EUROPEAN SUMMER SYMPOSIUM IN ECONOMIC THEORY

Generously hosted by
Study Center Gerzensee

Monday 29 June-Friday 10 July 2009

On Collective Action with Pairwise Externalities

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We are grateful to the following institutions for their financial and organizational support: Study Center Gerzensee and the Swiss National bank.

On Collective Action with Pairwise Externalities*

(preliminary and incomplete)

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June 2009

Abstract

In this paper we build a model to analyze two dimensions of collective action (distributional conflict and local public good provision) when the utility of each agent is heterogeneously influenced by other agents' utility. There are two dimensions of heterogeneity: who influences whom, and the strength of any such pairwise dependent influence. The pattern of bilateral influences takes the form of a weighted and directed network. Direct influences spread their effects through chains of connected agents in this network. We characterize the Nash bargaining solution for a particular divisible good and analyze how pairwise influences, and the indirect effects they generate, are internalized in shares and utilities obtained. Building on this analysis of distributional conflict, we analyze equilibrium and efficient behavior in a public good provision game with heterogeneous externalities and possibly uneven wealth distribution. The analysis relies on network centrality indexes that measure each agent's prominence due to his position in the influences structure. Our results have implications for the study of the location of urban public resources, government spending, and communities with social preferences.

*This paper subsumes two chapters of my Ph.D. thesis. I am specially indebted to my advisor Antoni Calvó-Armengol. His friendship and talents will be sorely missed. I am also specially grateful to Salvador Barberà, Ehud Kalai, Wolfgang Pesendorfer and Clara Ponsatí for valuable suggestions on previous versions. I also want to thank Eduard Alonso, Coralio Ballester, Caterina Calsamiglia, Julio González-Diaz, Rahmi Ilkiliç, Matt Jackson, Willemien Kets, Jordi Massó, David Pérez-Castrillo, Pedro Rey Biel, Joel Sobel, Jean-Marc Tallon, Sergio Vicente and John Weymark. Financial support from the Spanish Ministry of Education and Science through grant SEJ2006-09993/ECON and Juan de la Cierva program is gratefully acknowledged. All errors are of course mine.

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1 Introduction

The governement of any region provides public facilities to satisfy the needs of its population. Most of the times it is assumed that this public facilities reaches everyone in the same way. However, this is not always necessarily the case. As Olson (1969) notes:

> *One or more of the following of the three following logically possible relationships between the "boundaries" of a collective good and the boundaries of the government that provides it will apply: (1) the collective good reaches beyond the boundaries of the government that provides it; (2) the collective good reaches only a part of the constituency that provides it; or (3) the boundaries of the collective good are the same as those of the jurisdiction that provides it.*

In this work we deal with a variation of the second possibility. The task for the governement is easy if geographical cosntraints are not an important factor for citizens to enjoy these facilities. This would be the case with a *pure* public good, expressed in (1) and (3). However if the effects of these facilities are of a local nature, then deciding on the concrete allocation to implement becomes a more difficult task. Any decision implies that some agents receive a more direct benefit than others. A feature of this kind of local public goods is that they can induce spillover effects: even if some agents do not receive an immediate benefit from a particular allocation of resources, they can receive some positive (or negative) externality from it. Hence, if the government realizes that these spillover effects exist, it has to take them into account and the task of deciding between different possible allocations becomes a more complex task.

A city and its division into neighbourhoods provides a natural example. An activity in which spillover effects are clearly documented is the case of urban crime. There is a large amount of works in the criminology literature that highlights that there exist significant correlations among crime rates of different neighbourhoods of a city, and these correlations depend on social and geographic characteristics of the city (see, for example, Anselin and Messner, 1994, Anselin *et al.*, 2000, Bowers and Johnson, 2003, Morenoff *et al.*, 2001, Mears and Bhati, 2006). This suggests that there also exist significant positive spillovers when local resources to combat urban crime are implemented. Therefore, how these resources are allocated across the geography of the city becomes an important factor in the combat of urban crime and its positive implications for the citizenship. Those that live closer to where some resources are allocated will benefit more directly from them. Anyhow, because of the existence of such spillover effects, more distant individuals can also benefit from them.

Generally, when a jurisdiction is divided into smaller districts and these districts can have local representatives that ask for part of these resources, a negotiation process is necessary. The first part of the paper analyzes distributional conflict when there exist pairwise externalities, such as for example spatial spillovers among districts.

A second important question related to local public goods and spatial spillovers is where do the resources implemented come from. We analyze in the second part of this paper a public good provision game in which neighbourhoods provide part of their wealth to generate the resources that later on are divided following the Nash bargaining solution. We suppose there are two possible sources of heterogeneity: the pattern of spatial spillovers that the public good generates, and the distribution of wealth across the population. Neighbourhoods internalize in their equilibrium decisions both their level of wealth compared to that of the rest of the city and the positive benefits they receive both from the direct effects of the share of public resources received, as well as the spillover effects generated by the allocation of resources across the rest of the city.

Groups, like a family, whose members show social preferences provide us with another example of application of our framework.[1] Following the line of seminal work from Becker (1974), altruism and envy can be interpreted as influences of a very particular kind.[2] An agent is altruist for another if he is better off when the other is better off. Hence, this second agent is exerting a positive influence on the first one. Inversely, an agent is envious for another agent if he is worst off when the other is better off, and in this case the influence the second agent exerts on the first is negative. Our work applies then to the analysis of a bargaining game in the presence of pure altruism and envy effects, where the pattern of altruism and envy is variable in intensity across pairs of agents, as well as to the incentives to provide resources to the group taking into account that the benefits are both of a direct and indirect (through the altruism/envy linkages) nature.

The environment. There are two dimensions of heterogeneity in the model. On the one hand, the particular geometry of bilateral influences within the group. Not everybody necessarily exerts an influence on every other agent. The pattern of direct bilateral influences determines who exerts an externality on whom and how influences spread indirectly through the economy. On the other hand, the magnitude of each bilateral influence is pairwise dependent. The magnitude of an influence relation depends on exactly which agent exerts this influence and which agent receives it.

Direct influences generate indirect network effects. We call network externalities the sum of all these indirect effects. These are direct externalities derived from the assignment of resources, and measure how the level of consumption, not the utility, of an agent affects another agent's utility. Each pattern of pairwise influences determines a unique pattern of network externalities.

To better understand the spread of influences through direct bilateral influences we reinterpret at some points in the paper the model in terms of networks. The network links an agent to another whenever the second agent exerts a direct externality on the first one. Externalities spread then through the links of the network. We can keep track of all indirect influences generated from bilateral influences through paths and cycles in the associated network. The network metaphor is adequate in this setting due to asymmetric pairwise bilateral influences. Connections with well-known notions from social network analysis, such as centrality measures, arise as natural tools for the network reinterpretation of the model.

Direct bilateral influences within a finite set of agents are modeled with the use of a linear model that encompasses the possibility of positive and negative influences as well as asymmetric influences within pairs of agents. The model is characterized by a matrix that collects the possibly different levels of bilateral influences for each possible pair of agents, the primitives of the model.

A unit of a divisible resource has to be distributed among the agents of the economy. The utility that an agent obtains comes from the share of the resource that receives and also from the utility of agents that exert an influence on him, which enters his utility function proportionally to the intensity of the direct influence each of these other agents exert on him.

[1] See Fehr and Schmidt (2002) and Sobel (2005) for very comprehensive surveys of the theoretical literature and empirical evidence on social preferences. Levine uses a linear model similar to the one we develope in our work to analyze experimental results for some classes of games. Roth (1995) is a survey of experimental evidence of social preferences in bargaining games.

[2] There is also an extensive literature on (nonpaternalistic) intergenerational altruism: see Barro (1974) and Ray (1987).

In the example on crime, whenever direct influences exist among crime rates in different neighbourhooods, the assignment of a part of the resources against urban crime does not only have a direct effect on this neighbourhood. Its effects also spill over the rest of neighbourhoods in an heterogeneous manner. Hence, when the different neighbourhoods are engaged in dispute for these resources, and they understand that these spillover effects exist, they show heterogeneous preferences on possible assignments. Even if probably each neighbourhood would prefer to receive all the resource, if they realize they have to achieve an agreement with the rest of neighbourhoods, they would prefer that those neighbourhoods that overall exert a larger spillover effect on it receive more resources than those that exert less.

A linear structure of influence is assumed for tractability. It ensures for, almost, every economy with influences the existence of a unique solution to the system relating utilities. This solution provides the utility in terms of the allocation, instead of in terms of others' utilities. Hence, solving the model means to characterize the effect the share of the resource an agent receives changes others' welfare, and it internalizes the indirect effects bilateral influences generate.

When distributional conflict exists, the bargaining outcome is given by the Nash bargaining solution. With its use we ensure a unique well-defined outcome after the resolution of the conflict. Of course, this choice comes at a cost. By using a cooperative solution we abstract from institutional or environmental restrictions that can play a role during the bargaining process. These institutional restrictions are generally introduced defining a particular non-cooperative bargaining game that takes them into account. Then each particular application should be followed by a different non-cooperative game that specifies its particularities. This way we would lose some of the general conclusions that a more stylized model can give as general features of somewhat different situations.

The analysis of provision of public resources is based on a two-stage model: at the first stage, group members, endowed with some wealth, decide the level of contribution; at the second stage, all group members negotiate on how to divide the resources generated. The analysis of the second stage of the game strongly borrows from the first part, making use of the Nash bargaining solution as the solution concept to the distributional conflict. We provide close-form expressions for both equilibrium individual provision levels and socially optimal provision levels. Even in the case that all pairwise externalities are positive, there is a gap between equilibrium provision and socially efficient provision.

Results. We restrict our analysis to the more interesting case of regular economies. Regular economies are such that any allocation that exhausts resources is Pareto efficient. In regular economies, the Pareto frontier is non-degenerate.

For such economies, we characterize completely the Nash bargaining solution, providing closed-form expressions for the utilities and shares obtained.

The reason for which we restrict to regular economies is the following. In non regular economies the Pareto frontier is degenerate. This is so because the influence exerted by some given agent on others is much bigger than the influence he receives in return. Efficiency might require then that this agent receives all the resources available. If an individual in a group is so much loved by anybody else compared with other possible affective relations, which means that the influence this agent has on others' utility is very large, it could be efficient to give him all the resource.

Similarly, there might exists intermediate situations in which only some agents are allowed to receive a share of the resource for efficiency reasons. While our methods and analysis could be extended to such nonregular situations, we focus on the analysis of regular economies in which the pattern of influences excludes *de facto* some agents from the course for some part of the resources.

We first characterize regular economies. This characterization is twofold. First, regular economies are characterized by an upper bound on the aggregate level of bilateral influences every agent exerts on others. In terms of direct bilateral influences, we obtain a bound on the maximal level of aggregate direct influences an agent can exert. The economy is regular if and only if no agent exceeds this bound. Second, regular economies are characterized by conditions on the pattern of network externalities. More precisely, an economy is regular if and only if all agents are equally central in the network structure of influences. The relevant measure of network centrality is the Katz-Bonacich centrality index, pervasively used in the sociology literature, and that also arises naturally in other economic settings.

We next provide a constructive procedure to characterize the Nash bargaining solution. It is important to note that even if the economy is regular and, hence, the distributional conflict involves all agents in the economy, this does not exclude the possibility that some agents obtain finally no share of the resource. Externalities do not directly solve the bargaining problem but this does not mean that they can not be sufficiently asymmetric such that, after internalizing all influences, the Nash bargaining solution assigns nothing to some of the agents.

We also devote part of our work to the analysis of α-economies. In these economies all existent pairwise influences have intensity equal to α, and whenever an agent exerts an influence on another, this other agent also exerts an influence on the first one. One of the two dimensions of heterogeneity in the model, the possibly different levels of externality intensities across individuals, is kept to a minimum. The main source of heterogeneity is the geometry of the pattern of pairwise influences.

An analysis in depth of this family of economies gives a better picture of how the particular arrangement of pairwise relations, irrespective of the intensities of these, impacts on bargaining outcomes. In particular, utility is directly related with the number of connections an agent has. Those agents that receive and exert more influences are also rewarded with larger utility levels. However, this monotonicity does not necessarily translates into receiving larger fractions of the resource.

We also provide comprative results with respect to the primitives of the mode. We study how changes on the pattern of influences distort the Nash bargaining outcome. We analyze how changes in the levels of bilateral influences change the bargaining result. Furthermore, we also discuss how our framework can be used to describe and analyze situations in which some agents that are not involved in the bargaining game can affect the bargaining outcome.

