

# The Impact of Bargaining on Markets with Price Takers: Too Many Bargainers Spoil The Broth

David Gill

John Thanassoulis

Division of Economics

Dept. of Economics & Christ Church

University of Southampton

University of Oxford

March 2008

## Abstract

In this paper we study how bargainers impact on markets in which firms set a list price to sell to those consumers who take prices as given. The list price acts as an outside option for the bargainers, so the higher the list price, the more the firms can extract from bargainers. We find that an increase in the proportion of consumers seeking to bargain can lower consumer surplus overall, even though new bargainers receive a lower price. The reason is that the list price for those who don't bargain and the bargained prices for those who were already bargaining rise: sellers have a greater incentive to make the bargainers' outside option less attractive, reducing the incentive to compete for price takers. Competition Authority exhortations to bargain can therefore be misplaced. We also consider the implications for optimal seller bargaining.

**JEL Codes:** L13, D43

**Key words:** Bargaining; Price takers; List Price; Consumer Surplus

# 1 Introduction

Consumers often ask retailers or service providers for discounts off their list prices. This is particularly prevalent for large ticket items such as expensive electronic and household goods, cars, package holidays, estate agency (realtor) fees, etc. The United Kingdom's market for automobiles is a striking example where list prices are heavily advertised but discounts are common and large. As an example, discounts off the list price for Ford models sold in the UK in 1997/98 averaged 10.6% with reductions as large as 30% for some buyers. Similar discounts were available for other models (see Figure 1 from the UK's Competition Commission (2000) report on the car market.)

APPENDIX 7.1  
(referred to in paragraphs 7.32 and 7.36)

### Percentage discount given on make of car

	Number of sales																
	Vauxhall	Ford	Fiat	Peugeot	Toyota	Nissan	Citroën	Mitsubishi	Mazda	Renault	Rover	Honda	Audi	Volkswagen	Volvo	BMW	Mercedes-Benz
Percentage discount																	
0	4	55	12	16	4	2	44	0	26	29	64	30	42	71	33	158	74
0-5	9	53	16	53	21	25	31	19	29	140	47	49	117	191	73	234	167
5-10	51	144	36	58	59	52	43	35	50	82	60	73	72	113	19	75	17
10-15	27	96	12	44	65	16	23	6	10	14	13	13	13	2	2	6	1
15-20	5	17	6	9	2	6	4		10	16	8	1	2				
20-25	18	14	11	20		1	13		3	16	1						
25-30	7	23	5	8			6		1								
30-40	2	21	4	1													
Average	12.3	10.6	10.2	9.3	8.7	7.6	7.3	6.7	6.5	6.2	4.7	4.6	3.9	3.4	2.6	2.5	2.0
Standard deviation	7.5	8.7	8.6	7.5	3.8	4.4	7.7	2.9	5.9	6.2	4.9	3.6	3.3	2.9	2.7	2.5	2.4
Total number of sales	123	423	102	209	151	102	164	60	129	297	193	166	246	377	127	473	261
Percentage given discount	96.7	87.0	88.2	92.3	97.4	98.0	73.2	100.0	79.8	90.2	66.8	81.9	82.9	81.2	74.8	66.6	71.7

Source: Commission calculations based on OFT data.

Figure 1: Discounts on UK cars received by private purchasers from data sampled by the Office of Fair Trading between October 1997 and May 1998. The numbers in the table are the numbers of observations.

In this paper we model markets in which a proportion of the consumers bargain with sellers. To capture the effects of bargaining most succinctly, we introduce an original two-stage model of the price setting and subsequent bargaining process. In the first stage competing sellers establish a market price via simultaneous quantity choices. This market price acts as a publicly observed list price which will be the transaction price of the non-bargaining price takers. In the second stage the bargainers bargain with the sellers. An important innovation in this paper is in how

bargaining is modelled: the bargainers are assumed to approach one or more of the sellers and request a price better than the posted list price. The number of sellers given the opportunity to lower price by any given buyer is private information. This model of bargaining is an adaptation of Burdett and Judd's (1983) work on non-sequential search except here the upper bound of prices is endogenously decided by the first stage competition for the non-bargainers. Paralleling Burdett and Judd (1983), the sellers offered the chance to re-quote will select a price below the upper bound (here the list price) according to a probability distribution. In choosing what price to offer bargainers, the sellers trade off the possibility that (i) the bargainer may not approach any other firm and so a high price would be profitable for the seller; and (ii) the bargainer may approach multiple sellers for second quotes and so a seller would need to offer a low price to be successful. Our bargainers are restricted to taking or leaving any second quote offered to them. One can clearly imagine more complicated bargaining scenarios with other sequences of quotes and perhaps counter offers. However, our simple model is both tractable and captures four key features of bargaining: (a) bargaining involves actively requesting a seller to lower her initial price offer; (b) not all consumers bargain; (c) not all bargainers bargain with the same number of sellers; and (d) the bargained price is random with some buyers doing better than others.

This bargaining model highlights a key trade-off faced by sellers. On the one hand sellers wish to compete for the price takers in the standard way by increasing their market share which pushes the list price down. However, the list price acts as an upper bound on the bargained prices: the lower the list price the lower the prices which have to be offered to bargainers. Thus sellers wish to see a high list price to allow rents to be extracted even after bargained reductions are offered to the bargainers. As the proportion of bargainers changes, the relative importance of competing for price takers' business changes. If the proportion of bargainers goes up, competition for price takers becomes less desirable allowing the market price to rise towards collusive levels. Therefore consider the following not unusual policy recommendation made in the Office of Fair Trading (2004) report on estate agency:

"Greater shopping around and negotiation by consumers will increase competitive pressures on estate agents and result in better value for money in terms of both lower prices and higher service quality" (OFT 2004, §1.12)

We find that as the proportion of bargainers rises, consumers are split into three distinct

**Average discounts and standard deviation for some of the model ranges of the leading six suppliers, 1997 and 1998**

Supplier	Model range*	Average discount %	Standard deviation
Vauxhall	⎧	13.5	8.3
		12.2	7.6
Ford	⎧	12.2	10.2
		14.5	11.2
		10.1	4.6
Renault	⎧	4.4	3.7
		8.9	8.6
		8.9	5.7
Peugeot	⎧	8.2	6.7
		10.0	7.6
		11.9	9.6
Rover	⎧	3.5	4.1
		7.0	4.8
		6.0	4.9
Volkswagen	⎧	3.2	2.5
		2.7	3.0
		4.0	3.4

Source: Commission calculations on OFT data.

\*Includes model ranges with more than 20 observations.  
 †65 per cent of the observations were from the same dealer group.  
 ‡88 per cent of the observations were from the same dealer group.

Figure 2: UK Car Discounts at the Model Range Level from Competition Commission (2000)

camp: those who remain not bargaining, those who start bargaining, and those who were already bargaining. The new bargainers unambiguously gain as they swap a high list price for a lower bargained price. However, the list price and the bargained prices rise as the proportion of bargainers grows - thus the first and third camp of consumers experience higher prices. Overall this effect can dominate and at best an exhortation to bargain is a mixed bag.<sup>1</sup>

Our model of bargaining departs from the standard Nash and Rubinstein approaches as it offers a mixed strategy equilibrium and hence a probability distribution of prices. Evidence consistent with our predicted mixed strategy equilibrium can be found in Figure 1 displayed above, which shows that the discounts offered to customers in the UK car market vary widely. One might wonder whether this is an artefact of the aggregation of data to the manufacturer level, but Figure 2 from the same Competition Commission report shows that variance in discounts is similarly large at the model range level.

Our main model takes the proportion of bargainers as exogenous, deriving comparative statics as we change this proportion. In Section 5 we extend our model by introducing a cost to

<sup>1</sup> We note that the Internet may increase the proportion of bargainers by making bargaining easier. Zettelmeyer et al. (2006) find that customers who use the Internet reduce automobile dealers' gross margins by 22% and that much of this effect is driven by online referral services, such as Autobytel.com, which request quotes from dealers on behalf of buyers. Such services are used by about 20% of new car buyers in the sample.

bargaining and endogenizing the consumers' decision of whether to bargain or not. We find that the thrust of our results is preserved, and some of our results are actually strengthened, when the cost of bargaining is lowered to encourage some of the price takers to become bargainers. We also discuss how our results would extend to a model of sequential search.

To the best of our knowledge our work is unique in its focus on the relationship between the prices paid by non-bargainers and the equilibrium prices achieved by those who bargain in the same market, and on how the proportion of consumers who bargain affects consumer surplus through this relationship.

Much of the literature exploring bargaining by sellers considers the choice between committing to a fixed price and allowing consumers to bargain (see e.g., Bester (1993), Wang (1995), Arnold and Lippman (1998) and Camera and Delacroix (2004)). These models have the feature that no list price is set if the seller intends to bargain, so the interplay between the prices paid by bargainers versus non-bargainers cannot be explored.

A small number of papers allow bargaining below a list price, but to our knowledge none, bar Raskovich (2007), considers how the proportion of bargainers impacts on the list price. In Camera and Selcuk (2007) and Chen and Rosenthal (1996a, 1996b) there is only the bargaining type of consumer. More closely related to our paper, Desai and Purohit (2004) have two types of consumers, *hagglers* (who try to bargain) and *non-hagglers* (who accept the list prices). The hagglers are more price sensitive so retailers can use bargaining to screen consumers (an effect absent from our model). Two firms post prices and choose whether to commit to their posted price or allow bargaining. When both firms allow bargaining, Desai and Purohit only consider parameter values for which the posted prices do not act as effective outside options for the hagglers, so the posted prices are unaffected by the hagglers. Raskovich (2007) finds that when there are enough bargainers, list prices can jump from marginal cost to the monopoly price, but the mechanism which raises the list prices is different from ours. Raskovich assumes that firms setting higher list prices are weaker bargainers and so receive more business from bargaining consumers.

We have already noted that our model reinterprets and extends Burdett and Judd's (1983) analysis of price dispersion as the second part of a bargaining interface between sellers and

consumers.<sup>2</sup> Janssen and Moraga-Gonzalez (2004) extend Burdett and Judd by endogenizing the number of firms searched. They find that increasing rival numbers can lower welfare: the price distribution becomes more extreme as informed consumers see more prices, and fewer uninformed consumers search at all. Though we do not conduct the same analysis, we note that the Janssen and Moraga-Gonzalez result would not extend directly to our setting: introducing more sellers increases the incentive to push down the market price to win price takers, and so lowers prices to all types of buyers (taking into account the effect on the bargained prices).

