

# Advertising and Consumer Awareness of a New Product\*

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## Abstract

The increase of a new product's sales is usually attributed to consumers becoming informed about the existence of the product. Advertising can accelerate this consumer awareness process. This paper evaluates this effect. I develop and estimate a structural model in which the consumer purchase decision is specified using a discrete choice model with variation in the choice set, according to the information diffusion of the new products. I also model the optimal price and advertising decisions of the firm, taking into account the dynamic effect of advertising on future sales (via an increase in the proportion of consumers aware of the product). The model is estimated using Spanish automobile data. The estimates show that advertising enhances the three year it takes for the information diffusion of a new product to half as long, and reduces the loss of consumer welfare from the lack of information around by 37%.

**Key words:** New Product, Advertising, Consumer Awareness Process, Consumer Choice Set, Discrete Choice Model, Structural dynamic model.

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# 1 Motivation

The term product life-cycle has been used extensively to describe the standard evolution of new industries with reference to the number of producers, rate of product innovation and market shares (Klepper, 1996). This term is also used as a description of what happens to product sales over time in markets with a high product turnover due to intense innovation, such as automobiles, computers and digital camera (Levitt, 1965). The process of growth, diffusion and erosion of product's sales arises as a result of the interaction between consumers' and firms' behavior, although the demand side plays a crucial role to explain the initial evolution of the sales. In durable markets the initial growth of new product's sales is usually attributed to consumers becoming informed about the existence of the new product<sup>1</sup> (Horsky and Simon 1983 and Krishnan and Jain 2006). It means a consumer awareness process where the proportion of consumers aware of the new product increases over time. Advertising affects this information process by informing the consumers of the product's existence, although it can take place without advertising through other source of information such as consumers already aware about the new product (kwoka, 1996). The effect of advertising on sales via informing the consumers of the product's existence<sup>2</sup> is dynamic since advertising will affect the consumers aware of the product in the future, and therefore will affect future sales. This dynamic effect of advertising explains that firm usually advertises a product the most at the entry.

Evidence of such common strategy is found in the Spanish automobile market. The car models are heavily advertised during the first two years (when sales typically increase), even though sales are lower than in subsequent periods<sup>3</sup>. This behavior is especially remarkable in the first years when the average expenditure on advertising is around 15.1% higher than in the fourth year, while the average sales of a model are approximately 24.6% lower.

The goal of this paper is to evaluate the role of advertising in the information diffusion of a new product<sup>4</sup>. To address it I develop and estimate a structural discrete-choice model, in which the consumer is aware of only a subset of all products (her choice set), and her purchase decisions are only among the products in her choice set. The choice set evolves according to the information diffusion process, which is affected by the advertising expenditure. Furthermore, the model allows for advertising affects the consumer's preferences in her purchase decision. I also model the optimal price and advertising decisions of the firm,

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<sup>1</sup>Additional explanations are the reduction of the uncertainty about a new product that the risk averse consumer face (Byzalov and Schachar, 2004) and, for frequently purchased product, repeat purchases from own (satisfactory) consumption experience (Akerberg 2001).

<sup>2</sup>There are several effects of advertising on consumer's purchasing decision that could be also considered (see next subsection, Related Literature)

<sup>3</sup>Evidence suggests Spanish car models have a life-cycle with a sales that typically increase in the first two years and subsequently tend to decrease as time goes by (see Table 1 and Figure 1).

<sup>4</sup>The information diffusion of a new product is understood as the increase of the proportion of consumers aware of the new product until being up to the stationary proportion.

taking into account the dynamic effect of advertising on future sales (via an increase in the proportion of consumers aware of the product). Firms face a dynamic optimization problem in a oligopolistic market. Because it is computational unfeasible to explicitly solve this optimization problem for estimation purposes, I use the technique proposed by Berry and Pakes (2001). This technique is based on estimating the optimality conditions for the dynamic controls (advertising in my case), and it does not require an explicit solution for this optimization problem. I estimate the three equilibrium relationships (demand, price and advertising equations) using the Generalized Method of Moments and simulation techniques. I also propose a way to simulate consumer choice sets that deals with the dimensionality problem that arises due to the high number of products<sup>5</sup> commonly observed in differentiated product markets. As Berry, Levinsohn and Pakes (1995), henceforth BLP, the model incorporates consumer heterogeneity into differentiated product demand systems and takes prices as endogenous to obtain realistic predictions of elasticities and welfare estimates. Also similar is that my model is designed for aggregate data. My model departs from BLP in that consumer choice set are endogenous and advertising is considered (in addition to demand and price equation, I also model the dynamic firm decision on advertising).

I apply this analysis to the Spanish automobile market using a monthly panel data from January 1990 to December 2000 (132 months) , with “car model” as the elementary unit of analysis (257 distinct car models were sold in the market, offered by 33 multiproduct firms). During this period 180 car models that entered the market, and 93 exited with a mean age of around 8 years. The information gathered for each model includes price, new car registrations (sales), model age, brand, advertising expenditures, mechanical, design and equipment characteristics.

The results suggest advertising significantly enhances the information diffusion of new products, and reduces the loss of consumer welfare from the fact that potential consumers not aware of a product will purchase another one that provides less utility. The estimates show that advertising enhances the three year it takes for the information diffusion of a new product to half as long, and reduces the loss of consumer welfare from the lack of information around by 37%.

The next subsection reviews the related literature and the rest of the paper is organized as follows. I start by describing the data in Section 2. Sections 3 and 4 present the model (the consumer’s and firm’s problems, respectively); Section 5 discusses estimation issues; Section 6 presents the results and examines implications. Section 7 concludes.

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<sup>5</sup>The number of possible choice sets increases exponentially with the number of products giving  $(2^J - 1)$  raise to the so-called dimensionality problem.

## 1.1 Related Literature

The standard discrete choice models prevalent in IO literature, pioneered by McFadden (1974), assume that consumers are aware of all the products, and as a result those models only address variation in the choice sets across markets (in fact, it is an important source of identification in these models). However, there is some recent research in IO that focuses on other sources of variation to estimate more realistic demand specifications. Anupindi, Dada, and Gupta (1998) and Conlon and Mortimer (2007) study variations in consumer choice sets generated by the presence of stockouts. In Katz (2007), the variation comes from the fact that consumers restrict their attention to a subset of products before making a choice. This last paper is close to a large body of literature in marketing known as consideration set literature, focused on incorporating the variation in the consumer choice set into discrete choice models (Manski 1977 was the first to introduce it). In this literature, two interpretations of the choice set are possible. First, consumers might be unaware of the existence of some products, and their choice set consists of all the products they are aware of. Alternatively, consumers might face cognitive costs or constraints of having to consider a large number of products in their choice, and therefore they might restrict their attention to a smaller subset of products before making a choice. Both interpretations have been considered in the literature to study the effect of advertising on the consumer choice set. Goeree (2008) develops and estimates a model to analyze the effect of advertising on the consumer choice set for the PC market in the United States, where the consumer choice set is interpreted as all the products the consumer is aware of. Meanwhile, Chiang, Chib and Narasimhan (1998) assumes the second interpretation and proposes an integrated brand choice model that is capable of accounting for the heterogeneity in the choice set and in the parameters of the brand choice model. Both papers focus on the variation in the choice set across consumers according to consumer characteristics, and employ a static model.

My research considers the first interpretation of the choice set (all products a consumer is aware of), and the variation in the consumer choice set comes from the knowledge of the existence of new products. This variation, unlike in previous research, introduces dynamics in demand, which means a long-run effect of advertising through the consumer choice set.

One of the main problems of introducing variation in the consumer choice set (other than across markets) is the dimensionality problem that arises from the high number of possible choice sets. There are different strategies in the literature to deal with this problem, depending on the application and the data. For example, in Conlon and Mortimer (2007), the dimensionality problem arises from the fact that the exact timing of product stockouts (and therefore the choice set facing the consumer) is not observed. This feature of the data leads them to use the EM algorithm of Dempster, Lair, and Rubin (1977). Chiang, Chib and Narasimhan (1998) propose an estimation procedure using an approximation-free Markov

chain Monte Carlo procedure.

This paper proposes a way to simulate consumer choice sets suited to deal with the dimensionality problem when the variation in the choice set comes from being unaware of the new products. The method presented here is closely related to the proposed by Goeree (2008), simulating an important sampler of consumers choice set to use as simulator for the market share the average over individuals of the smoothed choice probabilities.

My paper is also related to the advertising literature (see Bagwell 2005 for a survey). There are three different views in order to explain how advertising affects the consumer choice. The persuasive view argues that advertising alter consumers' tastes (Dixit and Norman, 1978), while the complementary view considers that advertising is part of consumers' preferences in the same way that goods and that there are complementarities between advertising and goods (Becker and Murphy 1993). Common to the persuasive and the complementary view is that advertising affects the utility derived from consuming a product. The third approach in the literature views advertising as information about product's attributes or existence (Stigler, 1961). The informative view considers that advertising can affect both the utility and the consumer choice set<sup>6</sup>, therefore the effect of advertising on consumer choice set has been usually associated to the informative effect of advertising, although if consumers will restrict their attention to a subset of products before making a choice, a persuasive effect of advertising could arise in the choice set<sup>7</sup>. My research focuses on a different effect of advertising. I distinguish between the informative effect of advertising on the choice set when a product enters the market, and the effect of advertising on utility.

Both the effect on consumer utility and the effect on the choice set have been previously examined empirically in a static setting (Gooree 2008, Akerberg 2001, Nevo 2001). In a dynamic setting, the effect of advertising on the utility has been considered usually following Nerlove and Arrow (1962), taking goodwill to be a function of the stock of current and past advertising (Dube, Hitsch and Manchanda 2005, Akerberg 2003 or Tan 2006).

The dynamic effect of advertising through the consumer choice set, however, has never been considered in an empirical paper. In fact, Doraszelski and Markovich (2007) are the first to consider, in a theoretical framework, the long-run effect of advertising through the consumer choice set (which they call "awareness advertising"). They propose two different dynamic models of advertising, the goodwill and awareness advertising models, to study the effect of advertising on industry structure.

My research contributes to this literature by quantifying the long-run effect of advertising through the choice set when new products enter the market.

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<sup>6</sup>Butters (1977) and Grossman and Shapiro (1984) consider that advertising affects the consumer choice set to capture the fact that a consumer may be unaware of the existence of a product unless she sees it advertised. While Nelson (1974) and Milgrom and Robert (1986) consider that advertising affects the consumer utility to capture the fact that advertising signals information about the product (unobserved) quality.

<sup>7</sup>For example, if the consumer only considers the products from which she expects to derive higher utility.

Only a few papers have directly addressed the life cycle of products introduced by multiproduct firms in a differentiated product market. Moral and Jaumandreu (2007) is the closest to this paper in this respect. Working within a discrete choice demand framework they introduce time effect as a product characteristic to study the demand price elasticities over product live cycle in the Spanish automobile market. Another example for an industry with a high product turnover due to intense innovation is Bresnahan, Stern and Trajtenberg (1997). They study the impact of being a frontier (technological) brand in the demand price elasticities. In my paper, unlike theirs, the information diffusion over the product cycle is explicitly modeled by introducing variation in the consumer choice set in the same demand framework.