Related Literature. Our model bears a formal resemblance with previous work on interdependent utilities by Bergstrom (1999) and Bramoullé (2001). Bramoullé also interprets this type of systems in terms of weighted and directed networks, but focuses on some qualitative features of the mapping from bilateral influences to network externalities.[3] Here instead, we analyze the mapping from bilateral influences to Nash bargaining utilities and agreed shares, providing closed-form expressions for both.

A strategic model on status in networks that generates similar interdependency systems is provided in Rogers (2005). Rogers analyzes a network formation game in which agents with heterogeneous skills can choose with whom they want to contact and with which intensities they want that this contact is made. Hence, the pattern of influences is endogenously chosen. We analyze instead situations in which the struc-

[3]For a complementary approach to this mapping and a general characterization of Pareto efficiency in such a setup, see de Martí (2006).

ture of pairwise influences can not easily be affected by individual strategic decisions. For example, in the urban crime example no neighbourhood can do much to delimit influences among them, since these are largely determined by private decisions and actions of the population, which is an issue out of their control. A similar comment applies on the example on interdepartmental influences in a government. Also, altruism and envy are not only the result of strategic decisions but the effect of the embeddedness of individuals on a social environment they can not determine and control.

The model also resembles input-output models of linear economies (Leontief, 1951, Gale, 1960). However, input-output models only allow for positive bilateral influences, while here we do not impose sign restrictions of any sort. Of course, we also deal with different issues.

Some papers have analyzed multilateral bargaining with externalities from a non-cooperative viewpoint. Jehiel and Moldovanu (1995a, 1995b) consider a setup where one seller bargains with n potential buyers to decide which of them obtains the unit of an indivisible good. The acquisition of the good by one of the agents can exert a positive or negative externality on others. They analyze how the bargaining outcome is affected by this allocative externality.

In a political economy context, Calvert and Dietz (2004) explore how the introduction of externalities in a 3-agent economy alters the conclusions of the Baron and Ferejohn (1989) non-cooperative game of legislative bargaining. See Duggan (2004) for conditions about existence of equilibria in the n agents version of the Baron and Ferejohn game with externalities.

While our cooperative approach is less sensitive to possible particularities in the bargaining process, such as the particular mechanisms by which buyers and sellers bargain or the existence of a voting rule (the majority rule in Baron-Ferejohn models) in legislative bargaining, it allows for a general and tractable analysis of multilateral bargaining with one unique outcome prediction and an heterogeneous pattern of externalities.

Our work also borrows from the very active literature on networks in economics. However, we do not deal with the formation of social and economic networks, maybe the more extensively studied issue in the field, but on games played in a fixed network.[4] Other authors have also explored the interrelation of network structure and bargaining outcomes (see Calvó-Armengol, 2001, and Corominas-Bosch, 2004). The approach in these papers is different in many respects. Just to mention a few, bargaining is not among the many and the network represents communication restrictions and delimits the possible pairs of agents that can trade.

The Katz-Bonacich centrality measure was first defined by Katz (1953) and later on developed by Bonacich (1987). It is one of the more relevant centrality measures studied in the active field of social network analysis.[5] Another game played in a network, in this case not a bargaining game, where this centrality measure naturally arises is Ballester et al. (2006). Agents play a game with pairwise dependent strategic complementarities. In the unique equilibrium of the game each agent action is proportional to his Katz-Bonacich centrality index measured on this network of complementarities.

With regards to the analysis of provision of public resources, the work that is closest to our one is Bloch and Zenginobuz (2006). We differ from the analysis of in two different aspects: first, that in their model each jurisdiction/neighbourhood creates independently its own public good, from which other jurisdictions can benefit; second, we allow for heterogeneity in wealth. Hence, there are two dimensions of possible

[4]See Jackson (2005) for a very extensive survey of the field of networks in economics, and for an exhaustive list of references about games played in networks, including bargaining games.

[5]For an exhaustive survey of this literature see Wasserman and Faust(1994).

heterogeneity, spillovers and wealth distribution.

2 Bilateral Influences and Network Externalities

2.1 Modelling Bilateral Influences

In this section we propose a simple framework that allows to model positive and negative allocative influences across individuals.

Suppose that there is an amount of a certain resource to be distributed within a group of n individuals, $\mathcal{N} = \{1, \ldots, n\}$. Let c_i be the consumption of agent $i \in \mathcal{N}$. Let $b_{ij} \in \mathbb{R}$ be the magnitude of the influence agent j exerts on agent i. Then, an increase of one unit of welfare for agent j induces an increase of b_{ij} units of welfare for agent i. Given a profile $\mathbf{c} = (c_1, \ldots, c_n)$, the utility an agent obtains , $u_i(\mathbf{c})$, is equal to

$$u_i(\mathbf{c}) = c_i + \sum_{j \neq i} b_{ij} u_j(\mathbf{c}) \qquad i = 1, \ldots, n \tag{1}$$

This set of equations forms what we call the bilateral influences system. Note that the relation in this system is from outcomes to outcome.

In terms of the urban crime example, b_{ij} represents how the crime rate , not the share of public budget received, in neighborhood j affects the crime rate in neighborhood i.

Due to linearity, we can fix the sum of consumption levels to $\sum_{i=1}^{n} c_i$ to be equal to 1. Hence, we can interpret c_i as the share of the resource received by agent i.

Defining $b_{ii} = 0$ for all $i \in \mathcal{N}$, we gather all the b_{ij} in a matrix $\mathbf{B}$ of bilateral influences. An *economy* is completely characterized by its matrix of bilateral influences.

For a given economy $\mathbf{B}$, we can obtain from the structural system of bilateral influences to a reduced-form system where the utility of each agent can be directly expressed in terms of the shares profile, eliminating the dependency on other's utility.

The bilateral influence system in matrix form is equal to

$$\mathbf{u}(\mathbf{c}) = \mathbf{c} + \mathbf{B} \cdot \mathbf{u}(\mathbf{c}) \tag{2}$$

Hence, if $\mathbf{I}$ is the identity matrix, whenever $(\mathbf{I} - \mathbf{B})^{-1}$ exists we obtain the reduced-form system

$$\mathbf{u}(\mathbf{c}) = (\mathbf{I} - \mathbf{B})^{-1} \cdot \mathbf{c} \tag{3}$$

The first result we provide is a genericity result. An economy is characterized by $n(n-1)$ real values. Therefore, there is a one-to-one mapping from economies to elements of $\mathbb{R}^{n(n-1)}$. From this point of view, the set of economies in $\mathbb{R}^{n(n-1)}$ for which $(\mathbf{I} - \mathbf{B})^{-1}$ does not exist has (Lebesgue) measure zero. This implies that for almost every economy $\mathbf{B}$, the associated matrix $(\mathbf{I} - \mathbf{B})^{-1}$ exists (and, of course, is unique) and the next result then follows.

Proposition 1 *For almost every economy* $\mathbf{B}$ *the associated reduced-form system is uniquely characterized.*

In words, given a structural system of bilateral influences there is no indeterminacy in the obtaining of the associated reduced-form expression, except for a negligible set of economies.[6]

[6] See Bramoullé(2001) for structural models of a similar nature for which indeterminacy in the determination of the associated reduced-form system arises.

Let $\mathbf{E}(\mathbf{B}) = (\mathbf{I} - \mathbf{B})^{-1}$. Each entry $e_{ij}(\mathbf{B})$ expresses the magnitude of how the utility increases, if the entry is positive, or decrease, if the entry is negative, when the level of consumption of agent j increases. We call $\mathbf{E}$ the matrix of *network externalities*. An explanation for the choice of this name follows.

2.2 From Bilateral Influences to Network Externalities

Any economy $\mathbf{B}$ can be naturally represented by a network.

A network is formed by a set of nodes and a set of links that express a relation between the pair of nodes linked. While this is an abstract object, it is a useful metaphor to represent many varied situations in applied settings. In particular, in our case this metaphor can be applied to make nodes represent the agents involved in the structural system of bilateral influences, and make links represent the pattern of bilateral influences exerted across pairs of agents. A link in such an influence network is weighted, each link has an associated value that represents the strength of the influence this link represents, as well as a particular direction, since the influence agent i exerts on j does not necessarily coincides in strength with the influence agent j exerts on agent i, and hence we have to distinguish the link from i to j and the link from j to i.

Different conventions could be adopted to express the mapping from economies to networks. We adopt the following one. We say that there is a link from agent i to agent j whenever j exerts a, positive or negative, influence on i, and the weight for this link is then equal to the coefficient $b_{ij} \in \mathbb{R}$ of the structural system of bilateral influences. Since in our model there is no self-influence we do not allow for self-loops, links from an agent to itself. The set links that begin in i point to the agents that influence agent i.

Observe the weighted and directed nature of the network defined in this way: since we have not imposed any restriction on the possible values of the coefficients in the structural influence system, the weight of a link can take any real value; also, since we have not imposed symmetry on the levels of bilateral influence, it is possible that there exist both a link from i to j and another one from j to i and that their respective weights differ. Even more, it is possible that there exist a link from i to j while there is no link from j to i.

A weighted and directed network is defined by an adjacency matrix, where the entry (i, j) in this matrix is equal to the weight of the link from i to j. This weight equals the level of bilateral influence j exerts on i. Hence, given an economy $\mathbf{B}$ the adjacency matrix of its associated network, in the way we have defined this network, is also $\mathbf{B}$.

The following equality applies

$$\mathbf{E}(\mathbf{B}) = (\mathbf{I} - \mathbf{B})^{-1} \tag{4}$$

Whenever $\mathbf{B}$ is a contraction[7] we have that

$$(\mathbf{I} - \mathbf{B})^{-1} = \sum_{k=0}^{+\infty} \mathbf{B}^k \tag{5}$$

If j exerts an influence on i with weight b_{ij} and k exerts an influence on j with weight b_{jk}, k exerts an indirect influence on i with weight equal to $b_{ij}b_{jk}$. The matrix $\mathbf{B}^2$ keeps track of these second order network influences. The entry $b_{ik}^{[2]}$ of $\mathbf{B}^2$ computes the sum of weights of all paths of length two from i to k.[8]

[7] The matrix $\mathbf{B}$ is a contraction if and only if all its eigenvalues have norm smaller than 1. This will be the case for example for the set of *regular* economies, that we define later, if bilateral influences are positive.

[8] A *path* between i and j in network $\mathbf{G}$ is a sequence of agents $i_1, \ldots, i_K$ of calN, where an agent can appear several times in this sequence, such that $i_k i_{k+1}$ is a link of $\mathbf{G}$ for every $k \in 1, \ldots, K-1$, with $i_1 = i$ and $i_K = j$. The length of such a path is equal to $K - 1$, the number of links that form the path. In words, a path in g is an indirect connection from agent i to agent j through linked agents

More generally, for any $l \geq 1$ the matrix $\mathbf{B}^l$ keeps track of the l-order network influences: each entry $b_{ik}^{[l]}$ equals the sum of weights of all paths of length l from i to k.

Therefore, whenever the expression in equation (1.1) is valid, the entry $e_{ij}(\mathbf{B})$ of $\mathbf{E}(\mathbf{B})$ is the sum of weights of *all* paths from j to i in the network represented by the economy/adjacency matrix $\mathbf{B}$. The matrix $\mathbf{E}(\mathbf{B})$ computes the sum of indirect (network) effects that the pattern of bilateral influences generates. This sum of indirect effects of any order is what we denote *network externalities*, and this is why we call matrix $\mathbf{E}(\mathbf{B})$ the matrix of *network externalities*.

Each entry $\mathbf{e}_{ij}(\mathbf{B})$ represents by how much the consumption of agent j affects the utility of agent i not only through the direct bilateral influence agent j exerts on i, represented by b_{ij}, but also through the indirect influences resulting of all possible indirect network connections from j to i.

The following example is useful to understand how important are indirect network effects for the analysis of the mapping from allocations to utilities defined in the reduced-form system.

Example 1. There are three agents, $\mathcal{N} = \{1, 2, 3\}$, and the structural system of bilateral influences relating them is

$$
\begin{aligned}
u_1(\mathbf{c}) &= c_1 + \alpha u_2(\mathbf{c}) \\
u_2(\mathbf{c}) &= c_2 + \beta u_3(\mathbf{c}) \\
u_3(\mathbf{c}) &= c_3
\end{aligned}
$$

where $\alpha > 0$ and $\beta < 0$. This means that both agent 3 exerts a negative direct influence on agent 1, while agent 2 exerts a positive influence on agent 2. Besides, the values for these direct bilateral influences are α and β.