Cason et al. (2003) experimentally study the difference between markets with take-it-or-leave-it posted prices and ones where the consumers can haggle, using the posted prices as a starting point. They find that where the consumers can haggle, prices tend to be higher and efficiency lower. Similar results are found in the experimental study of Davis and Holt (1994). Our model provides a possible theoretical justification for these findings.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 derives the main results of the paper through comparative statics analyses involving the proportion of bargainers, the number of competitors and other variables. Section 4 analyses the sellers' optimal bargaining strategy. Section 5 extends the model by endogenizing the consumers' decision of whether or not to bargain. Section 6 concludes. All proofs are relegated to the appendix.

## 2 The Model

Consider a market in which  $N \geq 2$  firms compete to sell a homogeneous good. Each firm has the same constant marginal cost of production which we normalize to 0. There are  $M$  consumers who each purchase  $\frac{1}{M}$  units of the product if and only if their valuation  $v_i$  exceeds the price. Each consumer's valuation is drawn from the uniform distribution:  $v_i \sim U[0, 1]$ . Taking the limit as  $M \rightarrow \infty$ , so that each consumer is small and atomistic, the proportion of consumers buying at price  $p$  becomes deterministic and tends to  $\Pr[v_i \geq p] = 1 - p$ . Thus the aggregate demand will be

$$Q(p) = \lim_{M \rightarrow \infty} \frac{1}{M} \times M \times (1 - p) = 1 - p \quad (1)$$

<sup>2</sup> There are other related papers with a mixed strategy dispersed price equilibrium arising from differential search, including seminal examples such as Varian (1980) and Stahl (1989).

We also note that if a randomly drawn consumer is offered price  $p$  then their expected consumer surplus (CS) is given by

$$E[CS_i] \text{ at price } p = \int_p^1 (v_i - p) dv_i = \frac{1}{2}(1 - p)^2 \quad (2)$$

A proportion  $\mu \in (0, 1]$  of the population are ‘price takers’. The prevailing market (or list) price  $z$  is assumed to be the outcome of Cournot competition: each firm chooses a volume  $x_i$  to supply to the price takers, and the market price equates demand and supply. Each firm can adjust this market price by increasing or decreasing the volumes it supplies to the price takers, who do not seek to alter or bargain down from the market price  $z$ . We restrict  $\mu > 0$  to ensure that there are always some price takers so the Cournot market price is well-defined.

Together with the downward-sloping demand, our preferred interpretation is to consider the Cournot competition for the price takers as a reduced form of an oligopolistic interaction which captures the following key features: (i) a more aggressive action (modelled as a higher volume) raises the aggressor’s market share amongst the price takers without gaining the entire market, so competition is imperfect; (ii) by reducing the market price, the aggressive action increases the size of the total price taker market while having a cannibalizing effect on the aggressor’s existing sales to price takers; and (iii) the oligopolistic competition for price takers determines a market (or list) price which acts as an upper bound on the prices that can be quoted to any consumers who attempt to bargain.<sup>3</sup>

A second possible interpretation of our model is that the  $N$  competing firms have sales representatives who are each issued with a sales volume target: the volume target for the sales representative of firm  $i$  being  $x_i$ . The (Cournot) price of  $z$  is the market price which allows the sales representatives to sell exactly the volumes they have been told to achieve. The sales representatives are not however authorized to bargain. Thus the  $1 - \mu$  buyers who seek a price different from the market price must negotiate with the manager, and so sales to these consumers do not count against the sales representatives’ volumes. In either interpretation the  $\mu$  price takers are offered a market price of  $z$  which depends upon the strategic interaction of the competing firms.

---

<sup>3</sup> If we allowed firms to set their list prices directly and assumed homogeneous products, Bertrand-type price competition for the price takers would drive the list prices to marginal cost. With differentiated products, the analysis of the bargaining stage pricing distributions becomes intractable.

The other  $1 - \mu \in [0, 1)$  consumers are ‘bargainers’. After the Cournot market price is posted the bargainers each ask some firms for second quotes. A proportion  $q_k$  of the bargainers randomly approach  $k$  of the firms for second quotes, for  $k \in \{1, 2, 3, \dots, N\}$ . Of course  $\sum_{k=1}^N q_k = 1$ , and we assume that  $q_1 \in (0, 1)$  so some bargainers ask for only a single second quote while the others ask for more. If a seller is asked to offer a better price it is ignorant of how many other (if any) firms are being similarly approached by this consumer, and the price it quotes is assumed to be a final and binding take-it-or-leave-it offer to the consumer. Each consumer then buys the good at the lowest price from the  $k$  quotes (randomizing in the event of a tie) if and only if their valuation  $v_i$  exceeds this lowest price. The Cournot market price is assumed to be binding on the sellers - quotes cannot be offered above this level. The propensity to bargain with multiple firms is assumed independent of the consumers’ valuations. That is the distribution of valuations is independent of the distribution of the number of quotes requested  $\{q_1, q_2, \dots, q_N\}$ .<sup>4</sup> In Section 5 we relax these assumptions and endogenize the consumers’ decision of whether or not to bargain.

The firms have two strategic decisions to make. The first is to decide how aggressively to compete for price takers: this will dictate the market price  $z$  which the price takers receive in a Cournot type process. Should a firm be invited to offer a better price then it must decide what to bid. This is the second strategic choice. The firms seek to maximize their expected profits. The model’s parameters  $\{\mu, N, q_1, q_2, \dots, q_N\}$  and payoff functions are common knowledge. We restrict attention to symmetric equilibria and, where possible, to pure strategies.

### 3 The Consumer Surplus Effects of Bargaining

In this section, we analyse how the surplus of consumers is affected by changes in the proportion of bargainers and by the number of firms in the market. We start by stating the main results in Theorem 1. We then solve the model by backwards induction, looking first at the bargaining subgame given the market price, and then at the market price setting game. Next we provide intuitions and a sketch proof for Theorem 1, and finally we show that consumer surplus can be

---

<sup>4</sup> Note that empirically this may not always be the case. For example, the differences in bargained reductions in the UK car market between luxury cars (small reductions) and mass market cars (big reductions) highlighted in Figure 1 may, in part, be explained by the luxury brand buyers seeking multiple second quotes less often. This might be due to the greater value of time amongst luxury buyers and therefore a reduced propensity to bargain.

non-monotonic in the proportion of bargainers.

**Theorem 1**

1. *The price takers' consumer surplus is strictly decreasing as the proportion of price takers declines;*
2. *The bargainers' expected consumer surplus is strictly decreasing as the proportion of price takers declines;*
3. *If a small set of consumers swap from price taking to bargaining, they pay less;*
4. *More bargainers is:*
  - (a) *bad for overall consumer surplus if the bargainers seek multiple second quotes with low enough probability;*
  - (b) *good for overall consumer surplus if the bargainers seek multiple second quotes with high enough probability;*
5. *Increasing the number of competing sellers unambiguously raises overall consumer surplus.*

**3.1 The Bargaining Subgame**

We begin our analysis by considering the bargaining subgame. Suppose that the Cournot market clears at a price  $z > 0$  at which the price takers purchase. There then remains a measure  $(1 - \mu)M$  of bargaining consumers who each desire  $\frac{1}{M}$  units which they each value at a random draw from  $U[0, 1]$ . We search for a symmetric equilibrium in which each firm decides to set its quoted prices from some distribution  $F(p)$ . Prices in excess of  $z$  cannot be sustained in equilibrium in this subgame as the Cournot market price forms a binding upper bound by assumption. Hence the support of  $F$  is contained within  $[0, z]$ , so  $F(z) = 1$ .

The monopoly price in this model is  $\frac{1}{2}$ : if a monopolist offers a price  $p$  it will get a demand of  $1 - p$ , so expected profits  $p(1 - p)$  are maximized at  $\frac{1}{2}$ . Letting  $\hat{z} \equiv \min\{z, \frac{1}{2}\}$ , we get the following lemma.

**Lemma 1** *Given  $z > 0$ , any symmetric equilibrium must be mixed with (i)  $F(p)$  continuous, i.e., with no mass points in the density function at any price strictly above zero; (ii)  $F(p) < 1$  for  $p < \hat{z}$ ; and (iii)  $F(\hat{z}) = 1$ .*

In the mixed strategy equilibrium the firms are trading off the incentive to price high to extract surplus from the bargainers who ask for few second quotes against the incentive to price low to sell to customers who ask for many quotes. No pure strategy equilibrium can exist: if the price were positive the firms would just undercut it, and if the price were zero the firms would want to raise price towards  $z$  to extract surplus from the bargainers who ask for just one second quote.

The expected profit from the  $1 - \mu$  bargainers for a firm which sets a price of  $p \leq \hat{z}$  when asked for a second quote (that is a quote subsequent to the Cournot market price) is then given by

$$\frac{\pi(p)}{1 - \mu} = p \underbrace{(1 - p)}_{\substack{\text{probability} \\ \text{consumer} \\ \text{willing to} \\ \text{buy at price } p}} \cdot \left[ \sum_{k=1}^N q_k \underbrace{\frac{k}{N}}_{\substack{\text{probability} \\ \text{selected as} \\ \text{one of the} \\ k \text{ firms}}} (1 - F(p))^{k-1} \right] \quad (3)$$

For any  $p$  in the support of  $F$  we must have  $\pi(p) = \pi$  a constant implying that

$$\frac{N\pi}{(1 - \mu)p(1 - p)} = \sum_{k=1}^N kq_k (1 - F(p))^{k-1} \quad (4)$$

The right hand side is a smooth strictly increasing function of  $1 - F(p)$  and so must have a strictly increasing inverse function  $\Phi(\cdot)$  such that for any  $p$  in the support of  $F$ :<sup>5</sup>

$$F(p) = 1 - \Phi\left(\frac{N\pi}{(1 - \mu)p(1 - p)}\right) \quad (5)$$

Note that as  $p(1 - p)$  is strictly increasing in  $p$  for  $p < \hat{z}$  and  $\Phi$  is a strictly increasing function,  $F(p)$  must also be strictly increasing. Thus there can be no gaps in the density function.