The main contribution of this paper is to be the first one that estimates a dynamic effect of advertising that comes from the choice set. That is, the effect of advertising on the future consumers aware of a new product. This paper also contributes to the literature in the following ways: introducing in the standard discrete choice model variation in the consumer choice sets that evolve over time according to the information diffusion of new products, proposing a method to deal with the dimensionality problem that arises due to the high number of possible choice sets, and being the first paper that applies the Berry and Pakes (2001) technique to real data.

## 2 Data

The model is estimated for Spanish car market data. It is an unbalanced panel data on a monthly basis from January 1990 to December 2000 (132 months), with “model” as the elementary unit of analysis. The original sources of the data set are ANFAC (Asociación Nacional de Fabricantes de Automóviles y Camiones), “Guía del comprador de coches” magazine and “Infoadex”. Infoadex is a firm that computes advertising expenditure, by monitoring communication markets and their prices on a daily basis.

In the data for every model, the most representative (sold) version is considered, and the characteristics of the model are taken from it. This rule leads to 257 distinct models offered by 32 brands (multiproduct firms). Finally, treating a model/month as an observation, the total sample size is 16,362 observations. The information gathered for each model includes price, new car registrations (sales), segment, brand, advertising expenditures, mechanical design and equipment characteristics.

Infoadex reports total advertising expenditures on all models and automobile brands through the main media channels: newspapers, magazines, television, radio, cinema and billboards. In the automobile industry, it is common for firms to use both product and brand advertising. The (yearly) average expenditure on brand advertising is 18.2% of the total expenditure on advertising. Firms also use a combination of model-specific and group adver-

tising, with groups of varying sizes. The (yearly) average expenditure on group advertising is 11.7% of the total expenditure on advertising. I divide group advertising expenditures by the number of models which compose the group, and construct model advertising expenditures by adding model-specific expenditures to these weighted group expenditures.

Other sources of information are INE<sup>8</sup> and EPA<sup>9</sup>. From them I collect the number of Spanish households to proxy the potential market size<sup>10</sup> and the distribution of Spanish income per capita (annual mean and standard deviation<sup>11</sup>).

An interesting feature of the data is its high frequency, which helps to overcome the *data-interval-bias*. Clarke (1976) finds that estimated effects of advertising are sensitive to the frequency of data used, and calls it the data-interval-bias problem. The use of low-frequency advertising data when the effect of advertising on sales changes over a shorter period of time can lead to biased estimates of advertising effects.

The automobile industry is the second most important industry in the Spanish economy (after construction), representing almost 5 % of the GDP. After overcoming the second energy crisis, the eighties was a period of expansion for the Spanish car industry. Nevertheless, in the nineties, two environmental changes altered the evolution of the market: full integration into the EEC market, which implied a gradual reduction of tariff and non-tariff protection of the market (see Table 4), and the economic recession at the beginning of the decade (see Table 5).

Table 6 summarizes the evolution of the Spanish car market over the sample period. The market evolution during the nineties is characterized by (i) an increase in the number of car models, with a high rate of model introduction and turnover, (ii) an evolution of model sales closely related to the economic cycle, (iii) an important variation in the advertising expenditure, and (iv) stable prices, although model characteristics continuously improved (the real price for a car with the same characteristics fell by 19% in the 90s). The model attributes considered are: size (m<sup>2</sup>), auto cubic capacity per kg of model (cm<sup>3</sup>/kg), gas mileage (kms covered at a constant speed of 90 kph with a liter of gasoline), and maximum speed (as measures of size and safety, power, fuel efficiency and luxury, respectively).

The entry of car models seems to be motivated by the intention to replace old models and introduce models absent until this time. During the nineties, 180 models entered the market

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<sup>8</sup>Instituto Nacional de Estadística

<sup>9</sup>Encuesta de Población Activa

<sup>10</sup>The model market share is computed as unit sales of each model divided by the total market size (no. of households). Shares are annualized multiplying by 12 to facilitate comparability with the elasticities obtained with yearly data.

<sup>11</sup>The income distribution is assumed to be lognormal and its parameters are estimated from INE data. In particular, the estimated standard deviation  $\hat{\sigma}_y$  is 0.7 and the mean  $m_t$ , is the sample mean for each year.

and 93 exited, with a mean model age at exit of around 8 years.

Table 1 reports the estimated age effects on market share using a IV logit demand (taken price as endogenous). The estimated age effect on market share is also represent in Figure 2. Results clearly show that car models have life-cycle in the Spanish automobile market. Shares tend to increase until the third year in the life of a car model, and subsequently tend to decrease as time goes by. Model advertising and model age have a close relationship. The average annual advertising for a model is around 4 million euros 1995, which means about 4.65% of the revenue. Table 2 indicates that models are intensely advertised for the first years, and similar conclusions are found with a simple OLS regression of model advertising on product attributes (Table 3).

Falling prices of product over time is a standard behavior in durable good market with intense innovation<sup>12</sup>. This phenomena is also visible in the Spanish automobile market (see Table 3). One interesting relationship is found between advertising and price. Table 7 shows the range of advertising over revenue ratio and the associated model, where the associated model is one whose price is the closest to the mean of the percentile. This relationship suggests a higher effect of advertising on the demand for the lower segments of the car market<sup>13</sup> (see Barroso, 2006, for more evidence on this differentiated effect).

### 3 The Consumer's Problem

In the prevalent standard discrete choice model in IO, the assumption that the consumer is aware of all products implies that a consumer chooses the good which provides the highest utility,  $U$ , from all competing products. Then, the probability that consumer  $i$  purchases product  $j$  at time  $t$  is

$$P(U_{ijt} \geq U_{imt}, \forall m = 0, 1, \dots, J_t)$$

where alternatives  $m = 0, 1, \dots, J_t$  represent the purchase of the competing differentiated products at time  $t$ . Alternative zero, or the outside alternative, represents the option of not purchasing any of those products.

In reality, when a model enters the market, a certain proportion of consumers is not aware of the new product, and this proportion decreases over time with the diffusion of information. In this approach, the consumer maximizes her utility given the subset of the  $J_t$  products

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<sup>12</sup>Some recent recent research study this reduction and its implication on consumer behavior. For instance, Zhao (2007) examines price reduction in digital camera market and Gowrisankaran and Rysman (2007) develops and estimates a dynamic model where consumer takes into account this price behavior.

<sup>13</sup>The simplest (static monopoly) Dorfman-Steiner condition stipulates that the advertising exposure ratio,  $a$ , over revenue (price  $p$  multiplied by quantity  $q$ ) equals the advertising-demand elasticity ratio,  $\eta_a$ , over price-demand elasticity,  $\eta_p$ ,

$$\frac{a}{pq} = \frac{\eta_a}{\eta_p} \Rightarrow \eta_a = \eta_p \frac{a}{pq}$$

that she is aware of<sup>14</sup> (her choice set). Therefore, the probability that a product will be purchased by a consumer is the probability that the consumer is aware of the product and that it maximizes her utility, given the other products that the consumer is aware of. This probability can be written (see Appendix 1) as the sum of the probability that product  $j$  maximizes the consumer  $i$ 's utility given a choice set,  $S$ , multiplied by the (conditional) probability of this choice set, for all the possible choice sets at time  $t$  that include product  $j$ ,  $\Omega_{jt}$

$$s_{ijt} = \sum_{S \in \Omega_{jt}} \Pr(U_{ijt} \geq U_{imt}, \forall m \in S) \Pr_t(S | \{X_{js}, X_{-js}, Y_{is}\}_{s=1}^t) \quad (1)$$

where  $Y_i$  and  $X_j$  are consumer  $i$  and product  $j$  characteristics (including price and advertising) respectively, and  $-j$  denotes products other than  $j$ .

To obtain the probability that a product maximizes a consumer's utility, I postulate the following indirect utility for consumer  $i$  purchasing  $j$  at time  $t$ <sup>15</sup>

$$U_{ijt} = \delta_{jt} - \alpha_i p_{jt} + \epsilon_{ijt}, \text{ with } \delta_{jt} = \sum_k x_{jkt} \beta_k + \gamma a_{jt} + \xi_{jt}$$

where  $x_{jt}$  is a  $K$ -dimensional vector of observed product attributes other than price  $p_{jt}$  and advertising expenditure<sup>16</sup>  $a_{jt}$ , the variable  $\xi_{jt}$  represents product characteristics unobserved (to the econometrician), and  $\epsilon_{ijt}$  is a mean zero stochastic term which represents idiosyncratic individual preferences. As it is customary in the literature,  $\epsilon_{ijt}$  is assumed to be i.i.d. across products and consumers, with a type I extreme value distribution. Consumer income is assumed to be distributed as  $y_{it} \sim \log(m_t, \sigma_y^2)$  to model the price effect<sup>17</sup> as

<sup>14</sup>It is a model of product existence, and not of product content. It is assumed that once a consumer is aware of the existence of a product she is also aware of the product's attributes.

<sup>15</sup>Following BLP, this utility function comes from a Cobb-Douglas utility function in expenditures on other goods and services and characteristics of the good purchased:

$$U_{ijt}^* = (y_{it} - p_{jt})^\alpha G(x_{jt}, a_{jt}, \xi_{jt}) e^{\epsilon_{ijt}}$$

where  $G(\cdot)$  is assumed to be linear in logs, so that if  $u_{ijt} = \log(U_{ijt}^*)$ , then

$$u_{ijt} = \alpha \log(y_{it} - p_{jt}) + \sum_k x_{jkt} \beta_k + \gamma a_{jt} \delta_{jt} + \xi_{jt} + \epsilon_{ijt}$$

$$u_{i0t} = \alpha \log(y_{it}) + \xi_{0t} + \sigma_0 \nu_{i0} + \epsilon_{i0t}$$

If one normalize the  $[\alpha \log(y_{it}) + \xi_{0t}]$  to zero,  $U_{ijt} = u_{ijt} - [\alpha \log(y_{it}) + \xi_{0t}]$ , then

$$U_{ijt} = (\alpha/y_{it}) p_{jt} + \sum_k x_{jkt} \beta_k + \gamma a_{jt} \delta_{jt} + (\xi_{jt} - \xi_{0t}) + \epsilon$$

<sup>16</sup>There is a group of empirical studies that examine the impact of advertising on the price elasticity of demand, estimating interaction between price and advertising. Some examples are Krishnamurthi and Raj (1985) and Kanetkar, Weinberg and Weiss (1992). This research suggests that the effect of advertising on the price elasticity is difficult to determine and appears to vary across industries. However, this effect is usually associated with an advertising goodwill stock of brands and my research focuses on advertising of models where the long-run effect of advertising on the price elasticity intuitively seems less important.

<sup>17</sup>And so  $v_{iy} \sim N(0, 1)$ .

