \gamma^2}, \qquad (34)$$

which gives g^p_{high}.

Moreover, the comparison of V_{R1}^{pool} and V_{R1}^{ldev} provides another condition that has to be fulfilled for no upward deviation to occur:

$$g^p \geq [\frac{\gamma^2}{2c} + A\gamma - \frac{\gamma}{2c}\sqrt{4c[b+\gamma A + (\mu_R - \mu_L)^2] - \gamma^2}. \qquad (35)$$

As this condition is less strict than condition (30), g^p_{low} is given by equation (31).

■

Proof of Proposition 4

Case 1

Suppose that there exists another type of semi-separating equilibrium with two groups, in which one group, say group 1, chooses output g^1 and those policy-makers are reelected. The policy-makers in the other group, say group 2, choose effort level $e = \frac{\gamma}{2c}$ and are deselected. Output g^1 has to be larger than the maximal output produced by policy-makers in the second group, as otherwise policy-makers in the second group could mimic policy-makers in the first group at no cost.

Next we observe that minimal ability of agents in the first group is equal to maximal ability in the second group. Suppose however that a_1 belongs to group 1 and a_2 belongs to group 2 with $a_2 > a_1$. As a_1 is in group 1, his utility from producing g^1 is equal to or higher than that associated with choosing $e = \frac{\gamma}{2c}$, i.e.

$$\begin{aligned} U_{a_1}(g^1) &= b + \gamma(\frac{g^1}{\gamma} - a_1 + a_1) - c(\frac{g^1}{\gamma} - a_1)^2 + b + \frac{\gamma^2}{4c} + \gamma a_1 \\ &\geq \\ U_{a_1}(\frac{\gamma}{2c} + a_1) &= b + \frac{\gamma^2}{2c} + \gamma a_1 - \frac{\gamma^2}{4c} + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2 \end{aligned}$$

The opposite must hold for a_2:

$$\begin{aligned} U_{a_2}(g^1) &= b + \gamma(\frac{g^1}{\gamma} - a_2 + a_2) - c(\frac{g^1}{\gamma} - a_2)^2 + b + \frac{\gamma^2}{4c} + \gamma a_2 \\ &\leq \\ U_{a_2}(\frac{\gamma}{2c} + a_2) &= b + \frac{\gamma^2}{2c} + \gamma a_2 - \frac{\gamma^2}{4c} + \frac{\gamma^2}{2c} - (\mu_R - \mu_L)^2 \end{aligned}$$

This yields $(\frac{g^1}{\gamma} - a_1)^2 \leq (\frac{g^1}{\gamma} - a_2)^2$. From the first paragraph we can draw upon $g^1 > \gamma(\frac{\gamma}{2c} + a_2)$, so that $a_2 < \frac{g^1}{\gamma}$. Moreover, as no policy-maker will choose a lower effort than $\frac{\gamma}{2c}$, we have $g^1 > \gamma(\frac{\gamma}{2c} + a_1)$ and thus $a_1 < \frac{g^1}{\gamma}$. Hence $(\frac{g^1}{\gamma} - a_1)^2 \leq (\frac{g^1}{\gamma} - a_2)^2$ implies $a_2 < a_1$, which contradicts the assumption $a_2 > a_1$.

Case 2
Suppose that policy-makers completely separate themselves by choosing effort $e = \frac{\gamma}{2c}$ and all policy-makers with $a \geq 0$ are reelected. Such a constellation cannot be an equilibrium, as a policy-maker with $a = -\epsilon$ and $\epsilon > 0$ being small can increase his effort marginally and mimic a policy-maker with $a = 0$, thus securing reelection and generating a rise in his utility. Hence this deviation is profitable.

Case 3
Suppose that there exists a semi-separating equilibrium with three different groups, from which two groups pool, say group 1 and group 2. Pooling necessarily requires that office-holders are reelected, as otherwise every policy-maker will choose the same effort and produce a different output. Suppose that output in group 1 is g^1 and in group 2 it is g^2 with $g^1 > g^2$. Then policy-makers in group 1 can lower their effort and choose g^2, thus still securing reelection. Such a deviation is not profitable if $g^2 < \gamma(\frac{\gamma}{2c} + a)$ for all policy-makers with ability a in group 1, as $e_1 = \frac{\gamma}{2c}$ is the minimal effort any policy-maker will choose. Hence the ability of all policy-makers in group 2 is equal to or smaller than the lowest ability level in group 1.