From Lemma 1, the upper bound of the support of  $F$  is at  $\hat{z}$  so from (3) we can write profits

<sup>5</sup> Burdett and Judd (1983) make an equivalent argument.

from the bargainers as

$$\pi = \pi(\hat{z}) = \frac{1 - \mu}{N} \hat{z} (1 - \hat{z}) q_1 \quad (6)$$

As expected, profits are increasing in the proportion of bargainers who ask for just one quote. Profits are also increasing in the market price, but only up to  $z = \frac{1}{2}$ , after which profits remain constant as the firms do not wish to quote prices above the monopoly level.

Substituting (6) into (5), we get

$$F(p) = 1 - \Phi\left(\frac{\hat{z}(1 - \hat{z})q_1}{p(1 - p)}\right) \quad (7)$$

This tells us that given the Cournot market price and the bargaining strategy of consumers, the prices offered to bargainers are independent of the number of firms  $N$ .<sup>6</sup> Intuitively this is because a firm chooses a price to offer, when asked to lower its price below the market rate, optimally against the number of quotes it will be competing against in expectation *conditional on being chosen to give a second quote*. This depends on how consumers bargain and not on the total number of firms available.

In the subsequent analysis it will be useful to explicitly note the lower bound on prices,  $\underline{p}$ , in the bargaining phase. This can be deduced from the equal profit condition, so using (3) once more  $\underline{p}$  is given implicitly by<sup>7</sup>

$$\hat{z}(1 - \hat{z})q_1 = \underline{p}(1 - \underline{p}) \sum_{k=1}^N kq_k \quad (8)$$

### 3.2 The Market Price Setting Stage

By deciding how much volume to bring to the Cournot market for price takers each firm can influence the market price which bargainers see as their outside option. This has a knock on effect on expected profits from the bargaining stage, which the firms anticipate. The market price therefore depends on what proportion of consumers are bargainers  $(1 - \mu)$  and on how they bargain. The result of this analysis is:

<sup>6</sup> Of course, for a given bargaining strategy, we are not permitting  $N$  to fall below the highest  $k$  for which  $q_k > 0$  or the bargaining strategy would have to change.

<sup>7</sup> Note that  $\underline{p}$  is uniquely defined on  $(0, \hat{z})$  as  $p(1 - p)$  is falling as  $p$  falls below  $\hat{z}$  and  $\sum_{k=1}^N kq_k > q_1$ . Also,  $\underline{p}$  is increasing as  $\hat{z}$  rises and as  $q_1 \rightarrow 1$ ,  $\sum_{k=1}^N kq_k \rightarrow q_1$ , so  $\underline{p}(1 - \underline{p}) \rightarrow \hat{z}(1 - \hat{z})$  and hence  $\underline{p} \rightarrow \hat{z}$ .

**Lemma 2** *The unique equilibrium in pure strategies of the market price setting stage is symmetric with a market price given by*

$$z = \frac{\mu + q_1(1 - \mu)}{\mu(N + 1) + 2q_1(1 - \mu)} < \frac{1}{2}$$

Note first that as  $z < \frac{1}{2}$ ,  $z = \hat{z}$  so from this point on we can discard the notation  $\hat{z}$ . Note further that this result has the expected limiting properties. That is, as the proportion of price takers  $\mu$  tends to one (no one bargains) or the proportion of bargainers asking for just one quote  $q_1$  tends to zero (no profit can be made from bargainers), this market price tends to  $\frac{1}{N+1}$ , the "standard" Cournot price. As the proportion of price takers  $\mu$  tends to zero, the market price tends to the monopoly level of  $\frac{1}{2}$ : making profits from bargainers becomes key. The market price is falling in the number of competing firms  $N$ ; and as the number of firms tends to  $\infty$ , the market price goes to zero (firms' normalized marginal cost).

### 3.3 Intuitions For Theorem 1

The market price offered to price takers (non-bargainers) depends upon the proportion of consumers who will bargain and how effective they are at it (that is, how likely the buyers are to request second quotes from more than one seller). Lemma 2 and the distribution of prices in the bargaining stage established in (7) above are the foundations for proving Theorem 1. The proof remains technical however and so we confine it to the appendix. We instead provide an intuition and a sketch proof for the 5 parts of the theorem below.

Part 1 of Theorem 1 states that consumer surplus for the price takers goes down if the proportion of price takers declines. Algebraically this result follows quickly from Lemma 2. The intuition behind this very general result is as follows. Consider the bargaining stage first: the firms do not want the consumers to have a very good outside option. That is the firms do not want the consumers to have the option to buy at a low prevailing market price as this pulls down the expected profits from the bargaining consumers. More specifically, note that when the seller is approached by a bargainer, the Cournot market price must be in the distribution from which the seller chooses her price (if not, a firm setting the highest price, which only sells to bargainers asking for just one quote, could profitably raise price towards the monopoly level). If the seller offers the Cournot market price in the bargaining phase, it will sell only if the bargainer does

not approach any other firm for a second quote. As the Cournot market price declines the profit made from these consumers declines also, and hence the expected profits from bargainers falls as the seller is indifferent between all prices in her strategy. So to maximize profits from the bargainers, a high market price is required. However, a high market price implies that a seller could increase profits from price takers by unilaterally expanding output, lowering the market price, but taking market share from her rivals. As the proportion of bargainers grows, the lost profits from less active competition for the price takers become less significant and market prices are set with the bargainers more in mind. That is, for the price takers left behind, prices rise towards collusive levels and their consumer surplus falls.

Part 2 of Theorem 1 states that the incumbent bargainers' expected consumer surplus also declines as former price takers become bargaining consumers. To show this we prove that as the proportion of the consumers who bargain grows, the distribution of prices quoted in the bargaining stage is deformed in a first order stochastically dominant way. That is, the probability of a bargained price above any given level is higher if there are lots of bargainers than if there are few. It is clear from part 1 that as the proportion of bargainers grows the market price the price takers accept, and so the upper bound to bargained prices, also grows. Thus the expected profits from the bargaining consumers grow, and so the consumers must accept higher prices more frequently. This is captured formally in Lemma 4, stated in the appendix, which proves the first order stochastic dominance result. Given that bargained prices grow, in expectation, with more bargainers, it is then immediate that consumer surplus for the incumbent bargainers falls if the proportion of bargainers increases.

Part 3 of Theorem 1 follows from the fact that the bargained prices are strictly below the market clearing price. This is immediate from Lemma 1: sellers lower their prices in the bargaining phase as the buyer might solicit multiple second quotes ( $q_1 < 1$ ).

Therefore we've shown that if more people bargain then the people who haven't changed behaviour (stay bargaining or stay not bargaining) both lose out as prices for both groups rise relative to their positions before the change. This is of some policy interest as any exhortation to bargain - that is to actively give sellers the chance to revise their quoted prices downwards - creates losers amongst consumers who either ignore, or were already following, the advice. However, those consumers who do start to bargain swap a high market price for a lower bargained

price. Is overall consumer surplus raised or lowered in this case? This question is taken up in Part 4 of Theorem 1. This part of Theorem 1 offers an answer in the two extremes of consumers being either very good or very bad at bargaining. In this model good bargaining is when *multiple* sellers are offered the chance to revise their quotes downwards ( $q_1 \searrow 0$ ): this forces sellers to price down aggressively and results in low prices for bargainers. Bad bargaining is when buyers tend to only ask one seller for a better quote: the seller has little incentive to re-price below the prevailing market price in this case. Technically the proof considers the sign of the derivative of total consumer surplus with respect to the proportion of price takers in the extremes of good ( $q_1 \searrow 0$ ) and bad ( $q_1 \nearrow 1$ ) bargaining.

As the bargainers become bad at bargaining ( $q_1 \nearrow 1$ ), that is they tend to ask only one seller to move below the market price, there is little incentive to lower the prices below the market price level. The benefits of having a high market price are therefore more fully extracted during the bargaining phase. Raising the proportion of bargainers then strengthens the incentive to force a high market price for the remaining price takers in order to extract more from the bargainers. So all prices rise further, while the consumers switching to becoming bargainers only get a small discount, and hence aggregate consumer surplus falls.

The flip side of this reasoning is that if bargainers become adept at bargaining ( $q_1 \searrow 0$ ) and so often seek multiple quotes, then bargainers are able to force large discounts. This diminishes the value to the sellers of having a high market price which foregoes profit increases from unilateral moves to increase market share amongst the price takers as few rents are extracted from bargainers in any case. Therefore sellers become relatively keener to extract maximum gains at each others' expense from the price takers and so prices are pushed down towards regular Cournot levels. Thus raising the proportion of bargainers has little impact on prices, but the consumers who switch to bargaining get the much lower prices offered to bargainers and total consumer surplus therefore rises.

Part 5 of Theorem 1 notes that more sellers unambiguously raises consumer surplus whatever the proportion of bargainers. For price takers increasing the number of competitors lowers the Cournot market price in the standard way: any price reduction is internalized only for a smaller fraction of total demand and so sellers are keener to lower prices by increasing their volumes. For the bargainers, a lower Cournot market price provides a lower upper bound to prices in the

bargaining phase. Further, the lower maximum price pushes the distribution function of prices offered to bargainers down in a first order stochastically dominated way. Thus more probability weight is put on lower prices and so bargainers receive lower expected prices also. Hence all consumers gain if the number of competing firms rises.

### 3.4 Non-Monotonicity of Consumer Surplus

We have established that more bargainers can be detrimental to consumer welfare at the extremes of bargaining proficiency (Theorem 1, part 4). It remains unclear how overall consumer surplus changes in the proportion of bargainers at intermediate levels of consumer bargaining prowess. By example, we can show that changing the proportion of price takers while keeping bargaining ability fixed does not always alter overall consumer surplus in a monotonic way. Consumer surplus can, for some parameter values, be maximized at intermediate proportions of price takers versus bargainers. For example, for  $q_1 = q_2 = \frac{1}{2}$  and  $N = 5$  total consumer surplus is maximized at  $\mu^* = 0.39$ , i.e., where 39% of consumers take prices as given while the other 61% bargain. If  $q_1$  rises to  $\frac{3}{5}$  (and  $q_2$  falls to  $\frac{2}{5}$ ), the optimal proportion of price takers rises to 0.78, so we only want a handful of bargainers. Calculating these numbers is involved, but essentially straightforward, so the details are omitted. General results cannot be derived, as it is only possible to calculate an analytical solution for the pricing distribution  $F(p)$  in special cases.

## 4 The Sellers' Optimal Bargaining Strategy

This section aims to explore how the market price and sellers' choice of price distribution to offer to bargainers change in response to alterations in the proportion of bargainers ( $\mu$ ), the number of rivals ( $N$ ) or the consumers' bargaining strategy  $\{q_k\}$ . We start with Theorem 2 which looks at the impact of  $\mu$  and  $N$ , and then move on to Theorem 3 which analyses the impact of  $\{q_k\}$ .