$\alpha_i = \alpha/y_i = \alpha e^{-(m_t + \sigma_y^2 v_{iy})}$ , where the parameters  $m_t$  and  $\sigma_y^2$  are the observed mean and variance, respectively (exogenous data).

The consumer may decide not to purchase any of the goods; instead she may decide to purchase the "outside" good (that is nonpurchase or purchase of a used good<sup>18</sup>). The presence of an outside good endogeneizes aggregate sales as a response to uniform price changes. The indirect utility from the outside option,  $j=0$ , is

$$U_{i0t} = \sigma_0 v_{i0} + \epsilon_{i0t}$$

where  $\sigma_0 v_{i0}$  is consumer  $i$ 's taste for the outside good, and  $v_{i0}$  is an extra unobserved term that allows to account for the possibility that there is more unobserved variance in the idiosyncratic component of the outside good.

Therefore, the conditional (on the consumer characteristics,  $v_i = (v_{iy}, v_{i0})$ , and the choice set,  $S$ ) probability that product  $j$  maximizes consumer  $i$ 's utility in period  $t$  is given by the standard logit expression

$$f_{ijt|S}(v_i, \delta_t, X_t; \theta) = \frac{e^{\delta_{jt} + \alpha_i p_{jt}}}{e^{\sigma_0 v_{i0}} + \sum_{\forall m \in S} e^{\delta_{mt} + \alpha_i p_{mt}}} \quad (2)$$

To model the probability of a choice set, two assumptions are made. First, the probability that a product is included in the choice set does not depend on the consumer characteristics<sup>19</sup> and second it is also independent of the rival product attributes. Therefore, I assume there are no informational spillovers and that the probability a consumer is informed about a product is independent of the probability she is informed about other product<sup>20</sup>. So the probability of choice set  $S$  in period  $t$  is the product of the probabilities that included (in the choice set) products are known and the no-included products are not known

$$\Pr_t(S | \{X_{js}\}_{s=1}^t) = \prod_{m \in S} \phi_{imt} \prod_{n \notin S} (1 - \phi_{int}) \quad (3)$$

where  $\phi_{jt}$  is the probability that product  $j$  is in the choice set in period  $t$  (or awareness probability), that is assumed to be one for the outside good (consumers are always aware of the nonpurchase option). The awareness probability is modeled as a concave function of the awareness level of product  $j$  in period  $t$ ,  $\omega_{jt}$ ,

$$\phi_{jt} = \frac{\omega_{jt}}{1 + \omega_{jt}} \quad (4)$$

<sup>18</sup>As it is customary in the literature, the second-hand automobile market is captured through a outside option. Some research (Esteban and Shum, 2007 and Schiraldi, 2007) develops and estimates a model of the second-hand automobile market.

<sup>19</sup>These assumptions are necessary because I only have acces to aggregate data. Consumer level data could help to relax them.

<sup>20</sup>Allowing informational spillover greatly complicates the model, and a very important variation in the data is needed in order to identify its effect across products.

The awareness level of product  $j$  at time  $t$ ,  $\omega_{jt}$ , is a latent variable assumed to exhibit time dependence. The transition equation of the awareness level is given by

$$\omega_{j\ t+1} = \varphi\omega_{jt} + \psi\varsigma_{jt} \quad (5)$$

where  $\varphi$  is the effect of the previous level of awareness about the product, and  $\varsigma_{jt}$  is a random variable, multiplied by the parameter  $\psi$ , with a distribution function that depends upon the expenditure on advertising, so that

$$\varsigma_{jt} = \begin{cases} 1 & \text{with a probability } \varkappa a_{jt} / (1 + \varkappa a_{jt}) \\ 0 & \text{otherwise} \end{cases}$$

The initial awareness level when a product enters the market is assumed to be equal across products and denoted as  $\omega_0$ . The parameters  $\varphi$ ,  $\varkappa$ ,  $\psi$  and  $\omega_0$  will be parameters to be estimated.

This specification implies that the probability of being aware of a product is a function of the model age and the history of previous advertising expenditures. The parameter  $\varphi$  captures two effect. First, other source of information than advertising. For instance, consumers already aware about the new product who directly or indirectly disseminates the information of the new product's existence. Second, the fact consumers may be forget the product's existence. If the first effect is the largest, the fraction of consumers informed of a product (or the awareness probability) increases over time even if the product is never advertised (see Figure 2).

The predicted market share, or the awareness corrected market share, will be obtained by integrating out expression (1) over the distribution of consumers' characteristics and the random variable of the awareness transition equation

$$s_{jt}(P_0, \xi_t, X_t; \theta) = \int \sum_{S \in \Omega_j} \left( \prod_{m \in S} \phi_{mt} \prod_{n \notin S} (1 - \phi_{nt}) \right) f_{ijt|S} dP_0(v_i, \{\varsigma_t\}_{s=1}^{t-1})$$

where  $\phi_{jt} = \phi(\{\varsigma_{jt}\}_{s=1}^{t-1}, X_t; \theta)$ ,  $f_{ijt|S} = f(v_i, \delta_t(\xi_t, X_t; \theta), X_t; \theta)$ , and  $dP_0(v_i, \{\varsigma_t\}_{s=1}^{t-1})$  is the assumed distribution of  $(v_i, \{\varsigma_t\}_{s=1}^{t-1})$ . The consumer characteristics  $v_i$  are assumed to be independently normally distributed across the population and products with a mean of zero and a variance of one.

Demand for product  $j$  at time  $t$  is  $M_t s_{jt}$ , where  $M_t$  is the market size<sup>21</sup> proxied by the number of households.

<sup>21</sup>The market size is understood as all potential consumers. Some of them will choose not purchasing any of the products (outside option), and other enter the market purchasing a product (the market demand). In this models, advertising affects the market demand attracting consumers who will not purchase any of the products without advertising.

## 4 The Firm's Problem

The firm  $f$ 's problem is to choose, for the set of models produced  $\mathfrak{S}_f$ , a sequence of advertising expenditures,  $\{a_{f,t+\tau}\}_{\tau=0}^{\infty}$ , and prices,  $\{p_{f,t+\tau}\}_{\tau=0}^{\infty}$ , to maximize the expected discounted value of net cash flows, conditional on product attributes, price, advertising and the levels of awareness of all (own and competing) product<sup>22</sup>. So, the firm's problem is to choose the vector of advertising a price for own products,  $a_{ft}$  and  $p_{ft}$ , according to

$$\sup_{a_{ft}, p_{ft}} E \left[ \sum_{\tau=0}^{\infty} \rho^{\tau} \pi_{f,t+\tau} | x_t, \omega_t, a_t, p_t \right]$$

where the expectation is taken with the understanding that optimal actions will be taken in each future period, the parameter  $\rho$  is the discount rate, and the net cash flow is given by

$$\pi_{ft} = \sum_{r \in \mathfrak{S}_f} [p_{rt} - mc_{rt}] s_{rt}(x_t, \omega_t, a_t, p_t) M_t - a_{rt}$$

where  $mc_{rt}$  is the marginal cost and  $M_t$  is the potential market size.

I assume firm  $f$  is a Bertrand competitor (in a differentiated-product market) that takes as given the characteristics of its products and competing products. I also rule out asymmetric information. Then the awareness level of any product,  $\{\omega_t\}$ , will be observed by the firms<sup>23</sup> when the choices (advertising and prices) are to be made. Given the demand<sup>24</sup>, this approach implies that price is a multiproduct static decision that satisfies the first-order condition

$$s_{jt}(x_t, \omega_t, a_t, p_t) + \sum_{r \in \mathfrak{S}_f} (p_{rt} - mc_{rt}) \frac{s_{rt}(x_t, \omega_t, a_t, p_t)}{\partial p_{jt}} = 0 \quad (6)$$

where the marginal cost is specified using the "hedonic" approach (take cost as a function of a set of product attributes; see, for example, Rosen 1974). In particular as it is common in the literature, I approximate the marginal cost using the log of product attributes, which are decomposed into a subset which is observed by the econometrician  $w_{jt}$ , and an unobserved component  $\zeta_{jt}$ . Given these assumptions, the marginal cost of product  $j$  is written as

$$\ln(mc_{jt}) = \ln(w_{jt})\eta + \zeta_{jt} \quad (7)$$

where  $\eta$  is a vector of parameters to be estimated.

The dynamic firm decision about advertising is modeled following a technique introduced by Berry and Pakes (2001), an alternative to the Euler equation techniques. This technique has computational properties similar to the Euler equation technique since it is not required to solve explicitly for the value function and the policy function of the firm in order to use the

<sup>22</sup>This model take as exogenous the entry, exit and the characteristics of products.

<sup>23</sup>The realization of the awareness shock  $\zeta_{jt}$  is observed at the beginning of the period  $t$  for all firms.

<sup>24</sup>The demands are got from the consumer's problem presented in the previous section. There is nothing in the demand that implies dynamic in the price.

implications of the investment firm's choice in estimation. However, Euler equations cannot (in general) be derived for problems that involve interactions among agents (as in the current dynamic oligopoly model), while the Berry and Pakes (2001) technique can accommodate them (see Appendix 2).

To model the firm's dynamic behavior on advertising, I assume the firm is only uncertain about the future awareness level of its product and competing products<sup>25</sup>, although their distributions are known by all<sup>26</sup>. For estimation reasons I also make the assumption that each firm behaves as a single-product firm regarding advertising. That means that the firm does not consider the cross effect among its other own products. This last assumption allows me to derive an equilibrium relationship that could be estimated given the available data. I explain this procedure in more details later.

To apply Blackwell's theorem, the standard assumptions that  $0 \leq \rho < 1$ , the state variable, the vector of awareness levels for all products  $\omega_t$ , evolves as a Markov process conditional on the vector of action, and net cash flows are bounded from above are also made. Then the optimal advertising expenditure can be obtained from the value function that solves the Bellman equation

$$V_j(\omega_t) = \sup_{a_{jt}} \{ \pi_j(a_t, \omega_t) + \rho E_\omega [V_j(\omega_{t+1}) | \omega_t, a_t] \}$$

where  $\pi_j(a_t, \omega_t) = [p_{jt} - mc_{jt}] s_j(\omega_t, a_t, p_t) M_t - a_{jt}$  is the profit function, and the expectation, given the distribution of the awareness level<sup>27</sup>, is

$$E_\omega [V_j(\omega_{t+1}) | \omega_t, a_t] = \sum_{\omega_{-j, t+1}} \sum_{\omega_{j, t+1}} V_j(\omega_{j, t+1}, \omega_{-j, t+1}) K(\omega_{j, t+1} | \omega_{jt}, a_{jt}) K(\omega_{-j, t+1} | \omega_{-jt}, a_{-jt})$$

where  $K(\omega_{-j, t+1} | \omega_{-jt}, a_{-jt})$  is the Markov transition kernel for the vector  $\{\omega_{-j, t+1}\}$ , with  $a_{-jt}$  and  $\omega_{-jt}$  denoting the vector of advertising and level of awareness (respectively) of the rival products.