The network that represents this situation is

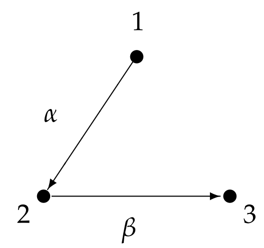


Figure 1

and the 3×3 bilateral influences matrix is

$$
\mathbf{B} = \begin{pmatrix} 0 & \alpha & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{pmatrix}
$$

in $\mathbf{B}$. We define the *weight* of a path $i_1, \ldots, i_K$ of $\mathbf{G}$ as the product $g_{i_1 i_2} \cdots g_{i_{K-1} i_K}$. This weight is different than zero because of the definition of path. A path such that $i = j$ is called a *cycle*.

Hence, the 3×3 matrix of network externalities, $\mathbf{E}(\mathbf{B}) = (\mathbf{I} - \mathbf{B})^{-1}$, is equal to

$$\mathbf{E}(\mathbf{B}) = \begin{pmatrix} 1 & \alpha & \alpha\beta \\ 0 & 1 & \beta \\ 0 & 0 & 1 \end{pmatrix}$$

Observe that in this case we can easily compute the matrices of indirect network effects, $\mathbf{B}^k$ for $k \geq 2$. The matrices of higher order network effects are

$$\begin{aligned} \mathbf{B}^2 &= \begin{pmatrix} 0 & 0 & \alpha\beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \mathbf{B}^k &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{for every } k \geq 3 \end{aligned} \tag{6}$$

Therefore, $\mathbf{E}(\mathbf{B}) = \mathbf{I} + \mathbf{B} + \mathbf{B}^2$. The expressions of utilities in terms of consumption are therefore

$$\begin{aligned} U_1(\mathbf{c}) &= c_1 + \alpha c_2 + \alpha\beta c_3 \\ U_2(\mathbf{c}) &= c_2 + \beta c_3 \\ U_3(\mathbf{c}) &= c_3 \end{aligned}$$

Observe that the weight of the network externality agent 3 exerts on agent 1, $e_{13} = \alpha\beta$, depends on the intensities of bilateral influences.

We omit the dependence of $\mathbf{E}$ on $\mathbf{B}$ when no confusion is possible.

3 The Set of Pareto Allocations

3.1 Characterization

From now on we will consider that there is a certain amount of a resource that, without loss of generality, we normalize to one. Before turning to the study of distributional conflict and how agents in an economy agree to divide this unit of resource among them, we have to make a clarification about the set of Pareto efficient allocations in an economy with influences. Externalities can have severe consequences on which allocations can be Pareto efficient. Our aim in this section is to characterize the set of economies for which distributional conflict is particularly strong.

Before providing an example of the peculiar situations that can arise in economies with influences we describe the utility possibility set for any economy $\mathbf{B}$, that we denote $\text{UPS}(\mathbf{B})$. Given an economy $\mathbf{B}$, and for any feasible allocation, we have that $\mathbf{u}(\mathbf{c}) = \mathbf{E} \cdot \mathbf{c} = \sum_{i=1}^{n} c_i \mathbf{e}^{(i)}$, where $\mathbf{e}^{(i)}$ is the i-th column vector of the matrix of network externalities. Since an allocation $\mathbf{c}$ is feasible if and only if $c_i \geq 0$ for every $i \in \mathcal{N}$ and $\sum_{i=1}^{n} c_i \leq 1$, we can conclude that the utility possibility set for the economy defined by $\mathbf{B}$ is the convex hull of the columns of the matrix of network externalities $\mathbf{E}$ plus the zero vector, that is

$$\text{UPS}(\mathbf{B}) = co\left\{\mathbf{e}^{(1)}, \ldots, \mathbf{e}^{(n)}, \mathbf{0}\right\}$$

This implies that the utility possibility set for any economy **B** is a simplex, and therefore it is a convex and compact set.

Example 2. Consider the economy represented by the following network

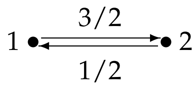


Figure 2

The 2×2 matrix of bilateral influences is

$$\mathbf{B} = \begin{pmatrix} 0 & 3/2 \\ 1/2 & 0 \end{pmatrix}$$

It follows that the matrix of network externalities for this economy is equal to

$$\mathbf{E}(\mathbf{B}) = \begin{pmatrix} 4 & 6 \\ 2 & 4 \end{pmatrix}$$

Here $\mathbf{e}^{(1)} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\mathbf{e}^{(2)} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ and the utility possibility set for this economy is the convex hull of these two vectors and the zero vector. We can depict UPS $(\mathbf{B})$

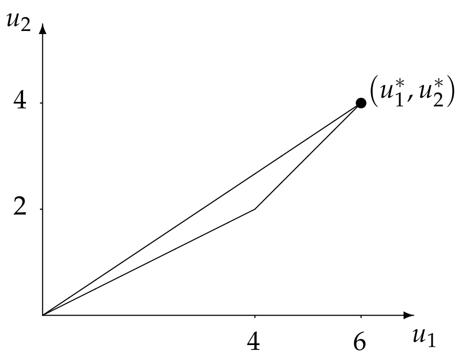


Figure 3

It turns out that the unique efficient allocation in this economy is (u_1^*, u_2^*), that corresponds to $(c_1^*, c_2^*) = (0, 1)$.[9] At the unique Pareto efficient allocation agent 2 receives all the resource. This is so because the magnitude of the externality agent 2 exerts on agent 1 is much larger than the one of the externality agent 1 exerts on agent 2. The effect of an increase in the level of consumption of agent 2 is then larger in agent's 1 utility than the effect of an increase in his own level of consumption.

When considering such kind of situations the implications on the solution to the Nash bargaining problem is immediate. Since the Nash bargaining solution has to be Pareto efficient and there is a unique Pareto efficient allocation, the Nash bargaining problem is trivially solved.

[9] Recall that we have normalized the total amount of resources to one. This is, of course, without lose of generality because of linearity.

We will disregard the kind of economies that we have just described. Indeed, we will concentrate from now on in the completely opposite kind of situations. We will only consider economies where any allocation that exhausts available resources is Pareto efficient. When this happens we say that the economy is *regular*.[10] The assumption of a regular economy ensures there is a nontrivial bargaining problem and there exists competition among all agents to obtain some share of the unit of resources.

The next result provides a complete characterization of regular economies in terms of the matrix of network externalities. Before stating it we define a useful notion for the analysis in the rest of the paper.

Definition *We say that an n-dimensional vector $\boldsymbol{\mu}$ is a strict system of weights if and only if $\mu_i > 0$ for every $i \in \mathcal{N}$ and $\sum_{i=1}^{n} \mu_i = 1$.*

Now we provide the first characterization result of regular economies.

Proposition 2 *An economy $\mathbf{B}$ is regular if and only if there exists a unique strict system of weights $\boldsymbol{\mu}$ and a positive constant $\kappa > 0$ such that $\boldsymbol{\mu} \cdot \mathbf{e}^{(i)}(\mathbf{B}) = \kappa$ for every $i \in \mathcal{N}$.*

The previous result characterizes regularity through the matrix of network externalities. Each element of column $\mathbf{e}^{(i)}$ expresses how large is the network externality agent i exerts on each agent. When we compute the weighted average of these elements we obtain a single value that expresses an overall measure of network externalities exerted by agent i. Hence, Proposition 2 says that regularity amounts to find normalized weights, which are unique and depend on the economy we are analyzing, such that this overall measure of network externalities each agent exerts is positive and equal for all agents.

It is also possible to provide a characterization of regular economies in terms of the primitives of the economy, i.e. the matrix $\mathbf{B}$ of bilateral influences.

Proposition 3 *A necessary and sufficient condition for an economy $\mathbf{B}$ to be regular is that*

$$\sum_{j=1, j\neq i}^{n} b_{ji} < 1 \quad for\ every\ i \in \mathcal{N}$$

This result provides a simple and direct way to check if an economy is regular, and it helps us to understand better which are the network forces that induce regularity. It states that regularity amounts to requiring that the aggregate level of bilateral influences each agent exerts on others, $\sum_{j=1, j\neq i}^{n} b_{ji}$, is not too large, in fact not larger than 1.

For example, consider an economy where all agents are connected to each other and the level of bilateral influence across any pair of individuals is equal to certain value α. In this case the necessary and sufficient condition for such kind of economy to be regular is that $\alpha < \frac{1}{n-1}$. this condition is satisfied if $\alpha < 0$, and only if bilateral influences are positive the condition bounds α. This is quite natural. When influences exerted are negative, distributional conflict naturally arises because each agent wants the rest of the economy to receive the smallest possible share. Any allocation that exhausts resources is in this case Pareto efficient. Instead, when influences are positive and sufficiently large each agent would prefer that other agents receive all the

[10]Observe that this is not the unique other possible situation. It could be that only a subset of agents should consume for an allocation to be Pareto efficient. In the next chapter we provide a generic characterization of all possible situations in terms of endogenous centrality measures derived from the position of each agent in the network of bilateral influences.

resource since this would increase more his utility than receiving himself part of it, and the Pareto frontier might degenerate to a single point.

Consider a regular economy **B**. Then we can compute easily its associated strict system of weights $\boldsymbol{\mu}$ and constant κ from Proposition 2.[11]

Lemma 1 *Let B be a regular economy. Define* $\delta_i = 1 - \sum_{j \neq i} b_{ji}$*,* $i \in \mathcal{N}$*, and* $\delta = \sum_i \delta_i$*. Then,* $\kappa = 1/\delta$ *and* $\mu_i = \kappa \delta_i$ *for all* $i \in \mathcal{N}$*.*

The value of δ_i is negatively related to the level of aggregate bilateral influences agent i exerts. Hence, μ_i is smaller the larger this aggregate level of bilateral influences is. The next section studies more in detail how these constants relate to the particular network structure of influences, and we provide interpretations for them.

3.2 Efficiency and Network Centrality

We can provide an alternative interpretation to the efficiency characterization in Proposition 2. To this end, we have to introduce some terminology derived from the literature on social networks.

Given a network we can try to measure the prominence of each agent due to her position in the network. There are several variables that can determine the prominence of an actor in a network. Furthermore, the definition of prominence may depend on the setting we are studying. It is not the same if we deal with directed or undirected networks, or with weighted or unweighted networks. Hence, there is not in the social network analysis literature a unique standard definition of prominence.

The more usual concept to analyze prominence in networks is centrality. It is fairly natural to associate prominence with connectivity and this is what centrality measures do. In the case of weighted and directed networks, as the ones we are considering in our analysis, a rough measure of centrality of agent i would be the sum of weights of the links that point to agent i, $S_i = \sum_{j \neq i} b_{ji}$.[12]This measure is called the degree centrality of agent i. In terms of our structural influence model, S_i measures the aggregate level of influence that emanates from agent i.

Note that $\delta_i = 1 - S_i$, which is a positive quantity whenever the economy **B** is regular, is then a complementary degree centrality index for agent i. Its value is smaller the largest is the degree centrality measure S_i. Therefore, μ_i is also a complementary centrality index, since it is a renormalization of δ_i to make the sum of the indices for all agents to add up to one.

While this degree centrality is informative of some kind of prominence derived by the way influences vary across pairs, it does not capture the value of how these influences spread indirectly along chains of bilateral influences.

Remember that, as we have explained before, given an economy **B** we have that for any $l \geq 1$ the matrix $\mathbf{B}^l$ keeps track of the l-order network externalities: each entry $b_{ij}^{[l]}$ equals the sum of weights of all paths of length l from i to j. Hence, to construct a more elaborate centrality measure we might include these indirect network effects subsumed in the sequence of matrices $\left\{\mathbf{B}^l\right\}_{l \geq 1}$. A natural way is to consider a decay factor

[11]The proof of Lemma 1 is contained in the proof of Proposition 3.

[12]This is an inner-centrality measure. Alternatively, we could define an outer-centrality measure by the sum of weights of the links that start in agent i. Since, as we will show in a moment, in our analysis the relevant centrality measure is an inner measure, we avoid this possible distinction in the text.