### Theorem 2

*If the proportion of bargainers increases, **or** if the number of competing sellers falls then:*

1. *The market price set for price takers rises;*
2. *The lowest price offered to bargainers rises;*

3. *The distribution of prices offered to bargainers places more weight on higher prices in a first order stochastically dominant fashion;*
4. *The spread of possible bargained prices grows.*

Theorem 2 is closely related to parts 1, 2 and 5 of Theorem 1. The intuition for the first part follows closely from the discussion in Section 3.3. If the proportion of bargainers increases, or if the number of competing sellers falls then the market price for the price takers rises. With fewer competing sellers this happens for the standard Cournot reason - each seller internalizes more of the industry losses from over supply to the market and so acts to restrict demand and drive up the price. With more bargainers the firms rank reducing profit loss from bargainers more heavily than reducing profit loss from price takers by under supply (too high a list price).<sup>8</sup> In either case the higher market price causes the upper bound of prices which can be quoted to bargainers to rise.

Parts 2 and 3 of Theorem 2 then follow as a seller is only willing to quote a price if the profits it makes in expectation are equal to the expected profits which the seller would make from not discounting and re quoting the market price. With the latter behaviour only those buyers who solicit a second quote from one seller will purchase, and as the market price goes up so do profits from this strategy. Hence the lower bound of the offered prices and the price distribution in general migrate up to higher price quotes.

Part 4 of Theorem 2 is a useful mnemonic for the optimal seller bargaining strategy. The result can be understood by noting that if the number of competitors were to rise high enough then the market price would approach marginal cost (0 here) and bargainers would enjoy no reduction: that is the spread of bargained prices would shrink to nothing. As the market price for the price takers rises, the lowest bargained price will rise too - but not as fast as the market price as it always lags behind. Part 4 confirms this insight as a robust general result.

Theorem 3 considers two bargaining strategies  $\{q_i^{\text{poor}}\}$  and  $\{q_i^{\text{good}}\}$ . The good bargaining strategy has buyers strictly less likely to solicit a second quote from only one seller, and weakly

---

<sup>8</sup> Note that in both interpretations of the Cournot competition for price takers, the list price rise is achieved by lowering the volumes / sales targets more than proportionately to the reduction in the measure of price takers.

more likely to solicit a second quote for any given multiple number of sellers. In particular:

$$\begin{aligned} q_1^{\text{good}} &< q_1^{\text{poor}} \text{ and} \\ q_k^{\text{good}} &\geq q_k^{\text{poor}} \quad \forall k \in \{2, 3, \dots, N\} \text{ with strict inequality for some } k \end{aligned} \tag{9}$$

In this case the following holds:

**Theorem 3**

*If the population of bargainers should move to using the good bargaining strategy  $\{q_i^{\text{good}}\}$  as opposed to the poor strategy  $\{q_i^{\text{poor}}\}$  then:*

1. *The equilibrium market price falls;*
2. *The lowest price offered to bargainers falls;*
3. *The distribution of prices offered to bargainers places more weight on lower prices in a first order stochastically dominated fashion.*

Note that it immediately follows from Theorem 3 that total consumer surplus rises as the bargainers become better at bargaining as all prices are falling while the proportion of bargainers remains unchanged. To understand Theorem 3 note that in the bargaining stage the sellers will only make quotes which give them the same profit level in expectation: otherwise some price quote would be suboptimal. The expected profits which can be made from the bargaining stage can therefore be deduced as the profit level if the seller should refuse to offer a reduction against the market price, in which case only those buyers who solicit a second quote from only that seller will purchase. Thus the expected profits from the bargaining stage depend only on the probability of soliciting one second quote ( $q_1$ ), and not on any of the other probabilities. Hence part 1 follows as  $q_1^{\text{good}} < q_1^{\text{poor}}$  and so if buyers use the good bargaining strategy sellers expect to make less profit from bargainers and so the incentive to overprice to price takers is reduced.

The lowest second quoted price to bargainers also falls under the good buyer bargaining strategy (part 2). This is because bargainers are interacting with more sellers on average, and so to maintain some probability of quoting the lowest second price amongst those sought, the seller will have to push her lowest quoted price down. Part 3 then confirms that this insight

applies to all prices which might be quoted: with the good buyer bargaining strategy (in the sense of condition (9)) the seller has to push prices down across the board.

## 5 Endogenizing the Decision to Bargain

Our main model takes the proportion of bargainers as exogenous, deriving comparative statics as we change this proportion. Conducting this analysis we have demonstrated that altering the proportion of bargainers in the population does not affect all consumers' surplus in the same direction. Some gain, others lose, and more bargainers can lower consumer surplus overall.

In this extension we explore the robustness of our insights when the decision to bargain is made endogenous. To this end we begin by introducing a cost of bargaining  $c > 0$ .<sup>9</sup> We suppose that three types of consumer are present in the population. A proportion of the consumers  $\gamma \in (0, 1)$  can decide to pay  $c$  to become a bargainer. We call these *potential bargainers*. If these consumers elect not to incur cost  $c$  and bargain then they can purchase at the market price should they wish. To ensure that the Cournot market and bargaining stage are well-defined, we retain a proportion of consumers  $\mu \in (0, 1)$  who always take the market price  $z$  as given (*price takers*) and a proportion  $1 - \mu - \gamma \in (0, 1)$  who always bargain (*costless bargainers*). We can think of the price takers as consumers whose cost of bargaining is so large that they would never consider bargaining, while the costless bargainers may actually enjoy the bargaining process.

Apart from the introduction of the potential bargainers, the model remains the same. In particular, a proportion  $q_k$  of the consumers who bargain ask for second quotes from  $k$  of the firms with  $q_1 \in (0, 1)$ . The potential bargainers choose whether to pay the one-off cost of bargaining  $c$  knowing the distribution of the number of second quotes sourced, but not knowing their own draw from this distribution. For instance, having chosen to become a bargainer by asking for an initial second quote, there may be uncertainty about how busy the consumer will turn out to be or about whether an opportunity to ask for a second quote from a given seller will arise. We assume that a potential bargainer who is indifferent chooses not to bargain.<sup>10</sup>

We start by considering the potential bargainers' decision given a market price  $z$  and a

<sup>9</sup> At the end of this section we also discuss the implications of making the number of second quotes sourced endogenous.

<sup>10</sup> Assuming instead that indifferent consumers bargain makes no qualitative difference to the analysis. The assumption does rule out mixing on the decision of whether to bargain or not.

symmetric pricing distribution  $F(p)$  for the firms when asked for a second quote. By the arguments of Lemma 1,  $F(p)$  is continuous; and if  $z \leq \frac{1}{2}$  (the monopoly price) then  $F(p)$  is supported on  $[\underline{p}_F, z]$ . Let  $\tilde{p}_F$  be the expected lowest of all second quotes that a bargainer expects to receive.

**Lemma 3** *Given a market price  $z$  and symmetric pricing distribution in the bargaining subgame  $F(p)$ :*

(i) *If  $c \geq z - \tilde{p}_F$  then none of the potential bargainers choose to bargain. Those with valuation  $v_i \geq z$  buy at the market price;*

(ii) *If  $c < z - \tilde{p}_F$  then there exists a  $\underline{v} < z$  such that potential bargainers with  $v_i > \underline{v}$  bargain while the rest do not buy at all.*

The result is proved formally in Appendix C, though the intuition is clear. If  $c \geq z - \tilde{p}_F$  the cost of bargaining exceeds the expected discount for consumers with  $v_i \geq z$ . Those with lower valuations benefit even less from bargaining as if they bargained but found that the best second quote they were offered was above their valuation  $v_i$ , they would not buy and so would not benefit from the discount. Overall, therefore, potential bargainers do not bargain. If we reduce  $c$  below  $z - \tilde{p}_F$  then potential bargainers with  $v_i$  close enough to  $z$  will choose to bargain, as of course will any with  $v_i \geq z$ .

We wish to consider the welfare consequences of a Competition Authority, for example, encouraging some of those consumers who buy at market prices instead to try and bargain sellers' prices down. In this extension, the consumers who are susceptible to the Competition Authority lowering the cost of bargaining are the potential bargainers (who all have the same cost of bargaining,  $c$ ). The following theorem therefore considers how all consumers are affected by a Competition Authority lowering the cost of bargaining sufficiently that potential bargainers strategically decide to move from price taking to bargaining (if their valuations are high enough to take part in the market at all).

**Theorem 4** *If the bargaining cost  $c$  is lowered sufficiently to move from an equilibrium in which the potential bargainers do not bargain to one in which some of them (those with  $v_i > \underline{v}$ ) do:*

1. *The market price rises, which reduces the price takers' expected consumer surplus;*

2. *The distribution of second quotes offered to bargainers places more weight on higher prices in a first order stochastically dominant fashion, which reduces the costless bargainers' expected consumer surplus;*
3. *The effect on the new bargainers' consumer surplus is ambiguous;*
4. *The overall effect on consumer surplus is:*
  - (i) *positive if bargainers seek multiple second quotes with high enough probability;*
  - (ii) *negative if bargainers seek multiple second quotes with low enough probability.*

Our results from the main model are actually strengthened. The thrust of our results is preserved: encouraging more bargaining hurts both the consumers who remain price takers and existing bargainers, and the overall consumer surplus effect is ambiguous. As before, encouraging more bargaining raises the market price (the price paid by price takers) as it becomes relatively more important for the firms that the bargainers face a poor outside option during the bargaining process. Once again the rise in the market price pushes up the prices offered in the bargaining phase.

However a second effect now operates to push the bargained prices up yet further, which was not present in our main model. When the costs of bargaining drop, not all the potential bargainers start to bargain: only those with valuations high enough that bargaining is expected to be worthwhile will elect to bargain. Those with lower valuations do not take part in the market. Therefore the demand from the bargainers overall (the potential bargainers who choose to bargain together with the costless bargainers) becomes less elastic than in our benchmark model. This is because for prices below the cut-off valuation of the potential bargainers (given a pricing distribution expected by the bargainers) lowering price does not increase the proportion of potential bargainers who choose to buy at all. Hence the incentive for a seller to choose the lowest prices in the pricing distribution falls and so bargained prices rise.