Note that advertising has a independent effect on both current profit, affecting the consumer utility, and on future profits, affecting the distribution of the state variable.

<sup>25</sup>It assumption implies that firm is not uncertain about the future car models and their characteristics in the market (included the entry and exit of the car models).

<sup>26</sup>Note that future states and actions of rival models affect the firm-model value function since it is an oligopolistic market.

<sup>27</sup>The transition awareness level of a product and the assumed distribution of the variables, implies the Markov transition kernel for the awareness level

$$k(\omega_{j, t+1} | \omega_{j, t}, a_{j, t}) = \begin{cases} \frac{\varphi a_{jt}}{1 + \varphi a_{jt}} & \omega_{j, t+1} = \varphi \omega_{jt} + \psi \\ 1 - \frac{\varphi a_{jt}}{1 + \varphi a_{jt}} & \omega_{j, t+1} = \varphi \omega_{jt} \end{cases}$$

that implies the Markov transition kernels for the awareness levels are independent across products, so

$$K(\omega_{t+1} | \omega_t, a_t) = \prod_{r=1}^J K(\omega_{r, t+1} | \omega_{rt}, a_{rt})$$

If the solution for  $a_{jt}$  is interior<sup>28</sup>, it must satisfy the first-order condition:

$$0 = \frac{\partial \pi_j(a_t, \omega_t)}{\partial a_{jt}} + \rho \sum_{\omega_{-j,t+1}} \sum_{\omega_{j,t+1}} V_j(\omega_{j,t+1}, \omega_{-j,t+1}) \frac{\partial K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}} K(\omega_{-j,t+1} | \omega_{-jt}, a_{-jt})$$

where the second term is an expectation over a function of the entire future net cash flows<sup>29</sup>, that means that the FOC can be rewritten as

$$0 = \frac{\partial \pi_j(a_t, \omega_t)}{\partial a_{jt}} + \sum_{\tau=1}^{\infty} \rho^\tau \pi_j(a_{t+\tau}, \omega_{t+\tau}) \frac{\partial \ln K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}} + \varepsilon_{jt} \quad (8)$$

where  $\varepsilon_{jt}$  is an expectations error

$$\varepsilon_{jt} = \rho E \left[ V_{jt+1}(\omega_{t+1}) \frac{\partial \ln K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}} \Big| \omega_t \right] - \sum_{\tau=1}^{T_j^{Exit}} \rho^\tau \pi_j(a_{t+\tau}, \omega_{t+\tau}) \frac{\partial \ln K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}}$$

and  $T_j^{Exit}$  is the period when product  $j$  exits the market.

The Rational Expectations assumption,  $E[\varepsilon_{jt} | \omega_t] = 0$ , describes the equation that underlies the estimation<sup>30</sup>

$$E \left[ \left( (p_{jt} - mc_{jt}) \frac{\partial s_{jt}(\omega_t, a_t, p_t)}{\partial a_{jt}} M_t - 1 \right) + \sum_{\tau=1}^{\infty} \rho^\tau \pi_j(a_{t+\tau}, \omega_{t+\tau}) \frac{1}{a_{jt}(1 + \kappa a_{jt})} \Big| \omega_t \right] = 0 \quad (9)$$

The equilibrium relationships (6) and (9) determine a system of  $J * 2$  equations since they must be satisfied for all  $J$  products. To obtain the price and advertising equilibrium equations in matrix notation, I define two  $J$  by  $J$  new matrices, the price effects matrix,  $\Delta_t^p$ , and the brand matrix,  $\Gamma_t$ . The price effects matrix collects the own and cross-demand effects of price; therefore, its  $(r, j)$  element is  $\Delta_{rj}^p = \frac{\partial s_{jt}}{\partial p_{rt}}$ . The brand matrix is one of ones and zeros, with the  $(r, j)$  element being the indicator of whether product  $r$  is produced by the same firm as product  $j$ . I also define the vector  $\Delta_t^a$ , which collects the own-demand effect of advertising where its  $j$  element (row) is  $\Delta_{jt}^a = \frac{\partial s_{jt}}{\partial a_{jt}}$ .

<sup>28</sup>Only 5 models are not advertised: Renault 4, Jan1990-Aug1993, Skoda 136, Jan1990-Dec1991, Daewoo Compact, Jul1999-Dec2000, Toyota MR2, Mar2000-Dec2000, and BMW M3, Jan1997-Dec2000. 5.4% of the sample report zero advertising (usually no consecutive months).

<sup>29</sup>Given

$$\frac{\partial K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}} = \frac{\partial \ln K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}} K(\omega_{j,t+1} | \omega_{jt}, a_{jt})$$

then

$$\frac{\partial E_w [V_{jt+1}(\omega_{t+1}) | \omega_t, a_t]}{\partial a_t} = E_w \left[ V_{jt+1}(\omega_{t+1}) \frac{\partial \ln K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}} \right]$$

<sup>30</sup>The level of awareness at the beginning of the period is given by  $\omega_{j,t+1} = \varphi \omega_{jt} + \psi \varsigma_{jt}$ , where the  $\varsigma_{jt} = 1$  with a probability  $\kappa a_{jt} / (1 + \kappa a_{jt})$  and  $\varsigma_{jt} = 0$  otherwise. So

$$\frac{\partial K(\omega_{j,t+1} | \omega_{jt}, a_{jt})}{\partial a_{jt}} = \frac{\partial \ln(\kappa a_{jt} / (1 + \kappa a_{jt}))}{\partial a_{jt}} = \frac{\partial (\ln(\kappa a_{jt}) - \ln(1 + \kappa a_{jt}))}{\partial a_{jt}} = \frac{1}{a_{jt}(1 + \kappa a_{jt})}$$

So, the equations (6) and (9) in matrix notation are

$$(p_t - mc_t) = [\Gamma_t \circ \Delta_t^p]^{-1} s_t \quad (10)$$

$$(p_t - mc_t)M_t \circ \Delta_t^a - I + \left( \sum_{\tau=1}^{T_j^{Exit}} \rho^\tau \pi_{t+\tau} \circ \frac{1}{a_t \circ (I + \varkappa a_t)} \right) = \varepsilon_t \quad (11)$$

where  $\pi_{t+\tau} = (p_{t+\tau} - mc_{t+\tau})M_{t+\tau} \circ s_{t+\tau} - a_{t+\tau}$ , the symbol  $\circ$  represents Hadamard's product, and  $I$  is a vector of  $J$  ones.

To compute the equation (11) I need to observe models which have exited during the sampling period, because for models that are still alive at the end of the sampling period  $\sum_{\tau=1}^{T_j^{Exit}} \rho^\tau \pi_{t+\tau} \circ \frac{1}{a_t \circ (I + \varkappa a_t)}$  is not observed. The firm problem could be written in this same framework for the case that advertising will be a multiproduct dynamic decision. To compute the advertising equilibrium relationship from this new assumption I will need to observe firms which have exited during the sampling period. Unfortunately, I do not observed any in my data, and, for this reason I model advertising as a single dynamic decision.

## 5 Estimation Strategy

The model consists of three equations, a demand, a price and an advertising equation, with the associated parameters  $\alpha, \sigma_0, \beta, \gamma, \eta, \varkappa, \psi, \omega_0$  and  $\varphi$  (the discount factor,  $\rho$ , is fixed to be 0.99) to be estimated. They are estimated simultaneously using the Generalized Method of Moments (GMM) and simulation techniques. I use moments arising from the demand and firms' pricing and advertising decisions, which express orthogonality between appropriate instruments and the unobservable components. Therefore, I consider the objective function  $\Lambda Z A_N^{-1} Z \Lambda$ , where  $A_N$  is a weighting matrix and  $Z = (Z_\xi, Z_\zeta, Z_\varepsilon)$  are instruments orthogonal to the composite error  $\Lambda = (\xi, \zeta, \varepsilon)$

$$Z \Lambda = \begin{bmatrix} \sum_j Z_{\xi j} \xi_j(s^n, P_{ns}, P_H; \theta) \\ \sum_j Z_{\zeta j} \zeta_j(s^n, P_{ns}, P_H; \theta) \\ \sum_j Z_{\varepsilon j} \varepsilon_j(s^n, P_{ns}, P_H; \theta) \end{bmatrix}$$

where the observed vector of sampled market shares is  $s^n$  (number of households sampled  $n$ ), the empirical distribution of  $ns$  simulation draws from the assumed distribution of consumer characteristics  $v_i$  is  $P_{ns}$ , and  $P_H$  is the empirical distribution of  $H$  simulation draws from the assumed distribution of the state of awareness shocks  $\zeta_{jt}$ .

To compute the objective function, the unobservable components are solved following the technique proposed in BLP. To estimate the model, for each choice set all purchase probabilities have to be computed. However, due to the large number of product in the automobile industry (in average, 220 models in the market), it is not feasible to calculate

$2^{J-1}$  purchase probabilities corresponding to each choice set for each product. If one could observe the choice set facing a consumer prior to purchase the computational burden would become substantially lower. Unfortunately, this data is not available and the choice set facing an individual is not observed directly. To solve this dimensionality problem from the high number of possible choice set, I propose to simulate consumer choice sets according to the awareness probabilities. I implement this solution in the standard BLP algorithm. For each  $\theta$ , I solve for unobservable components following the steps:

(i) Estimate, via simulation, the market shares implied by the model.

(a) Given a draw from the Bernoulli<sup>31</sup>  $\varsigma_{jt}$  with probability  $(\frac{\varkappa a_{jt}}{1+\varkappa a_{jt}})$ , compute the vector of awareness probabilities for all products<sup>32</sup>  $\phi_{jt}(\theta)$  according to the equation (4) and (5).

(b) For each product, I draw  $ns$  realizations from a uniform  $[0,1]$ ,  $(u_{j1}, \dots, u_{jn_s})$

(c) For each period  $t$ , simulate  $ns$  consumer choice sets as follows

Define a  $ns$  Bernoullis for each product,  $b_{jit}$  with mean  $\phi_{jt}(\theta)$ , following the rule

$$b_{jit} = \left\{ \begin{array}{ll} 1 & \text{if } \phi_{jt}(\theta) \geq u_{ji} \\ 0 & \text{otherwise} \end{array} \right\}$$

Define the  $ns$  choice sets taking the  $i$ th bernouilli from each product,  $S_{it} = (b_{1it}, \dots, b_{jit}, \dots, b_{Jit})$

(c) Draw  $ns$  vectors  $(v_{iy}, v_{i0})$  from a multivariate normal distribution with mean 0 and an identity covariance matrix

(d) Summarizing, for each period  $t$ ,  $ns$  consumers are simulated,  $(v_{iy}, v_{i0}, b_{1it}, \dots, b_{jit}, \dots, b_{Jit})$ , with the same characteristics but different choice sets over time.