$\lambda \in (0,1]$ and weight the l-order network effects by λ^l. This is the Katz-Bonacich centrality measure. The (unweighted) Katz-Bonacich inner-centrality[13] measure vector, $\boldsymbol{\kappa}(\mathbf{B};\lambda)$ is defined as:

$$\boldsymbol{\kappa}(\mathbf{B};\lambda) = \left(\sum_{l=0}^{\infty} \lambda^l \mathbf{B}^l\right)^t \cdot \mathbf{1}$$

Whenever this vector is well-defined we can rewrite it as:

$$\boldsymbol{\kappa}(\mathbf{B};\lambda) = \left[(\mathbf{I} - \lambda \mathbf{B})^{-1}\right]^t \cdot \mathbf{1}$$

A variation of this measure, called the weighted Katz-Bonacich centrality measure, is the following.

Let $\boldsymbol{\mu}$ be an strict system of weights. Then the $\boldsymbol{\mu}$-weighted Katz-Bonacich centrality measure, ˇ$_{\boldsymbol{\mu}}$ ($\mathbf{B}$;ˇ), is given by the following formula:

$$\boldsymbol{\kappa}_{\boldsymbol{\mu}}(\mathbf{B};\lambda) = \left[(\mathbf{I} - \lambda \mathbf{B})^{-1}\right]^t \cdot \text{¯}$$

In the unweighted Katz-Bonacich centrality measure all agents count the same when considering the sum of network effects generated by each one of them. In the $\boldsymbol{\mu}$-weighted Katz-Bonacich centrality measure the network effects generated by agent i are counted with weight μ_i. Some agents count more than others when aggregating the whole matrix of network effects $\mathbf{E}(\mathbf{B})$.

After this digression into the realm of social and economic networks, we can reinterpret the condition of proposition 3 making use of weighted Katz-Bonacich centrality measures. The condition is equivalent to say that there exists a unique strict system of weights ¯ such that

$$\left[(\mathbf{I} - \mathbf{B})^{-1}\right]^t \cdot \boldsymbol{\mu} = \kappa \mathbf{1}$$

with κ being a positive constant. The reader can immediately recognize the $\boldsymbol{\mu}$-weighted Katz-Bonacich centrality measure, with $\lambda = 1$, in the left handside of the last equation. Hence the regularity condition says that there exists a vector of weights for which the weighted Katz-Bonacich centrality measure is equal, and positive, for all agents. This individual index measures the aggregate level of network influence effects that i generates. These are represented by the paths on the network that finish on i, and this is exactly what the Katz-Bonacich centrality index takes into account.

Two comments are in order. First, observe that our model generates endogenously the unique system of weights $\boldsymbol{\mu}$ for which this centrality condition is satisfied. This is Proposition 3. Second, the decay factor is equal to 1, and hence direct and indirect influences count the same to compute this measure of prominence. Hence, we can rewrite proposition 2 as follows

Proposition 2′ *The economy* $\mathbf{B}$ *is regular if and only if there exists a unique strict system of weights* $\boldsymbol{\mu}$ *and a constant* $\kappa > 0$ *such that*

$$\boldsymbol{\kappa}^i_{\boldsymbol{\mu}}(\mathbf{B};1) = \kappa \qquad for\ all\ i \in \mathcal{N}$$

A general characterization, not only for regular economies, of Pareto efficiency in economies with pairwise influences by means of centrality measures can be in the next chapter.

[13]It is an inner measure of centrality because it measures weights of paths and cycles that end on each agent. An outer-centrality measure could be defined without transposing in the following equation.

4 Bargaining and Influences

4.1 The Bargaining Problem and its Solution

From now on, we consider only regular economies with influences. We turn to the study of distributional conflict for these economies.

We consider the classical and widely used Nash bargaining solution (Nash, 1950). Following this seminal work we define an n-person bargaining problem as a duple $\langle X, \mathbf{d} \rangle$, where $X \subset \mathbb{R}^n$ is a convex and compact set that expresses the utility possibility set in the economy, and $\mathbf{d} \in X$ is the disagreement point, that expresses the utilities each agent would obtain in case they are not able to reach an agreement. The disagreement point has to satisfy the following dominance condition: there exists $\mathbf{v} \in X$ such that $\mathbf{v}$ strictly Pareto dominates $\mathbf{d}$, i.e. $v_i > d_i$ for every $i \in \mathcal{N}$. The (symmetric)[14] Nash bargaining solution $\boldsymbol{x}^S = \left(x_1^S, \dots, x_n^S\right)$ to $\langle X, \mathbf{d} \rangle$ is the solution to the following maximization problem

$$\max_{x \in X} \prod_{i=1}^{n} (x_i - d_i)$$

Due to convexity of the utility possibility set X and strict convexity of the objective function this problem has a unique solution.

We want to analyze this Nash bargaining solution in the case the utility possibility set X is induced from a regular economy with influences. Observe this is possible since as we mentioned before UPS $(\mathbf{B})$ is convex and compact for any economy $\mathbf{B}$.

Given an economy $\mathbf{B}$, let $\mathbf{u}^{min} = \left(u_1^{min}, \dots, u_n^{min}\right)$ be the utility vector where each entry u_i^{min} is equal to the minimal utility agent i can obtain within the set of efficient allocations of economy $\mathbf{B}$.[15] Since we assume that the economy is regular, from Proposition 2 we know that there exist only one strict system of weights $\boldsymbol{\mu}$ and one positive constant κ such that $\boldsymbol{\mu} \cdot \mathbf{e}^{(i)} = \kappa$ for every $i \in \mathcal{N}$. Given a disagreement point $\mathbf{d}$ we relabel agents from 1 to n such that $\mu_1 \left(u^{min} - d_1\right) \geq \cdots \geq \mu_n \left(u_n^{min} - d_n\right)$. Finally, let

$$\psi^{(0)} = \frac{1}{n} \left(\kappa - \boldsymbol{\mu} \cdot \mathbf{d}\right)$$

and let

$$\psi^{(j)} = \frac{1}{n-j} \left(\kappa - \boldsymbol{\mu} \cdot \mathbf{d} - \sum_{k=1}^{j} \mu_l \left(u_k^{min} - d_k\right)\right)$$

for any $j \in \{1, \dots, n-1\}$.

Now we have all the necessary ingredients to characterize the Nash bargaining solution for any regular economy with influences. This is done in the following result.

Proposition 4 *Consider a regular economy* $\mathbf{B}$*. Then, there exists* $j \in \{0, \dots, n-1\}$ *such that the utility vector associated with the Nash bargaining solution,* $\mathbf{u}^S$*, is*

$$u_i^S = u_i^{min} \qquad \text{if } i \leq j$$

[14] To simplify the analysis we only consider the symmetric Nash bargaining solution. The analysis for asymmetric Nash bargaining solutions with heterogeneous bargaining power is completely analogous.

[15] In fact, $\mathbf{u}_i^{min} = \min\left\{e_{i1}(\mathbf{B}), \dots, e_{in}(\mathbf{B})\right\}$, so this vector of minimal utilities can easily be derived from the group influence matrix.

and

$$u_i^S = d_i + \psi^{(j)} \frac{1}{\mu_i} \qquad if\ i > j$$

The Nash bargaining solution allocation is equal to $\mathbf{c}^S = (\mathbf{I} - \mathbf{B})\,\mathbf{u}^S$.

This result characterizes the utilities and levels of consumption of the Nash bargaining solution for any regular economy. In particular, it characterizes corner, partially corner and interior solutions.

For an individual i that obtains positive gains, $u_i^S - d_i > 0$, these gains are inversely proportional to μ_i. If both i and j obtain positive gains we have that

$$\frac{u_i^S - d_i}{u_j^S - d_j} = \frac{1 - \sum_{k \neq j} b_{kj}}{1 - \sum_{k \neq i} b_{ki}} \tag{7}$$

The relative gains of agent i with respect to those of agent j uniquely depend on the level of aggregate influence exerted by agent i and agent j. In particular, the largest is the magnitude of aggregate influence that emanates from agent i compared with those that emanate from agent j, the largest the relative gains of i with respect to j.

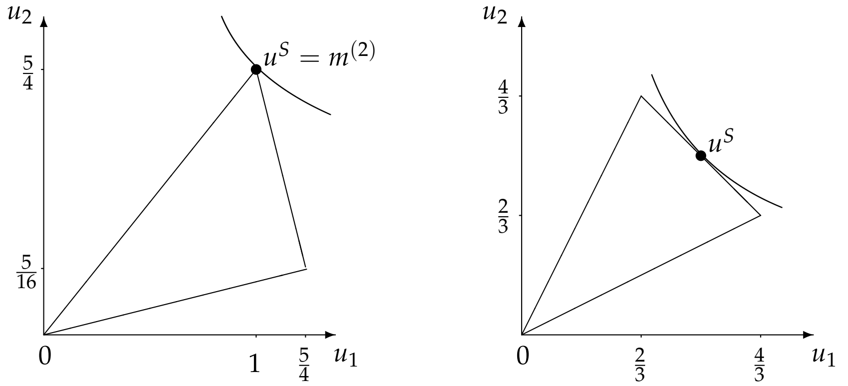
Aggregate influence levels determine relative gains for those agents that obtain positive gains. This does not mean that these levels form the unique relevant information from the structural influence model to characterize the Nash bargaining solution. The minimal utilities profile, $\mathbf{u}^{\min}$, and therefore the multiplier $\psi^{(j)}$, can not be expressed in terms of the aggregate influence levels. Indeed, minimal utilities internalize all levels of network influence effects, since u_i^{min} equals the minimal entry in i's row of matrix $\mathbf{E}(\mathbf{B})$. In non-interior solutions where some agents obtain no gains from bargaining the information from the matrix of network externalities is fundamental for the characterization of the Nash bargaining outcome.

The multiplier $\psi^{(j)}$ represents the remaining surplus, the remaining value of the available unit of resources in terms of utilities, once we subtract the minimal utilities some of the agents obtain (agents $k \leq j$). The rest of this remaining value is shared proportionally to the inverse of entries of $\boldsymbol{\mu}$.

Example 3. The analysis of the following two economies illustrate the characterization we have just provided in a 2-agents setting. Economy (a) is such that $b_{12} = 4/5$ and $b_{21} = 1/4$ while economy (b) is such that $b_{12} = b_{21} = 1/2$. The matrices of network externalities for each economy are

$$\mathbf{E}_{(a)} = \begin{pmatrix} 5/4 & 1 \\ 5/16 & 5/4 \end{pmatrix} \qquad \mathbf{E}_{(b)} = \begin{pmatrix} 4/3 & 2/3 \\ 2/3 & 4/3 \end{pmatrix}$$

The utility possibility set with the respective Nash bargaining solution depicted, when the disagreement point is $\mathbf{d} = \mathbf{0}$, in both cases are



(a) **Figure 4** (b)

In example (a) we obtain a corner solution. One agent receives all the resource as the solution to the distributional conflict. We have that $\delta_1 = 3/4$, $\delta_2 = 1/5$, and therefore $\delta = 19/20$. The associated constant and strict system of weights from Proposition 2 are $\kappa = 20/19$ and $\mu_1 = 15/19$, $\mu_2 = 4/19$. The minimal utility each agent can obtain in a Pareto efficient allocation is $u_1^{min} = 1$ and $u_2^{min} = 5/16$. In this case the j from proposition 4 equals 1, and the multiplier is $\psi^{(1)} = 5/19$. In the solution, agent 1 receives nothing and agent 2 receives all the resource. Agent 1 obtains his minimal utility, $u_1^s = 1$, while agent 2, instead, obtains $u_2^S = \psi^{(1)} \frac{1}{\mu_2} = \frac{5}{19}\frac{19}{4} = \frac{5}{4}$, that equals his maximal possible utility within the set of efficient allocation.

In example (b) we obtain an interior solution. Both agents obtain a utility within their minimal and maximal utility. In this case, we have that $\delta_1 = \delta_2 = 1/2$,and therefore $\delta = 1$. The associated constant and strict system of weights from proposition 2 are therefore $\kappa = 1$ and $\mu_1 = 1/2$, $\mu_2 = 1/2$. The minimal utility each agent can obtain in an efficient situation is $u_1^{min} = u_2^{min} = 2/3$. We get that the j from proposition 4 equals 0 and the multiplier is $\psi^{(0)} = 1$. Hence, each agent obtains a utility equal to $u_1^s = u_2^s = \psi^{(0)} \frac{1}{\mu_i} = 2$.