According to OAM, any policy-maker in group 1 would therefore be reelected if he chose $e_1 = \frac{\gamma}{2c}$ and produced output $\gamma(\frac{\gamma}{2c} + a)$, which is larger than g^2. We thus arrive at a contradiction, as we have assumed that such policy-makers will not pool.

■

Proof of Proposition 5
Consider a semi-separating equilibrium with $a^{cut} > 0$. Suppose that an office-holder with $a = 0$ expects to be reelected with certainty if he deviates. The maximal effort he would be willing to exert is given by $e^{dev}_{R1,a=0} = \frac{1}{2c}\{\gamma + \sqrt{4c[b + \gamma a^{cut} + (\mu_R - \mu_L)^2] - \gamma^2}\} := e^{dev}_{R1,a=0}(a^{cut})$. Suppose now that any policy-maker with $a \in [0, a^{cut})$ chooses $e^{dev}_{R1,a=0}(a^{cut})$. Such policy-makers will benefit from the deviation if voters believe that they actually have an ability of $a \geq 0$ and thus reelect them. Policy-makers with $a < 0$, however, are worse off by choosing $e^{dev}_{R1,a=0}(a^{cut})$, even if voters believe that they are of high ability and reelect them.

Hence equilibria with $a^{cut} > 0$ do not satisfy the Intuitive Criterion. Equilibria with $a^{cut} < 0$ do satisfy this criterion, as no office-holder with above-average ability would want to deviate.

■

Proof of Proposition 6

(i) We show that pooling equilibria in which all policy-makers are reelected with certainty do not exist for $m > \frac{1}{2}$. The expected ability of an office-holder in a pooling equilibrium is zero. The median voter is indifferent between reelecting the office-holder and electing a new candidate. This would imply that no office-holder can obtain a share of votes equal to m, as they get 50%.

We note that no pooling equilibrium exists in which policy-makers expect that they will not be reelected. Office-holders would choose the same effort level, but the outputs would be different, and voters could perfectly infer their ability. This is a contradiction.

(ii) We now look at semi-separating equilibria. With $m > \frac{1}{2}$, candidate R is reelected only if voter $i = 1-m$ prefers to vote for him, which implies that all voters with $i > 1-m$ will also prefer R to L. This leads to the following condition:

$$\gamma(\frac{\gamma}{2c} + \frac{A + a^{cut}}{2}) - (\mu_R - (1-m))^2 \geq \gamma(\frac{\gamma}{2c}) - (\mu_L - (1-m))^2. \tag{36}$$

Using $\mu_L = 1 - \mu_R$, we obtain

$$a^{cut} \geq \frac{2}{\gamma}(2\mu_R - 1)(2m-1) - A := a^{crit}. \tag{37}$$

■

Proof of Proposition 7

(i) We obtain the result of part (i) of the Proposition by inserting g^p_{high} for g^p in equation (17) and by inserting $-A$ for a^{cut} in equation (18).

(ii) The existence of an optimal value a^{cut*} is guaranteed, as $W^{sep}(a^{cut})$ is continuous and is maximized on the compact set $[-A, 0]$. The necessary condition for an interior solution is

$$\begin{aligned}\frac{\partial W^{sep}(a^{cut})}{\partial a^{cut}} &= \frac{\gamma}{2c}\Big(-a^{cut} - \sqrt{-\gamma^2 + 4c(b + a^{cut}\gamma + (\mu_R - \mu_L)^2)} \\ &\quad +(A - a^{cut})(1 + \frac{2c\gamma}{\sqrt{-\gamma^2 + 4c(b + a^{cut}\gamma + (\mu_R - \mu_L)^2)}})\Big).\end{aligned}$$

■

Proof of Proposition 8

We show that under the condition stated in Proposition 8 welfare in a semi-separating equilibrium with $a^{cut} = a$ is higher than with $a^{cut} = -A$ for some $a \in (-A, 0]$. We have

$$\begin{aligned}W^{sep}(a^{cut} = a) &= \frac{2\gamma^2 A}{c} + \gamma a(A - a) + \frac{\gamma(A - a)}{2c}\sqrt{4c[b + (\mu_R - \mu_L)^2 + \gamma a] - \gamma^2} \\ W^{sep}(a^{cut} = -A) &= \frac{2\gamma^2 A}{c} + \frac{A\gamma}{c}\sqrt{4c[b + (\mu_R - \mu_L)^2 - A\gamma] - \gamma^2} - 2\gamma A^2\end{aligned}$$