This procedure is repeated for  $H$  draws of  $\varsigma_{jt}$  to compute the simulation estimator of the market shares

$$s_{jt}(P_{ns}, P_R, \delta_t, X_t; \theta) = \frac{1}{H} \sum_{h=1}^H \left( \frac{1}{ns} \sum_{i=1}^{ns} f_{jt|S_{it}}(v_i, \varsigma_{ht}, \delta_t, X_t; \theta) \right) \quad (12)$$

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<sup>31</sup>The draws from the Bernoullis  $\varsigma_{jt}$  are computed in the following way

$$\varsigma_{jtr} = \left\{ \begin{array}{ll} 1 & \text{if } \varkappa a_{jt}/(1 + \varkappa a_{jt}) \geq u_{jr} \\ 0 & \text{otherwise} \end{array} \right\}$$

where  $u_{jr}$  is the  $r$  draw from a uniform random variable for product  $j$ .

<sup>32</sup>To compute the pre-sample awareness probability for models already existing at the beginning of the sample, I use the formula:

$$\omega_{j1/1990} = \omega_0 \varphi^{Age_j} + \psi \frac{\varkappa \bar{A}_j}{1 + \varkappa \bar{A}_j} \sum_{s=0}^{Age_j-1} \varphi^s$$

where  $\bar{A}_j$  is the average (monthly) advertising of model  $j$  in 1990 .

(ii) Solve for the demand unobservables, implied by the simulated and observed market share.

The demand unobservables are computed as

$$\xi_j(s^n, P_{ns}, P_H; \theta) = \delta_j(s^n, P_{ns}, P_H; \theta) - \left[ \sum_k x_{jkt} \beta_k + \gamma a_{jt} \right] \quad (13)$$

where I solve for the mean utility and advertising effect  $\delta_j(s^n, P_{ns}, P_H; \theta)$  recursively, using the contraction mapping suggested by BLP, which matches the model-predicted market share  $s_t$  and the observed market share  $s^n$  for all products in periods  $t$

$$\delta_t^{k+1} = \delta_t^k + \ln(s_t^n) - \ln(s_t)$$

and  $\delta_t^{k+1}$  is the vector of mean utilities computed in the step  $k + 1$ .

(iii) Calculate the vector of cost unobservables from the difference between the price and the markup computed from the shares. Using the equation (7), the cost unobservables can be written as

$$\zeta(s^n, P_{ns}, P_H; \theta) = [p - b(s^n, P_{ns}, P_H; \theta)] - \ln(w) \eta \quad (14)$$

where  $b(s^n, P_{ns}, P_H; \theta)$  is the markup calculated from the equation (10) as

$$[\Omega \circ \Delta^p(s^n, P_{ns}, P_H; \theta)]^{-1} s^n$$

(iv) Calculate the rational expectations disturbance  $\varepsilon$  from the equation (11)

$$\varepsilon(s^n, P_{ns}, P_H; \theta) = (p_t - mc_t) M_t \circ \Delta_t^a - I + \left( \sum_{\tau=1}^{T_j^{Exit}} \rho^\tau \pi_{t+\tau} \circ \frac{1}{a_t \circ (I + \kappa a_t)} \right) \quad (15)$$

The method outlined requires instruments that are correlated with specific functions of the observed data, but are not correlated with the unobservables and the expectations disturbance. I use instruments based on economic and econometric theory. A common assumption made in the literature is that the supply and demand unobservables are mean independent of both observed product characteristics and cost shifters,  $E[\xi_j \setminus (x, w)] = E[\zeta_j \setminus (x, w)] = 0$ . Then the characteristics and cost shifter of products can be employed as instruments in this equation. Note first that price and advertising are not included in the conditioning vector  $(x, w)$ . This is because price and advertising are determined in part by  $\xi$  and  $\zeta$ , and therefore advertising and price will likely be correlated with the error term, making these variables endogenous. It is also important to realize that the vector  $(x, w)$  includes the characteristics and cost shifters of all products. One natural feature of oligopoly is that markups respond differently to own and rival products. Therefore, I distinguish the instruments between the characteristics of products produced by the same multiproduct firm versus the characteristics of products produced by rival firms. The time dimension of my data is short in relation

to the variation space of product attributes, so the main source of the identification will be cross-sectional. However, sometimes this is not enough, so I also use the following instruments based on econometric theory: the differences in the prices with respect to their individual time means,  $\widetilde{p}_{jt} = p_{jt} - (\frac{1}{T})\sum_s p_{js}$ , lagged a number of periods. Instruments of this type were first proposed by Bhargava and Sargan (1983), and their moment restrictions have been studied in Arellano and Bover (1995). The error term of the equation from the firm's advertising decision, which underlies the estimation, is an expectation error. According to the rational expectation assumption it is not correlated with state variables. Therefore I use product's attributes and advertising as instruments in this equation.

To reduce computation time, I restrict the non-linear search over the parameters to a subset  $\{\alpha, \sigma_0, \omega_0, \varkappa, \varphi, \psi\}$ . I concentrate out the parameters  $\{\beta, \eta\}$  and minimize the GMM objective function with regard to  $\{\alpha, \sigma_0, \omega_0, \varkappa, \varphi, \psi\}$ . This search is performed using the Nelder-Mead (1965) non derivative simplex search routine<sup>33</sup>.

In order to obtain an optimal estimator, I use a consistent estimate of the weight matrix from a preliminary suboptimal GMM estimator (where  $A = ZZ'$ )

$$\widehat{A}_N = N \left( \widehat{\Lambda} \otimes ZZ' \right)$$

where  $\widehat{\Lambda}$  is the preliminary suboptimal GMM residual covariance matrix. As a result all the reported statistics are robust to heteroskedasticity and serial autocorrelation.

## 5.1 The Identification of the Effects of Advertising

The model follows the same identification strategy as BLP and the literature that follows. However, my model departs from BLP in that consumer choice set is endogenous and affected by advertising. Therefore, this subsection focuses on how the consumer choice set parameters are identified by the data.

The identification of the effect of advertising on the utility and consumer choice set comes primarily from examining how these effect change over the life of the car model. The probability that product  $j$  will be purchased by consumer  $i$  (1) can be written as (see Appendix 1):

$$\Pr(U_{ijt} \geq U_{imt}, \forall m \in S_t | j \in S_t, X_{jt}, X_{-jt}) \Pr(j \in S_t | \{X_{js}\}_{s=1}^t)$$

where advertising is included as a product's characteristic in  $X_j$ , and affects both probabilities.

Such a structure of the choice probabilities implies that an increase in advertising of product  $j$  will increase both terms. However, at the end of the information diffusion process,

<sup>33</sup>The Nelder-Mead method is a commonly used nonlinear optimization algorithm which uses the concept of a simplex (a polytope of N+1 vertices in N dimensions). A non derivative method like this is required in this estimation given that the objective function is not smooth with respect to the awareness parameters ( $\omega_0, \varkappa, \psi$  and  $\varphi$ ).

when almost all consumers are aware of the product or the stationary proportion of consumers aware about the product is reach, only the first term will be affected. In this case, the only effect of advertising is through the consumer utility. Therefore, the identification of the effect of advertising comes from the fact that car models that remain on the market a lot of time will be known by most of consumer (stationary proportion) and advertising of the car model will only affect the consumer's utility.

The effect of previous awareness coefficient  $\varphi$  is identified by the variance in the advertising expenditure among car models at the entry. The comparison of the initial growth of the sales between models heavily and scarcely advertised identifies this parameter.

## 5.2 Truncation Remainder

For models that are still alive at the end of the sampling period,  $\sum_{\tau=1}^{\infty} \rho^{\tau} \pi(\omega_{t+\tau}, a_{t+\tau}; \theta)$  is not observed. That is, I only have complete data on models which have exited. Therefore, the estimation strategy is to select data only from those models with complete data (models that exit before the last period of the sample) for the advertising equations (15). My data consists of an unbalanced panel of 257 models, and only 97 models exit before the last period of the sample (Dec.2000). It consists of 5,732 observations (out of the total of 16,362). The entire model live is observed for 45 of these models, which means they enter and exit during the sample period (there are 180 total model entries, so 135 new models stay on the market after the last period of the sample). The mean age of the model when it exits the market is around 8 years, although the heterogeneity is high (minimum age 2 years, and maximum age 20 years).

If one selects data only from those models which sell off before the end of the sample, one would be selecting models on the basis of the sample random draws that determine the residual from the estimating equation, and hence incur the possibility of a selection bias<sup>34</sup>. Of course, the truncation remainder will affect the results depending on the sample, and a priori one is not concerned about correcting for the truncation remainder. The select sample consists of 97 models. From these models, I observe the entire live of 45 model that enters during the sample period. The other 52 models entered the market before January 1990. Within these subsample of 97 models the mean age of exits the market is also very heterogeneous. It suggests that the truncation remainder will not be very important, although it could be verified using a semiparametric technique, and corrected for it if it is<sup>35</sup>.

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<sup>34</sup>Let  $T$  be the final year of the sample. The truncation remainder problem is that  $V(\omega_{T+1})$  is not observed for those firms which survive until  $T$ , and the firms which survive until  $T$  are selected (in part) on the basis of the realizations of the disturbance terms.

<sup>35</sup>Berry and Pakes (2001) introduces a semiparametric technique which allows us to do precisely that (checks and corrects it).

## 6 Empirical Results

In this section, the estimation procedure described is applied to the Spanish automobile market data set. First I begin by introducing different specifications (or scenarios) I will compare. In the first scenario it is assumed that consumers are aware of all the products, and advertising only affects the consumer utility. This represents a standard assumption in the differentiated products literature, which I report for a random coefficients logit model (a little different from those typically estimated in the IO literature since a static advertising equation<sup>36</sup> is included to deal with the endogeneity problem that arises from the effect of advertising on consumer utility). I will refer to this model as the standard model. In the second specification, I adjust the choice set for changes in the product awareness using the method outlined above. Therefore in this specification advertising affects both the consumer utility and the choice set (the awareness level of the products), and the firm's decision is estimated according to the equation (9), given the dynamic effect of advertising that arises from the effect of advertising on future consumer awareness. I will refer to this model as the consumer awareness model.

The results are presented as following: first, I discuss the substitution patterns present in the Spanish automobile market. Next, I highlight the importance of introducing the awareness process to evaluate the advertising effect over the product life. Finally, I discuss the effect of advertising on the consumer welfare, in particular, the effect on the loss of consumer welfare that comes from the lack of information.

Table 8 reports the parameter estimates for the two specifications. The estimates are broken down into three categories. First, the demand side parameters that include the effect of product attributes on the mean consumer utility ( $\beta$ ), the term on price ( $\alpha$ ), and the deviation of the consumer taste for the outside good ( $\sigma_0$ ). Second, the effect of (ln) attributes ( $\eta$ ) on the marginal cost of product are presented as the cost side parameters. Finally, the awareness parameters that underline the transition equation of the awareness level of products (which defines the awareness probabilities).