4.2 Discussion

4.2.1 The Disagreement Point

As stressed in Binmore et al. (1996), the choice of a particular disagreement point entails also some of the features of the bargaining process when considering the Nash bargaining solution. In our case two different possible disagreement points emerge as the more natural choices.

The first one is the choice of $\mathbf{d} = \mathbf{0}$. This would be the natural choice when in the bargaining situation there are time concerns. In this case, agents know that an agreement could be infinitely delayed and therefore that they could obtain no utility at all. This time concerns are modelled by means of the choice of the zero vector as the disagreement point.

Another possibility is to choose $\mathbf{d} = \mathbf{u}^{min}$, that is that d_i coincides with the minimal utility agent i can obtain in an efficient situation, u_i^{min}. In a regular economy, this vector of minimal utilities satisfies the property of domination that the disagreement point has to satisfy. Observe that this disagreement point derives endogenously from the pattern of influences expressed by matrix $\mathbf{B}$. It might be a natural selection when considering situations in which no time concerns exist. When Pareto efficiency is a requirement of the solution to the distributional conflict, as it is in the case of the Nash bargaining solution, agent i might make recognize the rest of members in the economy that he should not obtain less than u_i^{min}.

Hence, while our cooperative approach can not capture all of the features of particular applications, some of these features can be incorporated into the model, not by changing utilities but directly through the choice of the disagreement point.

Next result provides conditions under which we can ensure that the Nash bargaining solution is interior, meaning that all agents receive some share of the resource, for the two disagreement points we have just highlighted.

Corollary 1 *The Nash bargaining solution is always interior when* $\mathbf{d} = \mathbf{u}^{\min}$. *The Nash bargaining solution is interior when* $\mathbf{d} = \mathbf{0}$ *if and only if for all* $i \in \mathcal{N}$

$$\sum_{j \neq i} b_{ij} \frac{\delta_i}{\delta_j} < 1 \tag{8}$$

This condition resembles the condition for regularity stated in Proposition 3. However, it is different in two aspects. First, the set of pairwise influences that appear are in this case the ones that i receives, instead of those that i exerts. Second, these bilateral influences are weighted by the quotient

$$\frac{\delta_i}{\delta_j} = \frac{\left(1 - \sum_{k \neq i} b_{ki}\right)}{\left(1 - \sum_{k \neq i} b_{kj}\right)}$$

For example, in the case that all bilateral influences are positive this quotient is larger than 1 if $\sum_{k \neq i} b_{kj} > \sum_{k \neq i} b_{ki}$. The influence j exerts on i is weighted by larger values in condition (1.7) if j exerts a larger aggregate level of direct influences on others than i. Observe that this quotient was also present when computing relative profits obtained from bargaining across pairs of individuals.

When we fix a disagreement point $\mathbf{d}$ such that for some agent $u_i^{min} > d_i$ we also impose a value to the minimal gains that this agent is going to obtain from the bargaining situation. This value is equal to $u_i^{min} - d_i$. It might be possible that these minimal gains from bargaining can not be reconciled with the conditions imposed on relative gains across individuals in (1.6) when agents obtain a positive share of the resource. The conditions in Corollary 1 exactly account for this fact, and provide the expressions that ensure that this tension does not arises.

To better understand that interiority condition when $\mathbf{d} = \mathbf{0}$, we analyze in more depth the case of two agent economies. Given an economy

$$\mathbf{B} = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix}$$

the values of δ_1 and δ_2 are $\delta_1 = 1 - b_{21}$ and $\delta_2 = 1 - b_{21}$. Given the regularity condition, this two values are positive. The conditions for interiority expressed in the previous corollary are in this case,

$$\begin{aligned} \frac{1 - b_{12}}{1 - b_{21}} &> b_{12} \\ \frac{1 - b_{21}}{1 - b_{12}} &> b_{21} \end{aligned}$$

The following reasoning helps understand when we lose interiority. Fix a value $b_{21} < 1$. If $b_{12} = b_{21}$ the conditions reduce to $1 > b_{12}$ and $1 > b_{21}$ which are trivially satisfied because of regularity. When we increase b_{12} the second condition is still satisfied since the left-hand side increases. But the left-hand side of the first condition increases while the right hand side of this same condition increases. If we increase b_{12} enough it is possible that this first condition is not satisfied for the parameters. It becomes too difficult to control for both conditions. Agent 2 exerts a larger influence on agent 1 than the influence agent 1 exerts on agent 2. If this difference is sufficiently asymmetric, and this asymmetry is measured by the two conditions above, then one of the agents receives all the resource, even if the economy is regular.

4.3 Homogeneous Networks.

Here we study a family of networks with some particular properties.

Let $\alpha \in \mathbb{R}_+$. We say that an economy is homogeneous if whenever $b_{ij} \neq 0$, then $b_{ij} = b_{ji} = \alpha$. Hence, in an α-economy whenever there is a bilateral influence this influence is bidirectional and of same weight.[16]

In this family of economies the heterogeneity comes only from one source, the network geometry, and not from heterogeneous influence levels across pairs of agents. Therefore the analysis of this family of networks sheds some light on the isolated effect of the network geometry on the Nash bargaining outcome. In fact, as we show in the following lines, the characterization of the Nash bargaining solution becomes very transparent under some mild assumptions.

Fixed α and an α-economy $\mathbf{B}$, we define the degree of agent i, that we denote by $deg_i(\mathbf{B})$, as

$$deg_i(\mathbf{B}) = \frac{1}{\alpha} \sum_{j \neq i} b_{ij} \tag{9}$$

The degree of an agent is a measure of connectivity. It equals the number of connections an agent has in the network of bilateral influences. Due to the symmetric nature of α-economies, the degree of an agent computes at the same time to how many people this agents exerts a direct influence, and from how many people this agent receives a direct influence.

Suppose that if agents do not agree in a division of the resource the disagreement outcome is that no division is implemented and hence agents receive a utility equal to 0, i.e. $\mathbf{d} = \mathbf{0}$. Then, under the regularity condition $1 - (n-1)\alpha > 0$ that ensures that for a fixed α any α-economy is regular, we obtain the following characterization:

Proposition 5 *Let* $\mathbf{B}$ *be a regular* α*-economy. Then the Nash bargaining solution is interior and the utility each agent obtains is*

$$u_i^S = \frac{1}{n(1 - \alpha deg_i(\mathbf{B}))}$$

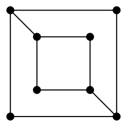
Hence,

$$c_i^S = \frac{1}{n(1 - \alpha deg_i(\mathbf{B}))} - \sum_{j \neq i} b_{ij} \frac{1}{n(1 - \alpha deg_j(\mathbf{B}))}$$

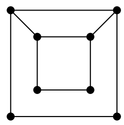
Hence, for α-economies the degree of an agent is the unique relevant element from the network to determine the utility an agent obtains, whereas the share an agent obtains does not only depend on its own degree but also on the degrees of agents to which he is connected.

[16] Observe that in the family of α-economies $\mathbf{G} = \mathbf{B}$, since matrix $\mathbf{B}$ is symmetric.

Example 5. Consider the following two networks.



$\mathbf{B}_a$



$\mathbf{B}_b$

In both networks there are four agents with three neighbours and four with two neighbours. However, if for example $\alpha = 0.1$, agents with three neighbours agents with three neighbours receive a larger share of the pie than in the second network, while the opposite applies for agents with two links. The following table provides the shares in both cases for the members of each class.

Nash Shares with $\alpha = 0.1$	$\mathbf{B}_a$	$\mathbf{B}_b$
Agents with 3 links	0.129	0.127
Agents with 2 links	0.121	0.123

This proves that the degree distribution is not a sufficient invariant to determine how the resource is distributed, it is also important the particular geometry of how agents are connected. This is expressed in the following corollary.

Corollary 2 *The share an agent obtains in the Nash bargaining solution increases with the number of neighbours he has and diminishes with the number of neighbours that his neighbours have.*

That is why we obtain different shares in the previous pair of networks. In the first one each agent that has three neighbours has one neighbour with three links and two with two links, while in the right one each agent with three links has two neighbours with three links and one with two. In accordance with this last corollary, this agents should obtain a smaller share in the network at the right, that is what we have observed before.

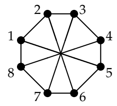
This last corollary provides in fact the main intuition on how the Nash bargaining solution internalizes influences. The share an agent receives depends on the aggregate level of bilateral influences provided at the local level. The more connected an agent is and the less connected his neighbours are, the more valuable is the share this agent receives for the spread of influence through the network. If, instead, his neighbours are also very connected, it is not necessary to give more to this agent. In this case other agents receive larger shares than before because they help more to spread the effect of influences all over the economy. The Nash bargaining solution with influences internalizes this indirect effects.

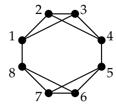
A particular case are degree-regular α-economies. In such economies all agents have the same degree. Here pattern does not matter. In fact, we have the following corollary.

Let $k \in \{0, \ldots, n-1\}$ and let B be a regular α-economy such that $\deg_i(B) = k$ for all $i \in \mathcal{N}$. Then all agents obtain the same utility and consume the same quantity in the Nash bargaining solution. In particular $c_i^S = 1/n$ for all $i \in \mathcal{N}$. and $u_i^S = \frac{1}{n(1-k\alpha)}$.

Therefore in the case of regular α-economies it does not matter how agents are connected but how many connections each agent has.

Example 6. The following two networks, with each link being bidirectional and with same weight α, differ in their particular geometry and lead to the same solution to the bargaining problem.





The network depicted in the right side has larger clustering[17] levels than the one in the left. This is not an issue in the determination of the Nash bargaining solution. This example shows that clustering plays no role in the resulting division that solves the distributional conflict under degree-regularity.

5 The Effect of Network Changes on Welfare and Consumption

In this section we explore how changes in the network of bilateral influences[18] translate into changes in welfare and consumption patterns. In particular, we center our attention in how a differential change on the weight of a link changes the utility and the level of consumption in equilibrium of each agent involved in the bargaining game. Hence, our results help to understand how changes in the magnitude of influences change the characteristics of the bargaining outcome.

For the sake of simplicity, we focus our attention in situations where the bargaining solution is interior, meaning that $c_i^S > 0$ and, hence, $u_i^S > u_i^{min}$ for every $i \in \mathcal{N}$.[19]

The following proposition provides conclusions on comparative statics related to the utility pattern of the Nash bargaining solution.[20]

Proposition 6 *Let **B** define an economy with influences such that the Nash bargaining solution is interior. Then:*
(i) $\frac{\partial u_i^S}{\partial b_{kl}} \geq 0$ *if* $l \neq i$*, with equality if and only if* $d_l = 0$.
(ii) $\frac{\partial u_i^S}{\partial b_{ki}} = sign\left(1 - \sum_{j\neq i} \delta_j d_j\right)$ *for* $k \neq i$.

The first part of the proposition states that the Nash bargaining solution utility of agent i generally increases when there is an increase on the magnitude of a bilateral influence across any two other agents, whoever these are. Increases on bilateral influences agents different than i exert on each other are beneficial for agent i. The second part of the proposition is a little bit more complex. It states that the increase on bilateral influences exerted by agent i are good for agent i if the term $\sum_{j\neq i} \delta_j d_j$ is sufficiently small (in fact, if it is smaller than one). Observe that this can happen either because the disagreement levels of agents different than i are small or because the levels of the δs are small for agents different than i. Agent j has a small level of δ_j whenever he exerts on the aggregate large positive bilateral influences on other agents. Hence, for agent i it is good to exert larger positive influences, if other agents exert large aggregate levels of bilateral influences. The intuition is that larger bilateral influences exerted by agent i can increase indirect

[17]Clustering measures if the agents to which an agent is connected are also connected within them.
[18]We make no distinction in defining the model in terms of networks or in terms of utilities. We refer to network changes because at some points this simplifies the necessary terminology.
[19]A more extensive analysis could be done to deal with corner solutions.
[20]If an economy is such that the Nash bargaining solution is interior, sufficiently small changes in the parameters of bilateral influences mantain interiority because of continuity. Hence, a comparative statics analysis in our setup is legitimated.

network influences from i to himself (through cycles in the network of bilateral influences) if other agents exert sufficiently high bilateral influences as well. If not, and for example other agents exert some bilateral negative influences, the indirect network effects can be negative for agent i and imply a decrease on utility.

We move now to comparative statics results related to consumption patterns. These are provided in the following result.