The new bargainers may or may not be better off: although they get a discount off the market price, all prices have jumped up and they have incurred the extra cost of bargaining. The overall effect on consumer surplus is once again ambiguous and depends on the bargaining skill amongst those who bargain. Just like in the main model, consumer surplus rises if bargainers approach

more than one firm for a second quote frequently enough and falls if they ask for just one second quote frequently enough.

We have not analysed the effect of assuming prices are such that some potential bargainers bargain, and then lowering the cost of bargaining incrementally to further raise the proportion of potential bargainers who bargain. Such a comparative static would not convert any consumers who had previously been willing to buy at the market price to bargaining, so there would be no impact on the market price.<sup>11</sup>

For simplicity, throughout we have fixed the distribution of the number of second quotes that bargainers source. However, allowing bargainers to choose sequentially how many second quotes to ask for does not affect the thrust of our results. Suppose we adopted Stahl's (1989) seminal model of sequential search to model bargaining in our core framework, instead of building on Burdett & Judd's (1983) model of non-sequential search. Stahl assumes that a proportion of consumers can search all firms for free while the rest see one price for free but have to pay a cost  $c$  for every further search undertaken. Let's return to our main model and maintain the assumption that the population is exogenously subdivided into different types of consumer. Building on Stahl, suppose we adjust the main model so that a fraction of the bargainers get to bargain with all firms for free (this first group have  $q_N = 1$ ) while the rest of the bargaining population bargain sequentially, receiving an initial second quote for free and then paying  $c > 0$  for each subsequent quote sourced. We can call the second group *hesitant bargainers*. The number of sellers to bargain with is therefore made endogenous for the hesitant bargainers. The price takers behave as in our main model.

Now suppose a Competition Authority (CA) gradually increases the proportion of bargainers by encouraging some price takers to become bargainers of the hesitant type. Arguably this is the most plausible case as if a CA increased the proportion of bargainers they would be changing those who didn't bargain, not into types who loved bargaining, but into types who could manage one request for a second quote but found subsequent bargains painful. Adapting standard arguments from the sequential search literature, it can be shown that in equilibrium the bargained price distribution will adjust so that the hesitant bargainers will not pay to

---

<sup>11</sup> Only potential bargainers whose valuations were so low that they were previously inactive would convert to bargaining. This idiosyncrasy of the model would vanish if potential bargainers had a range of costs of bargaining, though at a cost in tractability.

get further quotes: so they have  $q_1 = 1$ . (This is an implication of the fact that a firm will only be willing to offer a price at the top of the pricing distribution if some bargainers will accept it and stop searching.) As hesitant bargainers are created (by the CA for example), similarly to Stahl's Proposition 6, the bargained price distribution will shift up in a first order stochastically dominant fashion, so existing bargainers suffer. Initially however the top of the bargained price distribution will lie strictly below the standard Cournot price in the absence of bargainers and so bargaining will have no effect on the Cournot market price. As we raise the proportion of hesitant bargainers, the top of the bargained price distribution rises as a greater and greater proportion of the bargainers source second quotes from only one seller. Eventually the Cournot market price will become binding. From this point on the firms will once again have an incentive to raise the Cournot market price to make more profit from bargainers, so price takers also suffer from an increase in the proportion of bargainers and our core results continue to stand qualitatively.

## 6 Conclusion

In conclusion, using our relatively simple set-up we have been able to derive a rich picture of the impact of bargainers on markets with price takers. The model highlighted the strategic dependence of the market price (which the price takers see as final) on the rents which bargainers were able to extract. In the absence of bargainers, if market prices are high each firm has an incentive to try to take market share off its rivals by increasing output and accepting a small drop in the market price. However, when bargainers are present this price drop results in substantial extra profits being lost to those buyers able to bargain a reduction off the market price. Thus the presence of bargainers allows the market price to non-bargainers to be pushed closer to the collusive level. Of course bargainers do transact below the market price, but whether overall consumer surplus is benefited by bargaining depends on the parameters. Thus the best position for a consumer is to be one of few bargainers amongst a population consisting mainly of price takers. If more consumers start to bargain then the increase in prices to the remaining price takers and the incumbent bargainers can actually mean that total consumer surplus declines. Hence the title of this paper: too many bargainers can indeed spoil the broth.

The optimal proportion of bargainers for consumer surplus can be interior, or at an extreme

where nobody bargains or almost everybody does. When bargainers approach few sellers, consumers tend to be best off in the absence of bargaining. When almost all bargainers approach more than one firm, prices to the bargainers go to cost so consumers overall are best off if almost everyone bargains. Finally, increasing the number of sellers always lowers prices to both price takers and bargainers; and the more sellers the bargainers are likely to approach, the lower are all prices on average.

## Appendix

### A Proofs from Section 3

**Proof of Lemma 1.** Suppose first that there is a mass point at price  $p > 0$  in the density function. A firm could profitably deviate by lowering its quote price to  $p - \varepsilon > 0$  just below the mass point whenever it would have proposed  $p$ . This increases total sales by a discrete amount (when the consumer receives a lowest quote from a rival firm of  $p$  and has  $v_i \geq p$ ) in return for a vanishingly small loss and so is a profitable deviation. Suppose second that the support of  $F$  stops strictly below  $\hat{z}$ . Any firm charging the highest price could profitably deviate by raising price towards  $\hat{z}$  as in either case it will make a sale only if the bargainer asks for just one second quote, and expected profits from such a consumer of  $p(1 - p)$  are increasing as  $p$  rises towards  $\frac{1}{2}$ .<sup>12</sup> For part (iii) we need to show that sellers would never quote prices above the monopoly price of  $\frac{1}{2}$  to bargainers. To see this suppose for a contradiction that  $F(\hat{z}) < 1$ . As  $F(z) = 1$ , we must have  $z > \frac{1}{2}$  and  $\hat{z} = \frac{1}{2}$ . Then  $\exists p > \frac{1}{2}$  such that  $f(p) > 0$ . However the firm would do better to lower price towards  $\frac{1}{2}$ . Expected profits from a consumer asking for  $k$  quotes are  $p(1 - p)(1 - F(p))^{k-1}$ . These increase as  $p$  falls towards  $\frac{1}{2}$  as  $p(1 - p)$  goes up while  $F(p)$  falls.

■

**Proof of Lemma 2.** Suppose firm  $i$  elects to make a volume  $x_i$  available on the (first stage) Cournot market which is aimed at the price takers. Let  $X = \sum x_i$  be the total volume supplied. At a market price  $z \in [0, 1]$ , from (1) the price takers demand a quantity  $\mu(1 - z)$ ,

<sup>12</sup>This argument extends directly if the support of  $F(\cdot)$  is open so that  $\sup\{p : F(p) < 1\} = \bar{p} < \hat{z}$  as then a deviation from price  $p$  sufficiently close to  $\bar{p}$  that  $1 - F(p) < \varepsilon$  to a price of  $\hat{z}$  is profitable if  $\varepsilon$  is sufficiently small.

so given  $X$  the Cournot market price is given by

$$z(X) = \max \left\{ 1 - \frac{X}{\mu}, 0 \right\} \quad (10)$$

Using (6), the seller's total profits are therefore given by

$$\begin{aligned} \Pi(x_i) &= x_i z(X) + \text{expected profits from bargainers} \\ &= x_i z(X) + \frac{1-\mu}{N} \widehat{z}(X) (1 - \widehat{z}(X)) q_1 \end{aligned} \quad (11)$$

Recall that  $\widehat{z} \equiv \min \{z, \frac{1}{2}\}$ ; and note that  $\frac{\partial z(X)}{\partial x_i} = -\frac{1}{\mu}$  for  $z(X) > 0$ .

The next step is to determine firm  $i$ 's optimal volumes. For  $z > \frac{1}{2}$ ,

$$\frac{\partial \Pi(x_i)}{\partial x_i} = z(X) - \frac{x_i}{\mu} \quad (12)$$

$$\frac{\partial^2 \Pi(x_i)}{\partial x_i^2} = -\frac{2}{\mu} < 0 \quad (13)$$

For  $z \in (0, \frac{1}{2})$ ,

$$\frac{\partial \Pi(x_i)}{\partial x_i} = z(X) - \frac{x_i}{\mu} + q_1 \left( \frac{1-\mu}{N\mu} \right) (2z(X) - 1) \quad (14)$$

$$\frac{\partial^2 \Pi(x_i)}{\partial x_i^2} = -\frac{2}{\mu} + q_1 \left( \frac{1-\mu}{N\mu} \right) \left( -\frac{2}{\mu} \right) < 0 \quad (15)$$

At  $z = \frac{1}{2}$ , the right-hand side derivative is (12) while the left-hand side derivative is (14).

However, as  $2z(X) - 1 = 0$  these both equal (12). The right-hand side second derivative is (13)

while the left-hand side second derivative is (15).

In all these cases the optimal  $x_i^*$  is uniquely determined (possibly at the corner solution of  $x_i^* = 0$ ) so the equilibrium must be symmetric. Note also that the objective function is everywhere strictly concave.

Suppose first that  $z \geq \frac{1}{2}$  is an equilibrium. From (12) the first order condition gives  $x_i^* = \mu z$ . By symmetry,  $X = Nx_i^* = N\mu z$ . Thus (10) implies that  $z = 1 - \frac{X}{\mu} = 1 - Nz$  and so  $z = \frac{1}{N+1}$ . As  $N \geq 2$ , we therefore get  $z < \frac{1}{2}$ , a contradiction.

Suppose instead that  $0 < z < \frac{1}{2}$  is an equilibrium. From (14) the first order condition gives

$$Nx_i^* = \max \{N\mu z + q_1(1 - \mu)(2z - 1), 0\} \quad (16)$$

As  $z = 1 - \frac{X}{\mu}$  from (10), and  $X = Nx_i^*$  by symmetry,  $Nx_i^* = \mu - z\mu > 0$ . Equating this with (16), we get the market price for price takers given by

$$\begin{aligned} z [N\mu + 2q_1(1 - \mu) + \mu] &= \mu + q_1(1 - \mu) \Leftrightarrow \\ z &= \frac{\mu + q_1(1 - \mu)}{\mu(N + 1) + 2q_1(1 - \mu)} < \frac{1}{2} \end{aligned}$$

as required.