**The substitution patterns.** Regardless the model (the consumer awareness and the standard model), the estimated coefficients of attributes ( $\beta$ ) and advertising ( $\gamma$ ) imply a significant positive effect on the mean utility, while the estimated effect of price ( $\alpha$ ) is significant negative. According to the estimations, the (ln) attributes ( $\eta$ ) also have a positive impact on the (ln) marginal cost of a product. All of them are significant, with the exception of the size that is highly correlated with the product weight. Neither of the estimations do not change significantly across specifications. It could be as a result that the observations for models

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<sup>36</sup>If only the effect of advertising on the consumer utility is considered, advertising will be a static choice with a FOC:

$$\left[ (p_{jt} - mc_{jt}) \frac{\partial s_{jt}(a_t, \omega_t, p_t)}{\partial a_{jt}} M_t \right] - 1 = 0$$

with a age smaller than two year are only around 11% of the sample (1,764 observations of 16,362).

The demand-price elasticities and mark-up estimated reproduce the expected pattern in a (vertically) differentiated-product market. With a inverse relationship between the mark-up and demand-price elasticities (higher mark-up for car models with smaller demand-price elasticity). Both elasticity and mark-up are presented in Table 9 for the associated models to each percentiles of the price. The associated model is one whose price is the closest to the percentile. The reported elasticities and mark-up are for the consumer awareness model. For this model the demand-price elasticity is given by

$$\eta_{jt} = \frac{1}{H} \sum_{h=1}^H \left( \frac{1}{ns} \sum_{i=1}^{ns} \alpha e^{-(m_t + \sigma_y^2 v_{iy})} f_{jt|S_{it}}(v_i, s_{ht}, \delta_t, X_t; \theta) (1 - f_{jt|S_{it}}(v_i, s_{ht}, \delta_t, X_t; \theta)) \right) \frac{p_{jt}}{s_{jt}}$$

$$\eta_{k-jt} = \frac{1}{H} \sum_{h=1}^H \left( \frac{1}{ns} \sum_{i=1}^{ns} \alpha e^{-(m_t + \sigma_y^2 v_{iy})} f_{jt|S_{it}}(v_i, s_{ht}, \delta_t, X_t; \theta) f_{kt|S_{it}}(v_i, s_{ht}, \delta_t, X_t; \theta) \right) \frac{p_{jt}}{s_{kt}}$$

where  $\eta_{jt}$  is the own demand-price elasticities of product  $j$  in period  $t$ , and  $\eta_{k-jt}$  is the cross demand-price elasticities of product  $k$  at period  $t$  respect to product  $j$ .

The (model) mean demand-price elasticity is 3.11 and the (model) mean mark-up is 28.73. This results are along the lines to previous literature for the car market (see for example Berry, Levinsohn, and Pakes 1995, Verboven 1996 or Moral and Jaumandreu 2007).

Table 10 presents the own demand-advertising elasticities attributed to the effect of the advertising on the utility, for the consumer awareness models to each percentile of the ratio advertising over revenue. The associated model is one whose ratio is the closest to the mean of the percentile. These elasticities are estimates for the consumer awareness model. For this model the demand-advertising elasticities attributed to the effect of the advertising on the utility are given by

$$\varepsilon_{jt}^A = \frac{1}{H} \sum_{h=1}^H \left( \frac{1}{ns} \sum_{i=1}^{ns} \gamma f_{jt|S_{it}}(v_i, s_{ht}, \delta_t, X_t; \theta) (1 - f_{jt|S_{it}}(v_i, s_{ht}, \delta_t, X_t; \theta)) \right) \frac{a_{jt}}{s_{jt}}$$

where  $\varepsilon_{jt}^A$  is the own demand-advertising elasticities of product  $j$  in period  $t$ .

The (model) mean demand-advertising elasticity attributed to the effect of the advertising on the utility is around 0.18, and it seems to be larger for models that belong to lower market segments, where price is lower.

**The effect of advertising on the consumer awareness process.** The awareness parameters for the consumer awareness model are presented in the last rows of Table 8. The initial awareness level ( $\omega_0$ ) obtained is 0.34. This means, given the model, that the fraction of consumer aware of the product or the probability of knowing the product when it enters the market is around 0.25. The effect of the previous awareness level ( $\varphi$ ) is 1.18. This

parameter captures factors, other than advertising, that explain the information diffusion of new products. Note that this model does not impose that consumers do not forget, and this fact is also captured by the parameter associated to the previous awareness level. The parameters  $\varphi$  implies that the awareness level of a product to increase over time even if the product is never advertising. The awareness parameters of the advertising effect are  $\psi$  and  $\varkappa$ , both of them significant. To evaluate the effect of advertising in the information diffusion of a new product, a dynamic demand-advertising elasticity could be used. However, I compute the awareness probabilities over product age for two cases: without advertising and with advertising affecting the information diffusion of new products, in order to evaluate the effect of advertising on the consumer awareness process in a more intuitive way. The awareness probabilities over product age are computed using the estimated parameters and the transition equation (5). If advertising does not affect the information diffusion of a new product, the awareness level of a product only depends upon the pervious awareness level, and it will be computed using the transition equation

$$\omega_{j\ t+1} = \varphi\omega_{j\ t}$$

I take the (model) average expenditure on advertising observed in the data to compute the awareness probabilities with advertising. The results are represented in Figure 3. Without advertising the information diffusion of a new product will finish (most consumers know the product or the product awareness probability is close to one) around the third year, while with advertising most consumer are aware of the product around the one and half year, reducing the information process to half as long. Therefore this result support the idea that advertising significantly reduces the information diffusion of new products in the Spanish automobile market.

**The effect of advertising on the consumer welfare.** A major difference between the informative and persuasive advertising is with respect to the welfare effect. Informative advertising unambiguously improves consumer’s welfare, by providing them with more information, and thus allowing them to make better choices. The welfare implications of persuasive advertising are ambiguous<sup>37</sup>, and depend on the exact interpretation of the direct effect of advertising on utility. Akerberg (2003) evaluate the welfare implications of the “implicit” information (unobserved quality) that advertising signals. In contrast, I evaluate the welfare implication of the explicitly information of the product’s existence that advertising has. In particular, I evaluate the effect of advertising on the loss of consumer welfare from the lack of information. This loss of consumer welfare comes from the fact that potential consumers are not aware of a new product will purchase another one that provides lower utility. The question is by how much advertising reduces this loss of consumer welfare, given its effect on

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<sup>37</sup>The persuasive effect might imply that consumers’ preferences are unstable and can be manipulated by advertising. In this case it’s not clear how to interpret change in utility welfare-wise.

consumer awareness. To answer this question I measure the total consumer utilities for three case: full advertising, information diffusion without advertising, and information diffusion affected by advertising. The total consumer utilities for the three cases are computed as

$$TCU^{(*)} = \sum_{t=0}^T \sum_{j=0}^J \hat{s}_{jt}^{(*)} M_t \hat{U}_{jt}$$

where  $\hat{s}_{jt}^{(*)}$  is the simulated market share for product  $j$  at period  $t$  for the case  $(*)$ , and  $\hat{U}_{jt}$  is the estimated utility from the product  $j$  at time  $t$  using parameters estimated with the consumer awareness model,  $\hat{U}_{jt} = \sum_{i=1}^{ns} (\hat{\delta}_{jt} - \hat{\alpha}_i p_{jt} + \varepsilon_{ijt})$ .

The simulated market shares are given by

$$\hat{s}_{jt}^{(*)} = \frac{1}{H} \sum_{h=1}^H \left( \frac{1}{ns} \sum_{i=1}^{ns} f_{jt|S_{it}^{(*)}}(\hat{U}_t) \right)$$

where  $\hat{U}_t$  is the vector of the estimated utilities for all product at period  $t$ , and the consumer choice set  $S_{it}^{(*)}$  are simulated using the respective awareness probabilities for each case

Full information :  $\phi_{jt} = 1, \forall j \text{ and } \forall t$

ID without advertising :  $\phi_{jt}(age_{jt})$ , where  $\omega_{j\ t+1} = \varphi \omega_{j\ t}$

ID with advertising :  $\phi_{jt}(age_{jt}, \{a_{js}\}_{s=T_j}^t)$ , where  $\omega_{j\ t+1} = \varphi \omega_{j\ t} + \psi \varsigma_{jt}$

Given the total consumer utility for the three cases, I calculate the loss of consumer welfare without advertising ( $TCU^{(FI)} - TCU^{(ID \text{ without } adv)}$ ) and with advertising ( $TCU^{(FI)} - TCU^{(ID \text{ with } adv)}$ ). Comparing both losses, I conclude that advertising reduces the loss of consumer welfare from the lack of information by around 37% in the Spanish automobile market.

## 7 Conclusions

This paper evaluates the role of advertising in the information diffusion of a new product. Using a structural model in which consumer purchase decision is specified with a discrete choice model with variation in the choice set and a dynamic model of advertising taking into account the dynamic effect of advertising on future consumer awareness (future sales), my paper finds for the Spanish automobile market that advertising significantly enhances the information diffusion of new products and reduces the loss of consumer welfare that comes from the lack of information.

This model is also potentially applicable to other industries of differentiated and durable products with a high product turnover due to intense innovation. The computer, digital camera or electrical good industry are some examples of these market, where the product life-cycle and an important advertising expenditure at the product's entry are observed. One of

the main contributions of this paper is to show how the information diffusion of a product may be incorporated into an empirical model of consumer behavior in order to reproduce aspects of the product life-cycle. An alternative explanation<sup>38</sup> for the increase of new product's sales is a dynamic consumer's behavior. Consumers could delay their buying because the price usually drops in this kind of market within a short period of time (Gowrisankaran and Rysman, 2007). This dynamic consumer's behavior, however, does not explain the important advertising expenditure at the entry of new products.

This paper focuses on one of the most important variables that determine the information diffusion of new products, namely advertising. My model partially captures the effect of previous sales on the information diffusion through the model age. However, extensions of my model could implicitly address the effect of previous sales in the information diffusion of new products. In my approach it implies a more complicated firm's problem capable of capturing that firms internalize the dynamic effect of price and advertising that arise through the effect of previous sales in the information diffusion of a new product. The dynamic effect of price, through the effect of previous sales in the information diffusion of new products, will imply that firms chose a lower price during the information diffusion of new products in order to enhance it. However, this dynamic behavior of price is not observed in durable products because it is compensated by the drop of prices<sup>39</sup> as a result of consumer's forward-looking behavior and firm's dynamic policies (Zhao, 2007). Therefore, this extension of my model will also have to address the dynamic consumer's behavior in durable products. In this sense, my model is also potentially applicable to frequently-purchased product market in order to study the effect of previous sales and advertising in the information process of new products (Akerberg, 2001).

Of particular interest might be comparing estimates of advertising across different types of products, checking the role of advertising in it.

A central limitation of my model is that it does not consider a more realistic firm-side model that endogenizes entry, exit, product characteristics, and their interactions with advertising decision. These deficiencies point to further research on different directions.