Proposition 7 *Let* $\mathbf{B}$ *define an economy with influences such that the Nash bargaining solution is interior. If* $\mathbf{d} = \mathbf{0}$,[21] *then:*

(i) $\frac{\partial c_i^S}{\partial b_{ki}} > 0$ *if* $k \neq i$

(ii) $sign\left(\frac{\partial c_i^S}{\partial b_{kl}}\right) = -sign\left(b_{il}\right)$ *if* $k \neq i \neq l \neq k$

(iii) $\frac{\partial c_i^S}{\partial b_{ij}} > 0 \Leftrightarrow b_{ij} < -\delta_j$ *for* $j \neq i$

The first part of the proposition states a very simple and natural conclusion: agent i receives a larger share whenever the level of aggregate bilateral influences he exerts increases.[22] This generates a positive effect on several other agents in the economy through the spread of bilateral influences through network effects. We could also interpret this result in terms of bargaining power: since he exerts a larger aggregate level of bilateral influences his bargaining power increases, and that is why he gets a larger fraction of the resource.

The second part of the proposition states that if agent i receives a positive (resp. negative) direct bilateral influence from agent l then an increase of the direct bilateral influence agent l exerts on another agent k, different than i and l, implies that agent's i share diminishes (resp.increases). This is reminiscent of the result of the first part of the proposition, and hence the same kind of intuition applies.

Finally, the third part expresses that an increase in the weight of the direct bilateral influence agent l exerts on i implies an increase in agent i share if and only if the initial weight of this bilateral externality was sufficiently negative.

We recover example 3.(b) of section 4 to illustrate graphically how small changes on the levels of bilateral influences translate into changes on the utility and shares derived from the Nash bargaining solution. This example is the 2-person economy such that $b_{12} = b_{21} = 1/2$. If we increase b_{12} from the initial $b_{12} = 1/2$ to $\tilde{b}_{12} = 1/2 + \epsilon$, where $\epsilon < 1/2$ to satisfy the regularity condition, the new matrix of bilateral influences is

$$\tilde{\mathbf{B}} = \begin{pmatrix} 0 & 1/2+\epsilon \\ 1/2 & 0 \end{pmatrix}$$

and the new matrix of network externalities is

$$\tilde{\mathbf{E}}\left(\mathbf{B}\right) = \frac{4}{3-2\epsilon}\begin{pmatrix} 1 & 1/2+\epsilon \\ 1/2 & 1 \end{pmatrix}$$

The utility possibility set and the Nash bargaining solution behave as follows (dashed lines represent the initial situation and continuous lines represent the new one; u^S is the initial Nash bargaining solution and

[21] We consider this case since it is the more tractable one. In the proof the interested reader can find the exact expression of each one of the derivatives, no matter which disagreement point we consider.

[22] And the rest of bilateral influences do not vary.

$\tilde{u}^S$ is the new one)

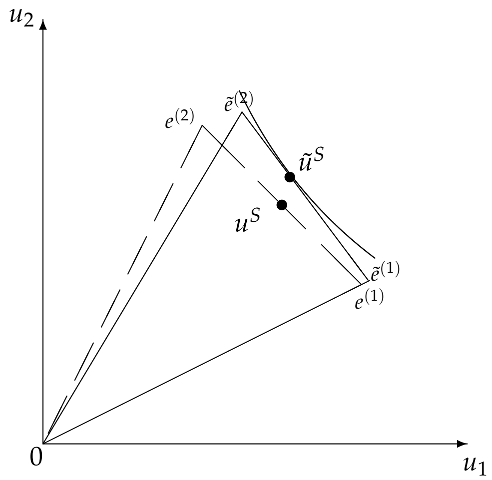


Figure 8

The increase on the level of altruism of agent 1 shifts the Pareto frontier upwards. Both agents obtain a larger utility in the new equilibrium of the bargaining game. This is consistent with the conclusions of proposition 7,[23] and now the division is no more half of the budget for each one. Indeed, the equilibrium point $\tilde{u}^S$ is closer to one of the extremes of the simplex than to the other. In particular it is closer to $\mathbf{e}^{(2)}$ which implies that c_2^S has increased while c_1^S has decreased. The increase of c_2^S is consistent with part (i) in proposition 8 while the decrease of $\mathbf{e}^{(1)}$ is consistent with part (iii), since $b_{12} = 1/2 > -1/4 = -\delta_2/2$.

6 Individual Provision of Global Resources

Preferences

A group is defined by its underlying structure of pairwise influences, gathered in matrix $\mathbf{B}$, and a wealth distribution $\mathbf{w} = (w_1, \ldots, w_n)$, with w_i being equal to the initial monetary endowment of agent i. Let $\bar{w}$ denote the mean income at the group level, i.e. $\bar{w} = (w_1 + \cdots + w_n)/n$. Each neighbourhood can decide to invest part of its income to the provision of public resources, c. We differ from the standard approach to public good provision, where everybody enjoys equally the public good. In our case, public resources are divided and allocated across group members. Each member enjoys a direct effect of the share of public resources she receives and she also can receive positive spillovers, measured by matrix $\mathbf{B}$, from the allocation of the public resources to the rest of members.

Each group member obtains utility from two distinct sources: the income that keeps after the investment in public resources, if she has invested at all, and the utility derived from the allocation of public resources to the rest of the group. Let τ_i denote the contribution of member i to the level of public resources, and let c denote the aggregate level of public resources produced. The production function of resources given a contributions' profile $\boldsymbol{\tau}$ is

$$c(\tau) = \ln\left(1 + \sum_i \tau_i\right)$$

[23]Observe that the increase in u_1^S is the conclusion of part (i) while the increase of u_2^S is the conclusion of part (ii), since $\mathbf{d} = \mathbf{0}$.

The level of public resources is an strictly concave function on the sum of contributions of all group members. Individual contributions are therefore perfect substitutes of each other.

The utility derived from wealth is equal to $\ln(1 + w_i - \tau_i)$. It is a concave function, increasing with respect to w_i and decreasing with respect to τ_i.

Let c_i be the share of public resources that agent i receives when resources are distributed among group members. The utility agent i obtains from the public resources produced depends on the allocation profile in the following form

$$u_i\left(c_1(\boldsymbol{\tau}), \ldots, c_n(\boldsymbol{\tau})\right)$$

In particular, and in accordance with previous sections, we assume a linear dependency on utilities of the following form:

$$u_i(\boldsymbol{c}(\boldsymbol{\tau})) = c_i(\boldsymbol{\tau}) + \sum_{j \neq i} b_{ij} u_j(\boldsymbol{c}(\boldsymbol{\tau}))$$

The total utility each agent obtains, that we denote v_i, is equal to the sum of the utility derived from wealth and the utility derived from the public good:

$$\tilde{v}_i(\boldsymbol{\tau}) = \ln(1 + w_i - \tau_i) + u_i(c(\boldsymbol{\tau}))$$

In this section we keep the assumption that the economy is regular, in the sense defined previously in text. Thus, if $b_i = \sum_{j \neq i} b_{ji}$ and $\delta_i = 1 - b_i$, we are assuming that $\delta_i \geq 0$ for all i. Lemma 1 shows that this condition, that bounds the sum of direct spillovers for each agent, is necessary and sufficient to ensure regularity.

6.1 The game.

The public good provision game has two stages. First, all group members chooses simultaneously an individual level of contribution to public resources, $\tau_i \in [0, w_i]$.

In the second stage of the game, group members negotiateon how to allocate the public resources obtained from individual contributions, $\ln\left(1 + \sum_{i=1}^{n} \tau_i\right)$. We assume that, if they don't reach an agreement the public good is not divided and the unique utility agents obtain derives from the remaining wealth once discounted its contribution.[24] Following our previous analysis, we assume that the solution to this distributional conflict is given by the Nash bargaining solution. Because of the no-division disagreement outcome, the disagreement point in the bargaining game is equal to

$$\mathbf{d}(\boldsymbol{\tau}) = \left(\ln(1 + w_1 - \tau_1), \ldots, \ln(1 + w_1 - \tau_1)\right)$$

The conditions on the matrix $\mathbf{B}$ ensure that the Nash bargaining solution. Since the Nash bargaining solution satisfies the property of *Scale Invariance*, we can immediately apply the results obtained in previous sections to characterize the corresponding utilities each group member is going to obtain in this second stage of the game. We do that in the form of a lemma.

[24]This assumption is not essential. We consider this disagreement outcome for analytical convenience since it leads to simple close-form expressions, but other disagreement outcomes could be accomodated in the model as well.

Lemma 2 *The final utilities of the public good provision game are*

$$v_i(\boldsymbol{\tau}) = \ln(1 + w_i - \tau_i) + \frac{c(\boldsymbol{\tau})}{n\delta_i}$$

6.2 Equilibrium analysis

Let $\tau_{-i} = \sum_{j \neq i} \tau_j$. The function v_i is strictly concave in τ_i.Since

$$\frac{\partial v_i}{\partial \tau_i} = -\frac{1}{1 + w_i - \tau_i} + \frac{1}{n\delta_i(1 + \tau)}$$

we obtain that the the best-reply function of agent i is

$$BR_i(\tau_{-i}) = \begin{cases} 0 & \text{if } 1 + \tau_{-i} \geq \frac{1+w_i}{n\delta_i} \\ \frac{1+w_i - n\delta_i}{1+n\delta_i} - \frac{n\delta_i}{1+n\delta_i}\tau_{-i} & \text{if } 1 + \tau_{-i} \in \left(\frac{1+w_i}{n\delta_i}, \frac{1}{n\delta_i}\right) \\ w_i & \text{if } 1 + w_i + \tau_{-i} \leq \frac{1}{n\delta_i} \end{cases}$$

We can clearly distinguish the dependencies of individual decisions on the two dimensions of heterogeneity of the problem (the wage distribution and the pattern of spatial spillovers). For example, an agent decides to not contribute at all if she is not wealthy enough or aggregate externalities it directly exerts to the rest of the group (measured by the parameter δ_i) are small . This last consequence is reminiscent of the use of the Nash bargaining solution as the solution to distributional conflict. The following proposition provides close-form expressions of the individual contributions to public resources.

Proposition 8 *If the public good provision game admits an interior equilibrium, it is unique and of the form*

$$\tau_i^* = \frac{1 + w_i - n\delta_i}{1 + n^2\bar{\delta}} + \frac{n^2}{1 + n^2\bar{\delta}}\left[(1 + \bar{w})\left(\bar{\delta} - \delta_i\right) - \bar{\delta}(\bar{w} - w_i)\right]$$

We can decompose the contribution of each group member in two different components. The first one, equal to $\frac{1+w_i-n\delta_i}{1+n^2\bar{\delta}}$, is essentially (with the exception of the common renormalization $\frac{1}{1+n^2\bar{\delta}}$ to all group members) an idiosyncratic component that only takes into account individual income, w_i, and aggregate direct externalities exerted by the agent, measured by δ_i.

The second term comprises more information about the whole group. Four elements are important: again individual income, w_i, and the aggregate level of direct externalities exerted by the agent, δ_i, and group's average income and average externalities exerted.

The contribution of a member increases the more wealthy she is with respect to the group's mean wealth. This effect is stronger the larger it is the mean of δ's, i.e. this effect is stronger the smaller are the direct externalities the group exerts on average. This is not surprising. When externalities are weak, group members have to invest more on the provision of public resources to receive its, direct and indirect, effects. In addition, the contribution of a member increases the smaller it is its associated δ_i compared with the mean of δ's, i.e. the larger are the aggregate direct spillovers this member exerts compared with the externalities the group exerts on average. Furthermore, this effect is larger the larger is the average income level of the whole group. This effect is reminiscent of the use of the Nash bargaining solution with the disagreement outcome of no public resource assignment.

Symmetric Case If $\delta_i = \bar{\delta} = 1 - \bar{b}$ and $w_i = \bar{w}$ for all i then

$$\tau_i^* = \frac{1 + \bar{w} - n\bar{\delta}}{1 + n^2\bar{\delta}}$$

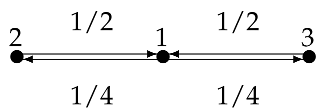
Some straightforward computations show that the contribution of each group member increases with wealth and with the level of externalities directly exerted.

Example Consider the following example, with two different groups. Both groups have three different members, $\mathcal{N} = \{1, 2, 3\}$. The matrix of pairwise influences is the same for both

$$\mathbf{B} = \begin{pmatrix} 0 & 1/4 & 1/4 \\ 1/2 & 0 & 0 \\ 1/2 & 0 & 0 \end{pmatrix}$$

The unique difference between these them is on their respective wealth distributions. The wealth distribution of the first group is $\mathbf{w}^1 = (2, 3/2, 3/2)$, while the one for the second group is $\mathbf{w}^2 = (2, 1, 1)$.