Finally, suppose that  $z = 0$  is an equilibrium, so  $\sum x_i \geq \mu$ . All firms make zero profit. Suppose first that there is a firm  $i$  for which  $\sum_{j \neq i} x_j < \mu$ . This firm can raise profits by lowering output and hence raising price above zero. If no such firm exists, we have an equilibrium with excess supply and zero price. However, we will disregard such equilibria as they are not robust to introducing any positive marginal cost, which would induce all the firms to deviate to zero output. ■

## A.1 Proofs of All Parts of Theorem 1

**Proof of Part 1 of Theorem 1.** If the Cournot market price is  $z$ , then the average consumer surplus per non-bargainer is given by (2) as  $\frac{1}{2}(1 - z)^2$ . The consumer surplus of the price takers falls if the market price,  $z$  rises. The proof then follows by showing that  $\frac{\partial z}{\partial \mu} < 0$  so that more bargainers (lower  $\mu$ ) implies a higher market price for the remaining price takers. In particular, using Lemma 2 we have

$$\begin{aligned} \frac{\partial z}{\partial \mu} &= \text{sign} [\mu + q_1(1 - \mu) + N\mu + q_1(1 - \mu)](1 - q_1) - [\mu + q_1(1 - \mu)](1 - q_1 + N - q_1) \\ &= -q_1(N - 1) < 0 \text{ as } N \geq 2 \text{ by assumption} \end{aligned} \quad (17)$$

■

To make further progress we require the following technical result which states that if the proportion of bargainers rises, then the distribution of prices to the bargainers shifts upwards

so that higher prices are always more likely.

**Lemma 4** *If the proportion of price takers falls from  $\mu_2$  down to  $\mu_1$ , then the distribution of prices to the bargainers under  $\mu_1$  (denoted  $F_1(p)$ ) first order stochastically dominates the distribution of prices to bargainers under  $\mu_2$  (denoted  $F_2(p)$ ).*

**Proof.** Define the Cournot market price with a proportion  $\mu_i$  of price takers as  $z_i$ . By the proof of part 1 of Theorem 1 we have that  $\mu_1 < \mu_2 \Rightarrow z_1 > z_2$ . This just says that with fewer price takers the prices to the remaining price takers rises. Recalling from Lemma 2 that the Cournot market price must satisfy  $z < \frac{1}{2}$  we must have that  $z(1-z)$  is an increasing function of  $z$ . Hence

$$z_1(1-z_1) > z_2(1-z_2) \quad (18)$$

Now denote the distribution function of prices with  $\mu_i$  price takers as  $F_i$ , and the lower bound of its support as  $\underline{p}_i$ . Using (7), this can be written

$$\begin{aligned} F_i(p) &= 1 - \Phi\left(\frac{z_i(1-z_i)}{p(1-p)}q_1\right) \text{ for } p \in [\underline{p}_i, z_i] \\ F_i(p) &= 0 \text{ for } p \leq \underline{p}_i; \quad F_i(p) = 1 \text{ for } p \geq z_i \end{aligned} \quad (19)$$

where  $\Phi(\cdot)$  is a strictly increasing function. From footnote 7  $z_1 > z_2 \Rightarrow \underline{p}_1 > \underline{p}_2$ . Then for any  $p \leq \underline{p}_2$ ,  $F_1(p) = F_2(p) = 0$ , for any  $p \in (\underline{p}_2, \underline{p}_1]$   $F_1(p) = 0 < F_2(p)$ , for any  $p \in (\underline{p}_1, z_1)$  (18) implies that  $0 < F_1(p) < F_2(p) \leq 1$  and for any  $p \geq z_1$   $F_1(p) = F_2(p) = 1$ . Thus  $F_1(p)$  first order stochastically dominates  $F_2(p)$ . ■

We can now analyse the consumer surplus of bargainers and prove part 2 of Theorem 1.

**Proof of Part 2 of Theorem 1.** Suppose that the proportion of price takers falls from  $\mu_2$  down to  $\mu_1$ . The distribution of prices for the bargainers with  $\mu_i$  price takers is given by  $F_i$ . By Lemma 4 we have  $F_1 \succ_{\text{FOSD}} F_2$ . If a bargainer gets  $k$  second quotes ( $k \in \{2, 3, \dots, N\}$ ), then the lowest price she receives has distribution function  $H_i^k(p) = 1 - (1 - F_i(p))^k$  and so we have

$$\begin{aligned} F_1 \succ_{\text{FOSD}} F_2 &\Leftrightarrow F_1(p) \leq F_2(p) \quad \forall p \quad \& \quad \exists p \text{ s.t. } F_1(p) < F_2(p) \\ &\Leftrightarrow H_1^k(p) \leq H_2^k(p) \quad \forall p \quad \& \quad \exists p \text{ s.t. } H_1^k(p) < H_2^k(p) \\ &\Leftrightarrow H_1^k \succ_{\text{FOSD}} H_2^k \end{aligned}$$

That is if there are lots of bargainers (and so few price takers, state  $\mu_1$ ), then the bargained prices tend to be high, and if two or more prices are drawn the minimum of these tends to be higher also.

Now note that with  $\mu_i$  price takers the consumer surplus per bargainer is given by

$$CS \text{ pp for bargainers} |_{\mu_i} = q_1 \underbrace{E_{F_i} \left[ \frac{1}{2} (1-p)^2 \right]}_{\text{Expected surplus if ask for 1 quote}} + \sum_{k=2}^N q_k \underbrace{E_{H_i^k} \left[ \frac{1}{2} (1-p)^2 \right]}_{\text{Expected surplus if ask for } k \text{ quotes}}$$

where pp stands for per person. Clearly  $(1-p)^2$  is a decreasing function of  $p$  for prices in the feasible range of  $[0, \frac{1}{2}]$  and so

$$F_1 \succ_{\text{FOSD}} F_2 \Rightarrow E_{F_1} \left[ \frac{1}{2} (1-p)^2 \right] < E_{F_2} \left[ \frac{1}{2} (1-p)^2 \right]$$

and similarly for  $\{H_i^k\}$ . Therefore we have

$$\mu_1 < \mu_2 \Rightarrow CS \text{ pp for bargainers} |_{\mu_1} < CS \text{ bargainers} |_{\mu_2}$$

which gives the result. ■

**Proof of Part 3 of Theorem 1.** Immediate from Lemma 1 as bargainers receive prices strictly below the market price which price takers accept, and  $z$  is a continuous function of  $\mu$  so as  $\Delta\mu \rightarrow 0$ ,  $\Delta z \rightarrow 0$ . ■

**Proof of Parts 4a. and 4b. of Theorem 1.** We define total consumer surplus (Tot  $CS$ ) as

$$\text{Tot } CS \equiv \mu \cdot (CS \text{ pp for price takers}) + (1 - \mu) \cdot (CS \text{ pp for bargainers})$$

The consumer surplus terms are continuous in  $\mu$ . Therefore for part (a) it is sufficient to show that

$$\lim_{q_1 \rightarrow 1} \frac{d\text{Tot } CS}{d\mu} > 0 \tag{20}$$

as then the result will hold for some range of high  $q_1$  as required. Now note that

$$\frac{d\text{Tot } CS}{d\mu} = \frac{(CS \text{ pp for price takers}) - (CS \text{ pp for bargainers})}{+ \mu \frac{\partial(CS \text{ pp for price takers})}{\partial\mu} + (1 - \mu) \frac{\partial(CS \text{ pp for bargainers})}{\partial\mu}}$$

First note that as  $q_1 \rightarrow 1$  the quoted price in the bargaining stage tends to  $z$  (see footnote 7): if consumers don't get multiple second quotes then there is no incentive to lower prices to bargainers. But in this case the consumer surplus per person for both the bargainers and price takers tend to equality as the prices they face tend to the same limit. Next by parts 1 and 2 of Theorem 1, proved above, we have

$$\frac{\partial(CS \text{ pp for price takers})}{\partial\mu} > 0, \quad \frac{\partial(CS \text{ pp for bargainers})}{\partial\mu} > 0$$

Therefore if these don't both vanish as  $q_1 \rightarrow 1$  we are done. Now note that  $\frac{\partial(CS \text{ pp for price takers})}{\partial\mu} = - (1 - z) \frac{\partial z}{\partial\mu}$  and

$$\lim_{q_1 \rightarrow 1} z = [2 + \mu(N - 1)]^{-1} < 1$$

And from (17)  $-\frac{\partial z}{\partial\mu} = \frac{q_1(N - 1)}{[\mu(N + 1) + 2q_1(1 - \mu)]^2} \rightarrow \frac{N - 1}{[2 + \mu(N - 1)]^2} > 0$  as  $q_1 \rightarrow 1$

Hence  $\lim_{q_1 \rightarrow 1} \frac{\partial(CS \text{ pp for price takers})}{\partial\mu} > 0$  and so (20) holds. This gives part 4a. of Theorem 1

Part 4b. of Theorem 1 is more straightforward. If all bargainers request more than one quote then the standard Bertrand pricing outcome follows for bargainers; prices fall to marginal cost and so all bargainers receive a price of 0 and have expected consumer surplus of  $\frac{1}{2}$ . Formally, as  $q_1 \rightarrow 0$  profits from the bargainers fall to zero from (6) which means that  $F(p) \rightarrow 1$  for all  $p$ . As the bargained price is independent of the Cournot market price this drops to its standard value  $(z = \frac{1}{N+1} > 0)$ . The total consumer surplus is therefore

$$\lim_{q_1 \rightarrow 0} \text{Tot } CS = (1 - \mu) \frac{1}{2} + \mu \frac{1}{2} \left(1 - \frac{1}{N + 1}\right)^2$$

which rises as  $\mu$  falls.<sup>13</sup> ■

**Proof of Part 5 of Theorem 1.** Suppose we compare two markets, one with  $N_1$  competing sellers, the other with  $N_2 > N_1$ . Let  $z_i$  denote the Cournot market price with  $N_i$  competitors. Then by Lemma 2 we have  $z_2 < z_1$ . Thus the price takers have greater consumer surplus with the larger number of competitors. Turning to the bargainers, we denote the distribution of prices they receive with  $N_i$  competitors as  $F_i$ . Following the proof of Lemma 4,  $F_1$  first order stochastically dominates  $F_2$ . That the bargainers have higher consumer surplus with the larger  $N_2$  competitors then follows identically to the proof of part 2 of Theorem 1. Combining we have the desired result. ■

## B Proofs from Section 4

**Proof of Theorem 2.** Part 1 requires us to confirm that  $\frac{\partial z}{\partial \mu} < 0$  which follows from the proof of Theorem 1, part 1. We also need to confirm that  $\frac{\partial z}{\partial N} < 0$  which is clear from Lemma 2. For part 2 revisit (8) to give

$$q_1 z (1 - z) = \underline{p} (1 - \underline{p}) \sum_{k=1}^N k q_k \quad (21)$$