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<sup>38</sup> Another explanation is the reduction of the uncertainty about a new product that the risk averse consumer faces. However, these effects could be implicitly captured by my model through the redefinition of awareness level as the minimum knowledge level needed to include a new product in her choice set.

<sup>39</sup> The drop of price is also observed in the Spanish automobile market (see Table 3).

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### Appendix 1. The choice probability.

Consumer  $i$  chooses among  $J + 1$  products, where  $j = 0$  denoting the outside alternative and each product  $j$  has observable attributes  $X_j$ . Consumer  $i$  has characteristics  $Y_i$  and a choice set  $S_i$  (subset of the  $J$  products that contains all the products consumer  $i$  is aware of, where the outside alternative is assumed to be always known by consumers). Product  $j$  is known by the consumer  $i$  (included in the choice set  $S_i$ ) with probability

$$\Pr(j \in S_i | Y_i, X_j)$$

that may vary across individuals and across products, and it may depend on interactions of consumer and product characteristics (for example, young consumers may be more likely to include the products most advertised on the Internet). Note the assumption below that  $\Pr(j \in S_i | Y_i, X_j)$  does not depend on rival product characteristics  $X_{-j}$ . This assumption might rule out some of the cognitive constraints stories behind the consumer choice set. For example, if the consumer only remembers the 3 most advertised products, this assumption will be violated.

The consumer chooses one of the products from her choice set, so the conditional probability of choosing  $j$  (for  $j \in S_i$ ) is

$$\Pr(j | S_i, Y_i, X_j, X_{-j})$$

where  $X_{-j}$  denotes the characteristics of all the products except  $j$ .

The choice probability for product  $j$  can be computed by integrating out all possible choice sets that include product  $j$ ,  $F_j$ ,

$$\Pr(j | Y_i, X_j, X_{-j}) = \sum_{S_i \in F_j} \Pr(j | S_i, Y_i, X_j, X_{-j}) \Pr(S_i | Y_i, X_j, X_{-j})$$

where

$$\Pr(S_i | Y_i, X_j, X_{-j}) = \prod_{m \in S_i} \Pr(m \in S_i | Y_i, X_m) \prod_{n \notin S_i} (1 - \Pr(n \in S_i | Y_i, X_n))$$

For the discussion of identification, it will be more convenient to rewrite the choice probability as

$$\Pr(j | Y_i, X_j, X_{-j}) = \Pr(j | j \in S_i, Y_i, X_j, X_{-j}) \Pr(j \in S_i | Y_i, X_j)$$

where

$$\Pr(j | j \in S_i, Y_i, X_j, X_{-j}) = \sum_{S_i \in F_j} \Pr(j | S_i, Y_i, X_j, X_{-j}) \Pr(S_{i,-j} | Y_i, X_{-j})$$

with  $S_{i,-j}$  denoting the elements of consumer choice set other than  $j$ , and

$$\Pr(S_{i,-j} | Y_i, X_{-j}) = \prod_{\substack{m \in S_i \\ m \neq j}} \Pr(m \in S_i | Y_i, X_m) \prod_{n \notin S_i} (1 - \Pr(n \in S_i | Y_i, X_n))$$

## Appendix 2. Berry and Pakes (2001) Technique

Berry and Pakes (2001) introduces an estimation technique based on the first-order conditions (optimality conditions) for continuous controls for dynamic models. It is applied to empirical models of markets where current decisions will have independent impact on future states, as well as on current profits. Examples include models with: learning by doing, experience, network or addictive goods, and collusion (applies both to models of single agents choosing policies in games against nature, and to dynamic games). This technique also allows to use the information on investment choice (advertising, R&D or investment in traditional capital stock) to help in estimation.

In the context of the single agent models, the investment equations are typically analyzed using the more standard Euler equation estimation techniques introduced into economics by Hansen and Singleton (1989). The technique proposed by Berry and Pakes (2001) is an alternative to this, with computational properties similar to those of Euler equation techniques. That is, neither of them requires the researcher to solve explicitly for the value function and/or the investment policy of the firm in order to use the implications of the investment choices in estimation. This alternative method has both advantages and disadvantages relative to the Euler equation technique. As advantages, (i) it allows us to analyze the dynamic game, (ii) it uses a stochastic accumulation model, and (iii) it allows future actions to be at the corner of the choice set, and so on. On the other hand, to use the Berry and Pakes (2001) method, one does have to specify the transition probabilities of the states conditional on the controls. It is likely, however, that this method is less sensitive to slight misspecifications in timing assumptions than is the Euler equation because it is based on the returns for investment over the entire future, while Euler equation techniques are only concentrated on returns over the period until the compensating perturbations are made.

Standard first-order conditions for continuous controls depend upon the derivative of the expected discounted value of future net cash flows. This expectation is not directly observed. However, under the rational expectation assumption, this can be expressed as a derivative of the discounted value of future net cash flows plus an error that will be mean independent of all variables known at the time the expectation is made. Then the first-order conditions will be a conditional expectation of known primitives, and a standard method of moments estimation algorithm can be used. To show the method, I present the firm's problem in a general setting.

The firm's problem is to choose a sequence of actions, say  $\{a_{t+\tau}\}$ , to maximize the expected discounted value of net cash flows,  $\{n_{t+\tau}\}$ , conditional on the information sets,  $\{\omega_{t+\tau}\}$ , that will be available when those actions are to be taken. So, the firm's problem is to choose  $a_t$  to

$$\sup_{a_t} E \left[ \sum_{\tau=0}^{\infty} \rho^{\tau} n_{t+\tau} | \omega_t, a_t \right] \equiv V(\omega_t)$$

where the expectation is assumed with the understanding that optimal actions will be taken

in each future period, and  $\rho$  is the discount rate. To apply Blackwell's theorem, we also make the standard assumptions that  $0 \leq \rho < 1$ ,  $\{\omega_t\}$  evolves as a Markov process conditional on the vector of action, and net cash flows are bounded from above. Then the optimal action can be obtained from the unique value function that solves the Bellman equation

$$V(\omega) = \sup_a \left\{ n(a, \omega) + \rho \int_{\omega'} V(\omega') k(\omega' | \omega, a) \right\}$$

where  $K(\omega' | \omega, a)$  is the Markov transition kernel for  $\{\omega_t\}$  conditional to the action  $a$ .

Two assumptions are made:

- Rational Expectations:

$$\sum_{\tau=0}^{\infty} \rho^{\tau} n_{t+\tau} = V(\omega_t) + \varepsilon_t$$

with,  $E[\varepsilon_t | \omega_t] = 0$

- Smoothness: (i)  $K(\omega' | \omega, a)$  has support which is independent of  $a$ , (ii) the density  $k(\omega' | \omega, a)$  differentiable in  $a$  for almost every  $\omega'$  and every  $\omega$ , and (iii)  $n(\omega, a)$  is a differentiable function of  $a$  for almost every  $\omega$ .

Given the Bellman Equation and the assumptions:

$$\begin{aligned} 0 &= \frac{\partial n(\omega_t, a_t)}{\partial a_t} + \rho \int V(\omega_{t+1}) \frac{\partial k(\omega_{t+1} | \omega_t, a_t)}{\partial a_t} d\omega_{t+1} \\ &= \frac{\partial n(\omega_t, a_t)}{\partial a_t} + \rho E \left[ V(\omega_{t+1}) \frac{\partial \ln k(\omega_{t+1} | \omega_t, a_t)}{\partial a_t} \middle| \omega_t \right] \end{aligned}$$

$$0 = \frac{\partial n(\omega_t, a_t)}{\partial a_t} + \sum_{\tau=1}^{\infty} \rho^{\tau} n_{t+\tau} \frac{\partial \ln k(\omega_{t+1} | \omega_t, a_t)}{\partial a_t} + \varepsilon_t$$

$$\text{where } \varepsilon_t = \rho E \left[ V(\omega_{t+1}) \frac{\partial \ln k(\omega_{t+1} | \omega_t, a_t)}{\partial a_t} \middle| \omega_t \right] - \sum_{\tau=1}^{\infty} \rho^{\tau} n_{t+\tau} \frac{\partial \ln k(\omega_{t+1} | \omega_t, a_t)}{\partial a_t}$$

So the equation which underlies the estimation of the parameters is  $E(\varepsilon_t | \omega_t) = 0$ , that is

$$E \left[ \frac{\partial n(\omega_t, a_t; \theta)}{\partial a_t} + \sum_{\tau=1}^{\infty} \rho^{\tau} n_{t+\tau}(\omega_{t+\tau}, a_{t+\tau}; \theta) \frac{\partial \ln k(\omega_{t+1} | \omega_t, a_t; \theta)}{\partial a_t} \middle| \omega_t \right] = 0$$

Table 1: IV Logit Demand

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Dependent variable:  $\ln(s_j) - \ln(s_0)$

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Constant	-16.909	(0.377)	-17.352	(0.380)
CC/Weight	2.076	(0.123)	2.169	(0.116)
Max.Speed	1.274	(0.114)	1.390	(0.115)
Km/l	0.603	(0.075)	0.617	(0.073)
Size	2.767	(0.354)	3.061	(0.342)
Age			0.060	(0.071)
Age <sup>2</sup>			-0.039	(0.017)
Age <sup>3</sup>			0.005	(0.002)
Age <sup>4</sup>			-2.18e-4	(0.84e-4)
Age <sup>5</sup>			0.03e-4	(0.01e-4)
Price	-1.822	(0.080)	-1.869	(0.083)

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Note: Seasonal, brand, and year controls are included in the equation.

Instruments: BLP instruments, differences with respect to the (indiv) time mean of price lagged one year, and the number of model and new models.

The standard errors (reported in parentheses) are robust to heteroskedasticity and serial correlation.

Table 2: Model Age and the Expenditure on Advertising a Model (model means)

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Interval of Model Age	Advertising (millions euros 1995)	Sales (units)	Advertising/revenue (%)
1 year	5.57	6,846.3	9.97
1-2 years	5.52	9,052.5	6.69
2-3 years	5.54	9,729.6	6.16
3-4 years	4.84	9,081.4	5.09
4-5 years	4.83	8,879.5	4.84
5-6 years	4.37	8,814.4	4.23

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Note: To avoid model selection problems, only models that exit the market after six years are considered (72, 75, 79, 91, 93 and 94 models). The mean age of models that exit the market is around 8 years.

Table 3: Age Effect for Advertising and Price

	OLS ln(price) on ln(w)		OLS Adv on w	
Constant	3.723	(0.078)	-12.168	(2.875)
CC/Weight	0.518	(0.009)	1.234	(0.293)
Max.Speed	1.032	(0.014)	7.474	(4.282)
Km/l	0.243	(0.009)	3.934	(0.307)
Size	0.198	(0.017)	-0.306	(1.334)
Weight	1.055	(0.012)	-2.728	(0.524)
Trent	-0.001	(3e-5)	-1.345	(0.423)
Age	-0.020	(0.003)	-0.629	(0.183)
Age <sup>2</sup>	0.006	(0.001)	0.066	(0.052)
Age <sup>3</sup>	-7e-4	(1e-4)	-0.003	(0.005)
Age <sup>4</sup>	0.3e-4	(0.05e-4)	1e-4	(3e-4)
Age <sup>5</sup>	-0.02e-4	(0.01e-4)	-0.02e-4	(0.05e-4)
N. Models			0.106	(0.032)
N. New Models			-0.029	(0.026)

Note: Brand, annual and seasonal controls are included in the equation.  
The standard errors are reported in parentheses.