To get a more graphical representation of these two cities we depict in form of a network the flows and intensities of direct spillovers from one neighbourhood to another:



In the case of the first group, it is easy to check that the unique equilibrium of the public good provision game is interior. In particular. $\boldsymbol{\tau}^* = (6/11, 1/22, 1/22)$. The difference in income translates into large differences between the contributions of the poor members compared to that of member 1, but it is not sufficiently large to preclude the participation of any of them.

In the case of the second group, income inequality is larger between the central member and peripheral members. With the use of the best-replies functions specified above we can easily derive that the unique equilibrium of the public good provision game for this group is that agent 1 contributes with half of her income and that the two other group members don't contribute at all, i.e. $\boldsymbol{\tau}^* = (1, 0, 0)$. All group members are interested in the benefit that the public resources generate but agents 2 and 3 are not wealthy enough to contribute to its provision and free-ride on agent 1.

6.3 The social optimum

We proceed to study the socially optimal contribution profile to public resources. We characterize this profile in case it is interior, and we proceed to compare it to the equilibrium of the provision game in case it is also interior. To describe this socially optimal provision rule, we need first to define the harmonic mean of a set of numbers. Let $\boldsymbol{x}$ be an n-dimensional real vector. The harmonic mean of the elements of this vector, that we denote $H(\boldsymbol{x})$, is equal to

$$H(\boldsymbol{x}) = \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$

Proposition 9 *If the social optimum is interior, the socially optimal level of contribution of agent i is*

$$\tau_i^s = \frac{1 + w_i - H(\delta)}{1 + nH(\delta)} + \frac{nH(\delta)}{1 + nH(\delta)} (w_i - \bar{w})$$

Again, the optimal provision level of each group member is formed by an intrinsic component that only depends on its own level of income and the (harmonic) mean of δ's of the whole group. This intrinsic component is larger the larger it is its own income level and the smaller it is the harmonic mean of δ's, i.e. whenever direct externalities are larger. The second component depends on the wealth distribution on the whole group. The larger the income level of one agent compared with the group's average income level, the more the agent should contribute. This effect is more severe when the harmonic mean of δ's is larger.

Proposition 10 *For the case of homogeneous spillovers and equally wealthy group members, there is always underinvestment with respect to the socially optimal provision level, and underinvestment decreases in δ for any $\delta > n^{-\frac{3}{2}}$. Furthermore, the difference between individual utility at the social optimum and individual utility at equilibrium is decreasing in δ for any $\delta > 0$.*

This results highlights that the collective action problem still exists even if there are no differences in wealth or in the pattern of spatial spillovers. As Olson (1971) notes, this is a natural feature on players' actions in most games of public good provision. Our model is not an exception. The presence of positive externalities does not dilute the underinvestment effect. To the contrary, the difference between socially optimal actions and equilibrium actions increase when aggregate direct externalities increase. This is also true when we compare differences in utilities.

7 Examples

[to be completed]

8 Conclusion

We have explored the outcome of the Nash bargaining problem with considering a simple model of interdependent behavior. Even if an economy is characterized by $n(n-1)$ variables, the model is tractable and we have been able to provide closed-form expressions for the bargaining outcome and comparative statics results. The network interpretation of the problem is helpful since it provides us with a set of tools that simplify the analysis and makes it more intuitive. It helps to understand the effect of heterogeneities in the model in all its dimensions, magnitude and pattern.

Part of the analysis in our work shows some similarities with previous work done by Kalai (1977).[25] Kalai interprets any agent that cares for a player in the bargaining problem but that do not participate in the bargaining problem, as a replica of this player. In our model, an agent can care for different players of the game, where this concern translates into influences as in the social preferences example in the introduction. The transmission of this concern is not done as a replication and its consequent change into the bargaining

[25]See also Lensberg and Thomson (1989) for some other work done with replicated agents in cooperative bargaining.

problem but through a pattern of different influences that affect players' behavior. In this sense, we allow for a more general pattern of interrelations and the transition is not done in a discrete manner, as replicas would do, but smoothly, since small changes in bilateral externality levels imply small changes in the levels of network effects.

Finally, our analysis borrows directly from the Nash bargaining solution. Different possible directions for further research are open. One possible direction could be to explore whether other cooperative solutions can be defined through some proper axioms adequate in a setting with heterogeneous influences such as the one developed in this work. Another possible direction is to go further in the study of non-cooperative bargaining models with an underlying structural pattern of bilateral influences. In particular, it might be valuable to study how the pattern of influences maps into equilibria of non-cooperative bargaining games that incorporate relevant features of particular applications, such as the voting rule in legislative bargaining, and how equilibria vary with respect to the case without influences.

With regards to the analysis of provision of public resources, an alternative approach to the production of public services could be to select heterogenous taxation levels for the different group members. Interpreting groups as a region, for example a city, this alternative would represent an intermediate approach in the theory of fiscal federalism from the two polar perspectives of purely homogeneous level of taxation at the city level and complete decentralization of control of public services to the different neighbourhoods (Oates, 1972, is a classical reference on this issue, and Besley and Coates, 2003, and Lockwood,2002, are modern approaches to that question). The introduction of spatial spillovers in a taxation model with externalities flowing from one neighbourhood to another would enrich the possible conclusions in such issues.

Of course, our model is based in several assumptions that require further consideration. We have here imposed a particular timing on the provision game. And, in the second stage of this game, we have considered a particular solution to the distributional conflict for resources. Moreover, we have neglected the role of free mobility of citizens. The possibility of a voting-with–their-feet mechanism (see Tiebout, 1956) is an important issue to be considered when dealing with local public good provision.

Appendix: Proofs

Proof of Proposition 1
The determinant of the matrix $\mathbf{I}-\mathbf{B}$ is a polynomial in $n\,(n-1)$ variables. The set of points of $\mathbb{R}^{n(n-1)}$ in which this polynomial vanishes forms an algebraic variety of dimension $n\,(n-1)-1$ at most, and hence it is a set with Lebesgue measure equal to zero in $\mathbb{R}^{n(n-1)}$. ■
Proof of Proposition 2
The following lemma is useful.

Lemma 3 *Given a regular economy $\boldsymbol{B}$, a feasible allocation $\boldsymbol{c}$ is Pareto efficient if and only if there exists a strict system of weights μ such that $\mu\cdot\boldsymbol{u}(\boldsymbol{c})\geq\mu\cdot\bar{\boldsymbol{u}}$ for every $\bar{\boldsymbol{u}}\in\mathrm{UPS}(\boldsymbol{B})$.*

Proof of Lemma 3 This is a slight variation of a well-known result relating Pareto efficiency to linear social welfare functions (see for example Proposition 16.E.2, pg.560, in Mas-Colell et al., 1995). The statement in terms of *strict* system of weights is valid because the shape of $\mathrm{UPS}(\mathbf{B})$ is a simplex, not simply a convex set. ■

Since there is no possibility of confusion we omit the dependence of $\mathbf{E}$ on $\mathbf{B}$. Observe that for any allocation $\mathbf{c}$, the vector of utilities is $\mathbf{u}(\mathbf{c})=\sum_{i=1}^{n}c_i\mathbf{e}^i$. If there exists a strict system of weights $\boldsymbol{\mu}$ and a strictly positive constant κ such that $\boldsymbol{\mu}\cdot\mathbf{e}^i=\kappa$ we have that for any allocation $\mathbf{c}$

$$\boldsymbol{\mu}\cdot\mathbf{u}(\mathbf{c})=\sum_{i=1}^{n}c_i\left(\boldsymbol{\mu}\cdot\mathbf{e}^{(i)}\right)=\kappa\sum_{i=1}^{n}c_i$$

Since $\kappa>0$, we have that $\boldsymbol{\mu}\cdot\mathbf{u}(\mathbf{c})$ is maximized whenever $\sum_{i=1}^{n}c_i=1$. Hence, any allocation such that $\sum_{i=1}^{n}c_i=1$ is Pareto efficient and the economy is regular.

Now, suppose any allocation such that $\sum_{i=1}^{n}c_i=1$ is Pareto efficient. Consider an interior allocation, i.e. such that $c_i>0$ for every $i\in\mathcal{N}$. The unique possible strict system of weights that can separate $\mathbf{u}(\mathbf{c})$ to the utility possibility set in the form of lemma 2 is the strict system of weights orthonormal to the hyperplane that contains the n columns of the matrix of network externalities. Obviously, this system of weights also separates $\mathbf{u}(\mathbf{c})$ to the utility possibility set when $c_i=0$ for some $i\in\mathcal{N}$. Hence, we have the unique candidate for the strict system of weights in the statement of proposition 2. From lemma 3 we know that in particular $\boldsymbol{\mu}\cdot\mathbf{u}(\mathbf{c})\geq\boldsymbol{\mu}\cdot\mathbf{u}(0)=0$. We can ensure that in fact this last inequality is strict since if it were equal to zero we would not be in a generic situation.[26]■

Proof of Proposition 2
From proposition 2 we know that there exists an strict system of weights $\boldsymbol{\mu}$ and a strictly positive constant such that $\boldsymbol{\mu}\cdot\mathbf{e}^{(i)}=\kappa$ for every $i\in\mathcal{N}$. In matrix terms this is equal to

$$\mathbf{E}^T(\mathbf{B})\cdot\boldsymbol{\mu}=\kappa\mathbf{1}$$

where $\mathbf{E}^T(\mathbf{B})$ is the transpose matrix of $\mathbf{E}(\mathbf{B})$ and $\mathbf{1}$ is the n-dimensional vector with all entries equal to 1. Hence, we have that, since the inverse matrix of $\mathbf{E}^T(\mathbf{B})$ is equal to $(\mathbf{I}-\mathbf{B})^T$,

$$\boldsymbol{\mu}=\kappa\left((\mathbf{I}-\mathbf{B})^T\cdot\mathbf{1}\right)$$

[26] If it were equal to zero this would imply that the columns of the matrix of network externalities are linearly dependent, and hence that the determinant of $\mathbf{E}$ is equal to zero. This would mean that we were considering a non solvable system of bilateral influences.

Therefore, $\mu_i = \kappa \left(1 - \sum_{j \neq i} b_{ji}\right)$. Since $\boldsymbol{\mu}$ is an strict system of weights, we have that

$$\kappa \sum_{i=1}^{n} \left(1 - \sum_{j \neq i} b_{ji}\right) = 1$$

and hence

$$\kappa = \frac{1}{\sum_{i=1}^{n} \left(1 - \sum_{j \neq i} b_{ji}\right)}$$

Let $\delta_i = 1 - \sum_{j \neq i} b_{ji}$, and let $\delta = \sum_{i=1}^{n} \delta_i$. Then

$$\mu_i = \frac{\delta_i}{\delta}$$

Since $\boldsymbol{\mu}$ is an strict system of weights, all entries of $\boldsymbol{\mu}$ have to be strictly positive, and this can only happen if either all δ_i's are strictly positive or all δ_i's are strictly negative. However, in the latter case κ would be negative, since $\kappa = 1/\left(\delta\right)$. Hence to obtain a regular economy it is necessary that $\delta_i = 1 - \sum_{j \neq i} b_{ji} > 0$ for every $i \in \mathcal{N}$.