As the market price ( $z$ ) is less than  $\frac{1}{2}$ ,  $z(1 - z)$  is monotonically increasing in  $z$ . As the lowest price offered to bargainers ( $\underline{p}$ ) is smaller than  $z$ , then  $\underline{p}(1 - \underline{p})$  is monotonically increasing in  $\underline{p}$  also. Hence  $z$  and  $\underline{p}$  move in the same direction giving part 2. Part 3 is a direct application of Lemma 4 for the effect of  $\mu$  and of the proof of part 5 of Theorem 1 for the effect of  $N$ . For part 4 differentiate (21) with respect to  $z$  to give

$$\frac{d\underline{p}}{dz} = \underbrace{\left[ \frac{q_1}{\sum_{k=1}^N k q_k} \right]}_{<1} \underbrace{\left[ \frac{1 - 2z}{1 - 2\underline{p}} \right]}_{<1} < 1$$

which gives the result. ■

**Proof of Theorem 3.** Consider the price takers first. The market price is determined

<sup>13</sup>Note that Tot  $CS$  is smooth in  $q_1$  as  $\Phi(\cdot)$  is and so

$$\frac{\partial}{\partial \mu} \left( \lim_{q_1 \rightarrow 0} \text{Tot } CS \right) = \lim_{q_1 \rightarrow 0} \left( \frac{\partial \text{Tot } CS}{\partial \mu} \right)$$

by a Taylor expansion around  $q_1 = 0$ .

in Lemma 2 and only depends on  $q_1$ , not on the value of  $q_k$  for  $k > 1$ . Further, by (9) we have  $q_1^{\text{good}} < q_1^{\text{poor}}$ . It is immediate from Lemma 2 that  $\frac{\partial z}{\partial q_1} =_{\text{sign}} \mu(1-\mu)(N-1) > 0$ . Therefore the market price for the price takers if bargainers use the good strategy is lower.

Turning to the bargainers, as the market price for the price takers is below the monopoly level of  $\frac{1}{2}$  by Lemma 2,  $z(1-z)$  is an increasing function of  $z$  and so

$$z^{\text{good}}(1-z^{\text{good}}) < z^{\text{poor}}(1-z^{\text{poor}})$$

Substituting (6) into (4), we can show that for any  $p$  in the support of  $F^i$  (the distribution of prices given  $\{q_1^i, q_2^i, \dots, q_N^i\}$ ) we have

$$q_1^i \left[ \frac{z^i(1-z^i)}{p(1-p)} - 1 \right] = \sum_{k=2}^N k q_k^i (1-F^i(p))^{k-1} \quad (22)$$

From (8)  $\underline{p}^i$ , the lower bound on the support of  $F^i$ , is defined by  $\underline{p}^i(1-\underline{p}^i) = \frac{z_i(1-z_i)q_1^i}{\sum_{k=1}^N k q_k^i}$ . Thus  $\underline{p}^{\text{good}}(1-\underline{p}^{\text{good}}) < \underline{p}^{\text{poor}}(1-\underline{p}^{\text{poor}})$  as (i)  $z^{\text{good}}(1-z^{\text{good}}) < z^{\text{poor}}(1-z^{\text{poor}})$ , (ii)  $q_1^{\text{good}} < q_1^{\text{poor}}$  and (iii)  $\sum_{k=1}^N k q_k^{\text{good}} > \sum_{k=1}^N k q_k^{\text{poor}}$  by (9). Hence as  $\underline{p} < z^i < \frac{1}{2}$  then  $\underline{p}(1-\underline{p})$  is an increasing function of  $\underline{p}$  and so  $\underline{p}^{\text{good}} < \underline{p}^{\text{poor}}$ . That is the lowest price a population of good bargainers can get is lower than the lowest price a population of poor bargainers can get.

We now aim to show that the prices a population of good bargainers would get are consistently lower than the prices a population of poor bargainers would get, in the sense of first order stochastic dominance. To see this note that for any  $p \leq \underline{p}^{\text{good}}$ ,  $F^{\text{poor}}(p) = F^{\text{good}}(p) = 0$ , for any  $p \in (\underline{p}^{\text{good}}, \underline{p}^{\text{poor}}]$ ,  $F^{\text{poor}}(p) = 0 < F^{\text{good}}(p)$ , for any  $p \geq z^{\text{poor}}$ ,  $F^{\text{poor}}(p) = F^{\text{good}}(p) = 1$ . In the range  $p \in (\underline{p}^{\text{poor}}, z^{\text{poor}})$  we must have  $F^{\text{poor}}(p) < F^{\text{good}}(p)$ . Suppose not. Then  $p$  is in the support of both  $F^{\text{good}}$  and  $F^{\text{poor}}$  so (22) must hold for both. The left hand side of (22) is smaller under good bargaining than under poor bargaining. However, by (9)  $k q_k^{\text{good}} \geq k q_k^{\text{poor}}$  for  $k \geq 2$ . Thus for (22) to hold we need  $1 - F^{\text{good}}(p) < 1 - F^{\text{poor}}(p)$ , a contradiction to the initial assumption. But this argument works for any  $p$  in  $(\underline{p}^{\text{poor}}, z^{\text{poor}})$  and so  $F^{\text{poor}}(p)$  first order stochastically dominates  $F^{\text{good}}(p)$ . Hence we have part 3. ■

## C Proofs from Section 5

**Proof of Lemma 3.** Let  $H_k(p)$  be the probability that the best second quote is less than  $p$  if  $k$  second quotes are sourced, so  $H_k(p) = 1 - (1 - F(p))^k$ , and let  $h_k(p)$  be the corresponding density function. A potential bargainer with  $v_i < \underline{p}_F$  will never choose to bargain as she will never be offered a second quote she would accept, and nor will she buy at  $z$  as  $\underline{p}_F \leq z$ . For a potential bargainer with  $v_i \geq \underline{p}_F$ , the expected gain from bargaining (gross of the bargaining cost  $c$ ) is given by

$$E[\text{Gain from bargaining}] = \sum_{k=1}^N q_k \int_{\underline{p}_F}^{\min\{v_i, z\}} (\min\{v_i, z\} - x) h_k(x) dx \quad (23)$$

This holds both for potential bargainers with  $v_i \in [\underline{p}_F, z)$ , who do not buy absent bargaining, and for those with  $v_i \geq z$ , who buy at  $z$  if they do not bargain.

Note that this gain is strictly increasing in  $v_i$  up to  $z$ , after which it is constant in  $v_i$ . As  $v_i \rightarrow \underline{p}_F$ ,  $E[\text{Gain}] \rightarrow 0$  and as  $v_i \rightarrow z$ ,  $E[\text{Gain}]$  tends to

$$z \sum_{k=1}^N q_k \int_{\underline{p}_F}^z h_k(x) dx - \sum_{k=1}^N q_k \int_{\underline{p}_F}^z x h_k(x) dx = z - \tilde{p}_F$$

Thus if  $c \geq z - \tilde{p}_F$  then none of the potential bargainers choose to bargain, while if  $c < z - \tilde{p}_F$  (which ensures  $\tilde{p}_F < z$ ) then there exists a  $\underline{v} \in (\underline{p}_F, z)$  such that potential bargainers with  $v_i > \underline{v}$  bargain where  $\underline{v}$  satisfies  $\sum_{k=1}^N q_k \int_{\underline{p}_F}^{\underline{v}} (\underline{v} - x) h_k(x) dx = c$ . ■

**Proof of Theorem 4.** Part (a) derives equilibrium properties when potential bargainers do not bargain. Part (b) does the same when some do. Part (c) uses these properties to prove the Theorem.

(a) Suppose first that we have an equilibrium in which none of the potential bargainers bargain. At the bargaining stage the firms only interact with costless bargainers. They therefore face exactly the same problem as in the main model, except that we replace the  $\mu$  term there with  $\mu + \gamma$ . Thus we get exactly the same  $F(p)$ , given by (7), and the bargaining stage profit function is given by (6) with  $\mu + \gamma$  replacing  $\mu$ . At the Cournot stage, the analysis from the

main model then tells us that using the first order condition

$$z^{\text{no barg}} = \frac{\mu + \gamma + q_1(1 - \mu - \gamma)}{(\mu + \gamma)(N + 1) + 2q_1(1 - \mu - \gamma)} < \frac{1}{2} \quad (24)$$

Let  $c^{\text{no barg}} \equiv z^{\text{no barg}} - \tilde{p}_F$  where  $\tilde{p}_F$  is calculated given (7) and  $\hat{z} = z^{\text{no barg}}$ . Iff  $c \geq c^{\text{no barg}}$  the potential bargainers indeed choose not to bargain (see Lemma 3). If  $c > c^{\text{no barg}}$  then the equilibrium exists locally, as  $F(p)$  and so  $\tilde{p}_F$  are continuous in  $z$ , so after a small change in  $z$  we can retain the bargaining stage equilibrium in which the potential bargainers do not bargain.

(b) Now suppose that we have an equilibrium in which potential bargainers with  $v_i > \underline{v}$  choose to bargain. From Lemma 3,  $\underline{v} < z$ . Let  $\mathcal{F}(p)$  be the new pricing distribution at the bargaining stage. In equilibrium,  $\mathcal{F}(p)$ ,  $\underline{v}$  and  $z$  are all interrelated. We begin by deducing some initial properties of the bargaining stage. Suppose the price takers face a market price of  $z \leq \frac{1}{2}$ . Expected profit from the bargainers at a second quote  $p \leq z$  is given by

$$\begin{aligned} \pi(p) = & (1 - \mu - \gamma)p(1 - p) \sum_{k=1}^N q_k \frac{k}{N} (1 - \mathcal{F}(p))^{k-1} \\ & + \gamma p \sum_{k=1}^N q_k \frac{k}{N} \underbrace{(1 - \underline{v})}_{\text{Prob bargain}} (1 - \mathcal{F}(p))^{k-1} \cdot \underbrace{\begin{cases} 1 & p < \underline{v} \\ \frac{1-p}{1-\underline{v}} & p \geq \underline{v} \end{cases}}_{\text{Prob buy given bargain}} \end{aligned} \quad (25)$$

which will be a constant throughout the price support so that firms are happy to mix.