Table 4: Tariff in the Spanish automobile market (%)

	1987	1988	1989	1990	1991	1992	From 1993 on
European foreign producers	28.4	22.9	17.4	12.8	8.2	3.6	0
Non-European foreign producers	34.3	24.5	23.6	19.3	15.08	12.8	10

Table 5: Spanish income per capita (euros 1995)

1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
7749,5	7863,1	7949,6	7824,8	7836,1	8234,7	8257,5	8450,4	8655,8	9093,1	9232,3

Table 6: Entries/Exits and Basic Statistics (yearly model means)

	No. of Models	Model Entries	Model Exits	Quantity (sales)	Price	Advertising	Size (m <sup>2</sup> )	Max. Speed	Kms/l	cc/w
1990	97	20	2	10,979.33	11,870	3.27	6.61	171.7	5.29	1.62
1991	105	10	5	8,730.8	11,740	2.64	6.66	173.1	5.32	1.61
1992	116	16	10	9,686.7	11,286	3.89	6.71	174.9	5.34	1.64
1993	117	11	8	6,615.8	11,557	4.72	6.74	175.9	5.34	1.63
1994	122	13	12	8,272.9	11,469	4.91	6.69	174.3	5.39	1.69
1995	127	17	11	7,316.3	11,844	4.18	6.70	175.1	5.49	1.56
1996	134	18	11	7,660.2	11,941	4.19	6.76	176.4	5.44	1.51
1997	152	29	11	7,902.1	11,907	4.13	6.84	178.3	5.54	1.48
1998	160	19	11	8,642.2	11,918	3.97	6.91	180.3	5.76	1.46
1999	160	11	7	9,317.3	11,741	3.89	6.97	182.3	5.81	1.44
2000	169	16	5	9,351.3	11,783	3.44	7.01	183.8	5.85	1.43
Total		180	93							

Note: Price (euros 1995) and characteristics are sales weighted means. Advertising is in millions of euros 1995.

Table 7: The Range of Advertising/Revenue (obs with age&gt;2)

Percentile	(%) Adv/revenue	Associated Model	Price	Model Adv
0	0	BMW M3	52,158.2	0
10	0.21	Chrysler G. Voyager	25,453.5	0.03
20	0.91	Mercedes 190	25,233.9	0.67
30	1.63	Audi 80	20,147.6	1.07
40	2.50	Volvo 940	19,931.7	1.65
50	3.23	Citroen Xantia	14,296.0	2.75
60	4.12	Ford Puma	13,702.9	3.92
70	5.29	Honda Concerto	12,552.8	4.18
80	6.72	Toyota Carina	13,842.1	5.43
90	9.35	Fiat Tipo	11,842.7	7.99
100	34.85	Suzuki Swift Sedan	9,879.7	34.86

Note: Associated model is one whose price is the closest to the mean of the percentile. Price (euros 1995) and advertising (in millions of euros 1995) are the annual means of the associated model. Only 5 models are not advertised (5.4% of the sample report zero investment).

Table 8: Estimated Parameters of the Demand, Pricing and Advertising Equations

		Standard Model		Consumer Awareness Model	
<hr/>					
Demand Side Parameters					
Means ( $\beta$ )	Constant	-8.561	(0.113)	-8.555	(0.084)
	CC/Weight	0.058	(0.019)	0.107	(0.022)
	Max.Speed	1.605	(0.017)	1.968	(0.076)
	Km/l	0.063	(0.028)	0.014	(0.003)
	Size	4.752	(0.048)	5.064	(0.088)
	Adv	1.692	(0.014)	1.742	(0.159)
Std. Dev of the outside good ( $\sigma_0$ )		3.112	(0.001)	3.361	(0.011)
Term on Price ( $\alpha$ )		-28.67	(0.554)	-30.76	(1.451)
Seasonal controls		yes		yes	
Brand controls		yes		yes	
Regulation plans controls		yes		yes	
<hr/>					
Cost Side Parameters					
Attributes ( $\eta$ )	Constant	4.299	(0.441)	4.292	(0.574)
	ln(CC/Weight)	0.566	(0.019)	0.579	(0.039)
	ln(Max.Speed)	0.820	(0.103)	0.818	(0.110)
	ln(Km/l)	0.206	(0.033)	0.219	(0.067)
	ln(Size)	0.121	(0.097)	0.112	(0.115)
	ln(Weight)	0.988	(0.020)	0.983	(0.076)
	Trend	-0.003	(0.001)	-0.002	(0.001)
Brand controls		yes		yes	
<hr/>					
Awareness Parameters					
Initial Awareness ( $\omega_0$ )				0.342	(0.001)
Previous Awareness ( $\varphi$ )				1.181	(0.002)
Advertising Parameter ( $\varkappa$ )				0.753	(0.009)
Awareness Random Vb Parameter ( $\psi$ )				4.778	(0.192)

Note: the instruments are BLP instruments and differences of prices with respect to their (indiv) time mean lagged one year, and the number of model and new models. The standard errors (reported in parentheses) are robust to heteroskedasticity and serial correlation.

Table 9: Demand-Price Elasticities

Percentile	Price	Associated model	Price Elast	Markup (%)
0	0.491	Marbella	4.910	15.0
10	0.729	Micra	4.143	18.9
20	0.890	Logo	3.113	29.3
30	1.089	Scoupe	3.296	25.8
40	1.212	Vento	3.124	29.5
50	1.374	Marea	2.812	33.7
60	1.636	Lybra	2.490	38.7
70	1.888	Swift	2.596	38.4
80	2.192	Galaxy	2.995	31.4
90	2.528	325	2.396	42.1
100	6.919	TT	3.220	32.2

Note: Estimated elasticities from the Consumer Awareness Model parameters.  
Associated model is one whose price is the closest to the mean of the percentile.

Table 10: Demand-Advertising Elasticities from the Effect on Consumer Utility

Percentile	Adv/revenue (%)	Associated Model	Consumer Awareness Model
0	0	BMW M3	0.000
10	0.21	Chrysler G. Voyager	0.027
20	0.91	Mercedes 190	0.045
30	1.63	Audi 80	0.029
40	2.50	Volvo 940	0.188
50	3.23	Citroen Xantia	1.093
60	4.12	Ford Puma	1.639
70	5.29	Honda Concerto	0.215
80	6.72	Toyota Carina	0.301
90	9.35	Fiat Tipo	0.183
100	34.85	Suzuki Swift Sedan	0.975

Note: Observations for model age bigger than two years are only considered.



Figure 1: Estimated model age effect on market share, Table 1.

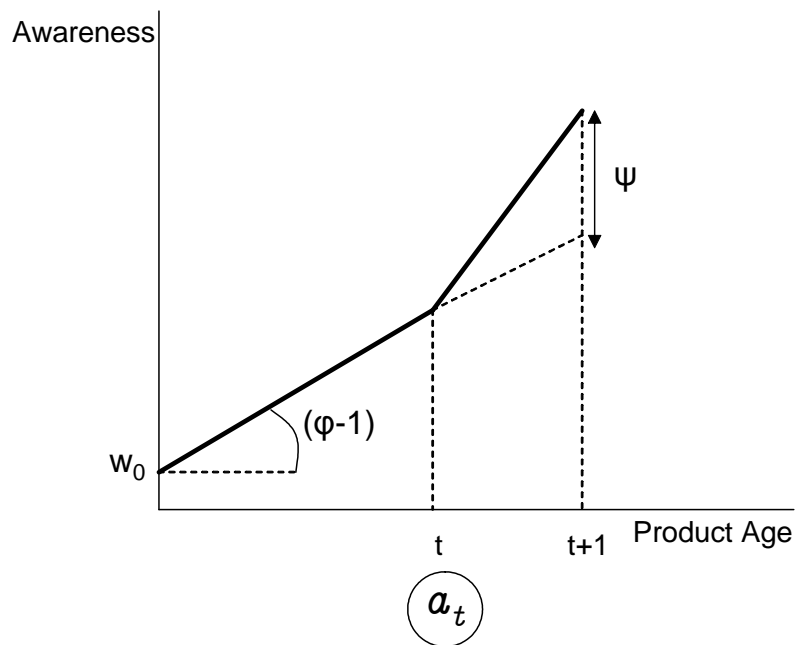


Figure 2: Product awareness level for the case  $\varphi > 1$ .

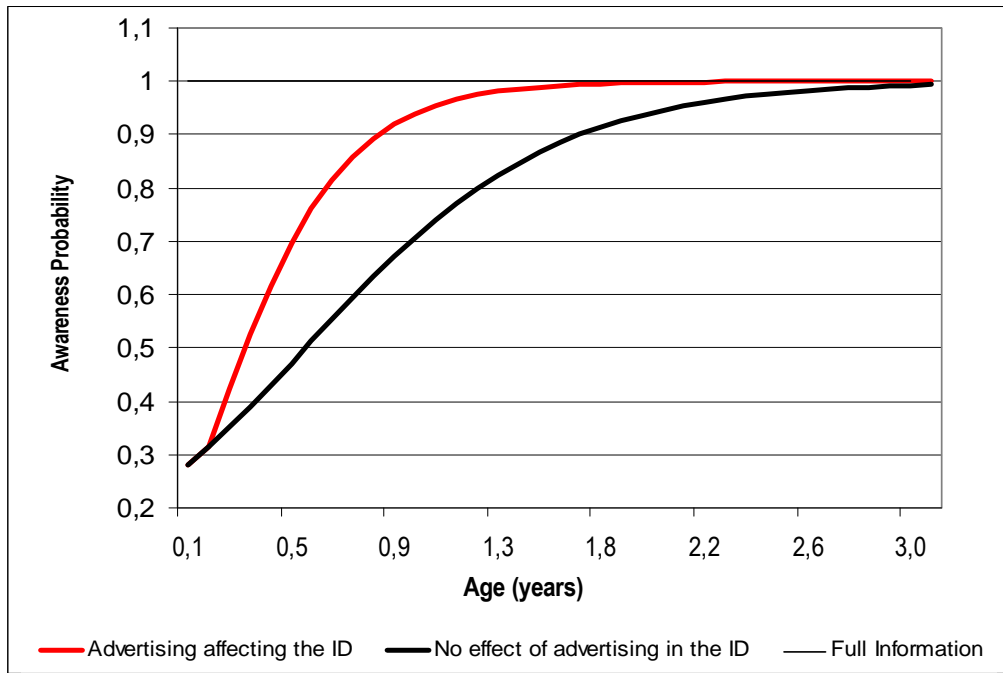


Figure 3: Awareness Probabilities.