Hence the difference $\Delta = W^{sep}(a^{cut} = a) - W^{sep}(a^{cut} = -A)$ is given by

$$\begin{aligned}\Delta &= \gamma(2A^2 + Aa - a^2) \\ &\quad - \frac{\gamma a}{2c}\sqrt{4c[b + (\mu_R - \mu_L)^2 + \gamma a] - \gamma^2} \\ &\quad + \frac{A\gamma}{c}\Big[\frac{1}{2}\sqrt{4c[b + (\mu_R - \mu_L)^2 + \gamma a] - \gamma^2} - \sqrt{4c[b + (\mu_R - \mu_L)^2 - A\gamma] - \gamma^2}\Big]\end{aligned}$$

The first summand is positive, as $-A < a \leq 0$: $2A^2 + Aa - a^2 = A^2 - a^2 + A(A + a)$. The second summand is positive, as $a \leq 0$. Hence if the third summand is positive, then $\Delta > 0$, i.e. $a^{cut} = -A$ is the worst semi-separating equilibrium. The third summand is positive if

$$\begin{aligned}\tfrac{1}{2}\sqrt{4c[b + (\mu_R - \mu_L)^2 + \gamma a] - \gamma^2} &> \sqrt{4c[b + (\mu_R - \mu_L)^2 - A\gamma] - \gamma^2} \\ \Leftrightarrow \quad 4c[b + (\mu_R - \mu_L)^2 + \gamma a] - \gamma^2 &> 4\Big(4c[b + (\mu_R - \mu_L)^2 - A\gamma] - \gamma^2\Big) \\ \Leftrightarrow \quad \tfrac{3}{4}\gamma^2 + 4c\gamma A + c\gamma a &> 3c[b + (\mu_R - \mu_L)^2]\end{aligned}$$

As $a > -A$, the LHS is greater than $\frac{3}{4}\gamma^2 + 3c\gamma A$. Using this observation yields the sufficient condition (equation (19)) stated in Proposition 8. The upper boundary for benefits b can be obtained by rearranging terms.

■

Proof of Proposition 9

For $m > \frac{1}{2}$ expected welfare $W(a^{crit}(m))$ is given by

$$
\begin{aligned}
W(a^{crit}(m)) &= \int_{a^{crit}}^{0} \frac{1}{-a^{crit}} W^{sep}(a^{cut}) da^{cut} \\
&= \int_{a^{crit}}^{0} \frac{1}{-a^{crit}} \Big\{ \frac{2\gamma^2 A}{c} + \frac{\gamma A}{2c}\sqrt{4c[b+\gamma^{cut}+(\mu_R-\mu_L)^2]-\gamma^2} \\
&\quad -\frac{\gamma a^{cut}}{2c}\sqrt{4c[b+\gamma^{cut}+(\mu_R-\mu_L)^2]-\gamma^2} + \gamma a^{cut} A - \gamma a^{cut2} \Big\} da^{cut}
\end{aligned}
$$

If $m = \frac{1}{2}$, expected welfare is given by

$$\int_{a^{crit}}^{0} \frac{1}{-a^{crit}+g^h-g^l} W^{sep}(a^{cut}) da^{cut} + \int_{g^l}^{g^h} \frac{1}{-a^{crit}+g^h-g^l} W^{pool}(g^P) dg^P$$

We next observe that there exists an arbitrarily small $\epsilon > 0$ and $\hat{m} \in (\frac{1}{2}+\epsilon, 1)$ such that expected welfare is higher for $\hat{m}$ than for any $m \in [\frac{1}{2}, \frac{1}{2}+\epsilon]$. The reason is that a slightly higher m than $m = \frac{1}{2}$ eliminates the pooling equilibrium and the worst semi-separating equilibria. Accordingly, welfare is increasing for m arbitrarily close to $m = \frac{1}{2}$, which proves the observation. So, we can find the optimal m by maximizing $W(a^{crit}(m))$ on $[\frac{1}{2}+\epsilon, 1]$. $W(a^{crit}(m))$ is continuous on the compact interval $[\frac{1}{2}+\epsilon, 1]$, which guarantees the existence of on optimal m^* with $m^* > \frac{1}{2}$.

■

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