As  $z \leq \frac{1}{2}$ , Lemma 1 continues to apply to  $\mathcal{F}(p)$ . Consider pricing at the top of the distribution at  $z$ . As  $\mathcal{F}(z) = 1$ , using (25) and the fact that  $\underline{v} < z$  from Lemma 3,  $\pi(z)$  is given by  $\frac{1}{N}(1 - \mu)z(1 - z)q_1$  which matches (6). At the Cournot stage, the analysis from the main model then tells us that, using the first order condition for volumes to the price takers:

$$z^{\text{barg}} = \frac{\mu + q_1(1 - \mu)}{\mu(N + 1) + 2q_1(1 - \mu)} < \frac{1}{2} \quad (26)$$

For  $p \geq \underline{v}$ ,  $\pi(p)$  given by (25) is the same as that given by (3). Equating to (6) therefore gives  $\mathcal{F}(p)|_{p \geq \underline{v}}$  as (7) yet again, with  $\hat{z} = z^{\text{barg}}$ .

For  $p < \underline{v}$ , equating (25) to (6) gives

$$(1 - \mu) z^{\text{barg}} (1 - z^{\text{barg}}) q_1 = p \{(1 - \mu - \gamma)(1 - p) + \gamma(1 - \underline{v})\} \sum_{k=1}^N q_k k (1 - \mathcal{F}(p))^{k-1}$$

so, using the function  $\Phi$  from the main model

$$\mathcal{F}(p)|_{p < \underline{v}} = 1 - \Phi \left( \frac{(1 - \mu) z^{\text{barg}} (1 - z^{\text{barg}})}{p \{(1 - \mu - \gamma)(1 - p) + \gamma(1 - \underline{v})\}} q_1 \right) \quad (27)$$

To complete our equilibrium the cutoff  $\underline{v}$  must be consistent, so using (23) we require

$$-c + \sum_{k=1}^N q_k \int_{\underline{p}_{\mathcal{F}}}^{\underline{v}} (\underline{v} - x) h_k(x) dx = 0 \quad (28)$$

We finally wish to confirm that a solution to the simultaneous equations (27) and (28) exists. Letting  $\underline{v}$  be the primitive variable we can consider  $\underline{p}_{\mathcal{F}}$  and  $h_k(x)$  as functions of  $\underline{v}$  via  $\underline{v}$ 's effect on  $\mathcal{F}(p)$ . Let  $\tilde{p}_{\mathcal{F}}(\underline{v}, z)$  denote  $\tilde{p}_{\mathcal{F}}$  as a function of  $\underline{v}$  and  $z$ . If  $c < c^{\text{barg}} \equiv z^{\text{barg}} - \lim_{\underline{v} \rightarrow z^{\text{barg}}} \tilde{p}_{\mathcal{F}}(\underline{v}, z^{\text{barg}})$  then there must exist a solution to (28) with  $\underline{v} < z^{\text{barg}}$  (though the solution is not necessarily unique). We can see this as follows. First, note that  $\mathcal{F}(p)$  is continuously varying in  $\underline{v}$ , so  $h_k(x)$  and  $\underline{p}_{\mathcal{F}}$  are too. If  $\underline{v} = \underline{p}_{\mathcal{F}}$  with  $\mathcal{F}(p)$  given by (7) then  $\mathcal{F}(p) = (7)$  throughout, so  $\underline{v} = \underline{p}_{\mathcal{F}}$  and the left hand side of (28) =  $-c$ . As  $\underline{v} \rightarrow z^{\text{barg}}$ , the left hand side of (28)  $\rightarrow -c + c^{\text{barg}} > 0$ . Thus there must be at least one  $\underline{v} < z^{\text{barg}}$  which satisfies (28).

We have found the properties of any equilibrium in which some potential bargainers bargain. For  $c < c^{\text{barg}}$  we have local existence: after a small change in  $z$  a bargaining stage equilibrium in which some potential bargainers choose to bargain continues to exist, and profits from such an equilibrium are continuous in  $z$ . There can be no equilibrium with market prices above  $\frac{1}{2}$ , the monopoly level.<sup>14</sup>

(c) Part 1. Using (17) and (26),  $\frac{\partial z^{\text{barg}}}{\partial \mu} < 0$ . We can think of  $z^{\text{no barg}}$  (given by (24)) as  $z^{\text{barg}}$

<sup>14</sup>If  $z > \frac{1}{2}$  with the potential bargainers strictly preferring to bargain, then it can readily be shown that the firms would always have a profitable deviation, either to lower the Cournot market price or to not price at the upper bound of the pricing distribution. For technical reasons our derivation of this result does require that those potential bargainers indifferent between bargaining and not elect not to bargain. Dropping this assumption could result in prices making a jump change in response to a small deviation on the Cournot market in certain circumstances, which is technically difficult to analyse.

but replacing  $\mu$  by  $\mu + \gamma > \mu$ , so  $z^{\text{no barg}} < z^{\text{barg}}$ . The expected consumer surplus  $E[CS]$  of the price takers, given by (2), falls as  $z$  has risen.

Part 2. In the first equilibrium, in which the potential bargainers don't bargain,  $F(p)$  is given by (7). Suppose that after the increase in market price to  $z^{\text{barg}}$ ,  $\mathcal{F}(p)$  was also given by (7) for all  $p$ . Then from the proof of Lemma 4, the distribution of prices under  $z^{\text{barg}}$  would first order stochastically dominate that under  $z^{\text{no barg}}$ . However, we get a second effect as from (27),  $\mathcal{F}(p)|_{p < \underline{v}} < (7)$  for  $\hat{z} = z^{\text{barg}}$  as  $\Phi$  is a strictly increasing function, further strengthening the stochastic dominance result. By the same reasoning as in the proof of Part 2 of Theorem 1, costless bargainers' expected consumer surplus declines.

Part 3. Follows from the proof of Part 4.

Part 4. (i) As  $q_1 \rightarrow 0$ ,  $z^{\text{barg}} \rightarrow \frac{1}{N+1}$  and  $z^{\text{no barg}} \rightarrow \frac{1}{N+1}$  from (26) and (24). Therefore  $(z^{\text{barg}} - z^{\text{no barg}}) \rightarrow 0$ , so the change in the price takers'  $E[CS]$  goes to zero. Profits from the bargainers in both types of equilibrium go to zero (remember  $\pi(z)$  is given by (6), with  $\mu + \gamma$  replacing  $\mu$  in the first type of equilibrium). Therefore  $F(p) \rightarrow 1$  for all  $p$  in the first equilibrium and similarly for  $\mathcal{F}(p)$ , so the change in the costless bargainers'  $E[CS]$  goes to zero. Potential bargainers who switch to bargaining from price taking must do strictly better, as they could choose to remain a price taker, and those who switch to bargaining from not buying also do better, so potential bargainers'  $E[CS]$  strictly rises. For a given reduced  $c < \frac{1}{N+1}$ , this rise does not go to zero as  $q_1 \rightarrow 0$ .

(ii) As  $q_1 \rightarrow 1$ ,  $z^{\text{barg}} \rightarrow \frac{1}{\mu(N-1)+2}$  and  $z^{\text{no barg}} \rightarrow \frac{1}{(\mu+\gamma)(N-1)+2} < \frac{1}{\mu(N-1)+2}$  from (26) and (24). Therefore the change in the price takers'  $E[CS]$  is strictly negative. A straightforward generalization of the argument in Footnote 7 shows that in the first equilibrium  $\underline{p}_F \rightarrow z^{\text{no barg}}$  and in the second  $\underline{p}_F \rightarrow z^{\text{barg}}$ . Thus the discounts the bargainers receive go to zero, so the rise in  $z$  causes both the costless bargainers' and the potential bargainers' change in  $E[CS]$  to be strictly negative (note that for any  $q_1$  we can still create some new bargainers by pushing  $c$  very close to zero). ■

## References

- [1] Arnold, M.A., Lippman, S.A., 1998. Posted prices versus bargaining in markets with asymmetric information. *Economic Inquiry*, 36, 450-457.
- [2] Bester, H., 1993. Bargaining versus price competition in markets with quality uncertainty. *American Economic Review*, 83, 278-288.
- [3] Burdett, K., Judd, K.L., 1983. Equilibrium price dispersion. *Econometrica*, 51, 955-970.
- [4] Camera, G., Delacroix, A., 2004. Trade mechanism selection in markets with frictions. *Review of Economic Dynamics*, 7, 851-868.
- [5] Camera, G., Selcuk, C., 2007. Price dispersion with directed search. Mimeo.
- [6] Cason, T.N., Friedman, D., Milam, G.H., 2003. Bargaining versus posted price competition in customer markets. *International Journal of Industrial Organization*, 21, 223-251.
- [7] Chen, Y., Rosenthal, R.W., 1996a. Asking prices as commitment devices. *International Economic Review*, 37, 129-155.
- [8] Chen, Y., Rosenthal, R.W., 1996b. On the use of ceiling-price commitments by monopolists. *RAND Journal of Economics*, 27, 207-220.
- [9] Competition Commission, 2000. *New Cars: A Report on the Supply of New Motor Cars within the UK*. Cm 4660, Competition Commission, United Kingdom.
- [10] Davis, D.D., Holt, C.A., 1994. The effects of discounting opportunities in laboratory posted-offer markets. *Economics Letters*, 44, 249-253.
- [11] Desai, P., Purohit, D., 2004. "Let me talk to my manager": Haggling in a competitive environment. *Marketing Science*, 23, 219-233.
- [12] Janssen, M.C.W., Moraga-Gonzalez, J.L., 2004. Strategic pricing, consumer search and the number of firms. *Review of Economic Studies*, 4, 1089-1118.
- [13] Office of Fair Trading, 2004. *Estate Agency in England and Wales*. OFT 693, Office of Fair Trading, United Kingdom.

- [14] Raskovich, A., 2007. Competition or collusion? Negotiating discounts off posted prices. *International Journal of Industrial Organization*, 25, 341-354.
- [15] Stahl, D., 1989. Oligopolistic pricing with sequential consumer search. *American Economic Review*, 79, 700-712.
- [16] Varian, H., 1980. A model of sales. *American Economic Review*, 70, 651-659.
- [17] Wang, R., 1995. Bargaining versus posted-price selling. *European Economic Review*, 39, 1747-1764.
- [18] Zettelmeyer, F., Scott Morton, F., Silva-Risso, J., 2006. How the Internet lowers prices: Evidence from matched survey and automobile transaction data. *Journal of Marketing Research*, 43, 168-81.