The sufficiency result is almost immediate. Consider the weights and κ defined in Lemma 1 in the text. Then by construction these coefficients satisfy the regularity condition in proposition 2. ■

Proof of Proposition 4

Let $\boldsymbol{\mu}$ and κ be the strict system of weights and constant from Proposition 2 associated to the economy. Let $\mathcal{J} \subseteq \mathcal{N}$ be the set of agents for which $d_j \geq u_j^{min}$. The Nash bargaining problem with network externalities is equal to

$$\max_{\mathbf{u} \in \mathrm{UPS}(\mathbf{B})} \sum_{i=1}^{n} \ln\left(u_i - d_i\right)$$

subject to

$$\sum_{i=1}^{n} \mu_i u_i = \kappa \tag{10}$$

$$u_i \geq d_i \quad if\ i \in \mathcal{J} \tag{11}$$

$$u_i \geq u_i^{min} \quad if\ i \notin \mathcal{J} \tag{12}$$

We know that the solution to this problem is unique. We denote this solution $\mathbf{u}^s$. Let $\bar{\psi}$ be the multiplier associated to restriction (1.9). The Kuhn-Tucker conditions of the problem are

$$\frac{1}{u_i^S - d_i} \leq \bar{\psi}\mu_i \qquad with\ equality\ if\ u_i^S > d_i \quad (i \in \mathcal{J}) \tag{13}$$

$$\frac{1}{u_i^S - d_i} \leq \bar{\psi}\mu_i \qquad with\ equality\ if\ u_i^S > u_i^{min} \quad (i \notin \mathcal{J}) \tag{14}$$

From (1.12) we obtain that for each $i \in \mathcal{J}$ we must have $\frac{1}{u_i^S - d_i} = \psi\mu_i$. If not, the value of the objective function in the solution would be $-\infty$. Hence, if, for simplicity, we denote $\psi = 1/\bar{\psi}$, we have

$$u_i^S = d_i + \psi\frac{1}{\mu_i} \qquad for\ every\ i \in \mathcal{J}$$

On the other hand, we obtain from (1.13) that, for every $i \notin \mathcal{J}$, u_i^S must satisfy

$$u_i^S = \max\left\{u_i^{min}, d_i + \psi\frac{1}{\mu_i}\right\} \quad \textit{for every } i \notin \mathcal{J}$$

Observe in particular that, for every $i \notin \mathcal{J}$ it holds that $u_i^S = u_i^{min}$ if and only if $\mu_i\left(u_i^{min} - d_i\right) \geq \psi$. Using this fact, we proceed to provide and algorithm that at most in n steps provides the solution to the problem. As we stated in text, we suppose without loss of generality that $\mu_1\left(u_1^{min} - d_1\right) \geq \cdots \geq \mu_n\left(u_n^{min} - d_n\right)$

Step 0:
Suppose $u_i^S = d_i + \psi^{(0)}\frac{1}{\mu_i}$ for every $i \in \mathcal{N}$. The multiplier $\psi^{(0)}$ is equal to $\psi^{(0)} = \frac{1}{n}\left(\sum_{i=1}^n \mu_i\left(u_i^S - d_i\right)\right) = \frac{1}{n}\left(\kappa - \bar{} \cdot \mathbf{d}\right)$. If $\mu_1\left(u_1^{min} - d_1\right) < \psi^{(0)}$, then $\psi = \psi^{(0)}$ and $\mathbf{u}^S$ is the utility vector associated to the Nash bargaining solution, and we are done. If not, go to step 1.

Step 1:
Suppose $u_1^S = u_1^{min}$ and $u_i^S = d_i + \psi^{(1)}\frac{1}{\mu_i}$ for every $i > 1$. The multiplier $\psi^{(1)}$ is equal to $\psi^{(1)} = \frac{1}{n-1}\left(\sum_{i=2}^n \mu_i\left(u_i^S - d_i\right)\right) = \frac{1}{n-1}\left(\kappa - \bar{} \cdot \mathbf{d} - \mu_1\left(u_1^S - d_1\right)\right)$. Observe that

$$(n-1)\psi^{(1)} = \kappa - \bar{}\mathbf{d} - \mu_1\left(u_1^S - d_1\right) \leq n\psi^{(0)} - \psi^{(0)}$$

Hence, $\psi^{(1)} \leq \psi^{(0)}$, and therefore we know for sure that $\mu_1\left(u_1^{min} - d_1\right) \geq \psi^{(1)}$. If $\mu_2\left(u_2^{min} - d_2\right) < \psi^{(1)}$, then $\psi = \psi^{(1)}$ and $\mathbf{u}^S$ is the utility vector associated to the Nash bargaining solution, and we are done. If not, go to step 2.

Step k ($2 \leq k < n$):

Suppose $u_i^S = u_i^{min}$ for $i \leq k$ and $u_i^S = d_i + \psi^{(k)}\frac{1}{\mu_i}$ for $i > k$. An analogous reasoning to the one in the previous step establishes that $\psi^{(k)} \leq \psi^{(k-1)}$. In fact

$$\begin{aligned}(n-k)\,\psi^{(k)} &= \kappa - \mu \cdot \mathbf{d} - \sum_{l=1}^{k} \mu_l\left(u_l^S - d_l\right) \\ &\leq (n-k+1)\,\psi^{(k-1)} - \psi^{(k-1)} = (n-k)\,\psi^{(k-1)}\end{aligned}$$

The last inequality follows from the previous step of the procedure. If

$$\mu_{k+1}\left(u_{k+1}^{min} - d_{k+1}\right) < \psi^{(k)}$$

then $\mathbf{u}^S$ is the utility vector associated to the Nash bargaining solution, and we are done.

This process finishes at most in step $n-1$ since in this case we get that

$$\psi^{(n-1)} = \kappa - \bar{} \cdot \mathbf{d} - \sum_{i=1}^{n-1} \mu_i\left(u_i^{min} - d_i\right) > \mu_n\left(u_n^{min} - d_n\right)$$

This last inequality follows from the fact that $\kappa = \mu \cdot \mathbf{u}^S > \mu \cdot \mathbf{u}^{min}$, since $\mathbf{u}^{min}$ can not be the total vector of utilities associated to an efficient allocation. Thus, if we arrive to step $n-1$, we can ensure that the utility vector associated to the Nash bargaining solution is $u_i^S = u_i^{min}$ for $i < n$ and $u_n^S = d_n + \psi^{(n-1)} \frac{1}{\mu_n} > u_n^{min}$. ■

Proof of proposition 5

Fix α. The network in which the minimal utility of an agent is maximal is the complete network, where all pair of agents are connected. The matrix $\mathbf{E}$ for the complete network has entries $\frac{1-(n-2)\alpha}{(1+\alpha)(1-(n-1)\alpha)}$ in the diagonal and $\frac{\alpha}{(1+\alpha)(1-(n-1)\alpha)}$ outside the diagonal. The minimal utility an agent can obtain is the minimum of this two numbers, which coincides with the coefficient outside the diagonal, given the regularity assumption $1-(n-1)\alpha > 0$. Hence for any α-economy $\mathbf{B}$ we have that

$$u_i^{min}(\mathbf{B}) \leq \frac{\alpha}{(1+\alpha)(1-(n-1)\alpha)} \tag{15}$$

If $\mathbf{d} = \mathbf{0}$ we have that the condition to stop in the first step of the algorithm provided in the proof of Proposition 4 is

$$(1 - deg_i(\mathbf{B})\alpha)\, u_i^{min}(\mathbf{B}) \leq \frac{1}{n} \tag{16}$$

Given the regularity condition we know that $\alpha/(1+\alpha) < 1/n$ and therefore

$$\frac{\alpha}{(1+\alpha)(1-(n-1)\alpha)} < \frac{1}{n(1-deg_i(\mathbf{g})\alpha)} \tag{17}$$

Hence the condition in the first step of the algorithm provided in the proof of Proposition 4 is satisfied, and we are done. ■

Proof of Proposition 6

We can rewrite the total utility an agent obtain in an interior solution as

$$u_i^s = d_i + \frac{1}{n\delta_i}\left(1 - \sum_{j=1}^{n} \delta_j d_j\right) \tag{18}$$

If $i \neq j \neq k$, straightforward calculus yields to

$$\frac{\partial u_i^s}{\partial b_{kj}} = \frac{d_j}{n\delta_i} \tag{19}$$

and the result of the first part of the proposition follows, since in any regular economy $\delta_i > 0$ for all $i \in \mathcal{N}$.

If $i \neq j$ we have that

$$\frac{\partial u_i^s}{\partial b_{ji}} = \frac{1}{n\delta_i^2}\left(1 - \sum_{k\neq i} \delta_k d_k\right) \tag{20}$$

Again, by the regularity condition, the result follows. ■

Proof of Proposition 7

When $\mathbf{d} = \mathbf{0}$ the share agent i obtains in the Nash bargaining solution when it is interior is

$$c_i^s = \frac{1}{n\delta_i} - \sum_{j\neq i} b_{ij} \frac{1}{n\delta_j} \tag{21}$$

Let $k \neq i$. If we differentiate the expression in (1.22) with respect to b_{ki} we obtain

$$\frac{\partial c_i^s}{\partial b_{ki}} = \frac{1}{n\delta_i^2} > 0$$

and the first part of the proposition follows.

If i, j and k are pairwise different we have that

$$\frac{\partial c_i^s}{\partial b_{kj}} = -\frac{b_{ij}}{n\delta_j^2}$$

Hence, $\frac{\partial c_i^s}{\partial b_{kj}} b_{ij} \leq 0$ with equality if and only if $b_{ij} = 0$.

Finally, if $i \neq j$ we have that

$$\frac{\partial c_i^s}{\partial b_{ij}} = -\frac{1}{n}\left(\frac{1}{\delta_j} + b_{ij}\frac{1}{\delta_j^2}\right)$$

Hence,

$$\frac{\partial c_i^s}{\partial b_{ij}} > 0 \Leftrightarrow \delta_j < -b_{ij}$$

■

Proof of Proposition 8

If there exists an interior equilibrium, in which $\tau_i^* \in (0, w_i)$ for all i, it solves the following system of equations:

$$\tau_i^* + \frac{n\delta_i}{1+n\delta_i}\tau_{-i}^* = \frac{1+w_i-n\delta_i}{1+n\delta_i} \qquad i \in \mathcal{N}$$

Let $s_i = \frac{1+w_i-n\delta_i}{1+n\delta_i}$, and let $a_i = \frac{n\delta_i}{1+n\delta_i}$. Let $\mathbf{A}$ be the matrix with ones in the diagonal and $a_{ij} = a_i$ if $j \neq i$. Then the interior equilibrium is equal to

$$\boldsymbol{\tau}^* = \mathbf{A}^{-1} \cdot \mathbf{s}$$

It is a simple exercise of linear algebra to prove that

$$\mathbf{A}_{ij}^{-1} = \begin{cases} (1+n\delta_i)\frac{1+n\sum_{k\neq i}\delta_k}{1+n^2\bar{\delta}} & \text{if } i=j \\ -\left(1+n\delta_j\right)\frac{n\delta_i}{1+n^2\bar{\delta}} & \text{if } i \neq j \end{cases}$$

Therefore, $\tau_i^* = \sum_{j=1}^{n} \mathbf{A}_{ij}^{-1} s_j = \frac{1+w_i-n\delta_i}{1+n^2\bar{\delta}} + \frac{n}{1+n^2\bar{\delta}}\sum_{j\neq i}\left[\delta_j(1+w_i) - \delta_i\left(1+w_j\right)\right].$

Some algebra shows that

$$\sum_{j\neq i}\left[\delta_j(1+w_i) - \delta_i\left(1+w_j\right)\right] = n\left[(1+\bar{w})\left(\bar{\delta}-\delta_i\right) - \bar{\delta}(\bar{w}-w_i)\right]$$

■

Proof of Proposition 9

The first-order conditions of the social planner problem, in the case that the social optimum is interior are:

$$\tau_i^s + \frac{H(\delta)}{1+H(\mathbf{ffi})}\tau_{-i}^s = \frac{1+w_i-H(\delta)}{1+H(\delta)} \qquad i \in \mathcal{N}$$

where $\tau_{-i}^s = \sum_{j\neq i} \tau_j^s$. These first-order conditions form a system of linear equations with a unique solution. Let $r_i = \frac{1+w_i-H(\delta)}{1+H(\mathbf{ffi})}$, and let $\alpha = \frac{H(\delta)}{1+H(\delta)}$. Let $\mathbf{Z}$ be the matrix with $z_{ii} = 1$ for all $i \in \mathcal{N}$

and $z_{ij} = \alpha$ whenever $i \neq j$. The solution to the above system of linear equations is:

$$\boldsymbol{\tau}^s = \mathbf{Z}^{-1} \cdot \mathbf{r}$$

Some algebra leads to:

$$\mathbf{Z}_{ij}^{-1} = \begin{cases} \frac{(1+H(\delta))(1+(n-1)H(\delta))}{(1+nH(\delta))} & \text{if } i = j \\ -\frac{(1+H(\delta))H(\text{ffi})}{(1+nH(\delta))} & \text{if } i \neq j \end{cases}$$

Plugging this back in $\tau_i^s = \sum_{j=1}^{n} \mathbf{Z}_{ij}^{-1} r$, and rearranging terms we obtain the desired result. ■

Proof of Proposition 10

We omit the proof. The result follows from direct comparison of the expressions obtained. ■

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