Majority-efficiency and Competition-efficiency in a Binary Policy Model

Stefan Krasa Mattias Polborn*

March 16, 2007

Abstract

We introduce a general framework in which politicians choose a (possibly infinite) sequence of binary policies. The two competing candidates are exogenously committed to particular actions on a subset of these issues, while they can choose any policy for the remaining issues to maximize their winning probability. Citizens have general preferences over policies, and the distribution of preferences may be uncertain. We show that a special case of the model, the weighted-issue model, provides a tractable multidimensional model of candidate competition that can generate (i) policy divergence in pure and mixed strategies, (ii) adoption of minority positions, and (iii) inefficient outcomes.

JEL Classification Numbers: D72, D60.

Keywords: Multidimensional policy, voting, citizen-candidate, normative analysis of political competition

*Address of the authors: Department of Economics, University of Illinois, 1206 South 6th Street, Champaign, IL 61820 USA,

E-mails: skrasa@uiuc.edu, polborn@uiuc.edu.

We would like to thank Ernesto Dal Bo, Matthias Messner, Dan Bernhardt, Cagdas Agirdas and seminar participants at UC Berkeley, University of Toronto, Simon Fraser University and Michigan State University for helpful comments. Stefan Krasa gratefully acknowledges financial support from NSF grant SES-0318394. The most current version of this paper is available at https://netfiles.uiuc.edu/polborn/www/

1 Introduction

The one-dimensional policy model based on the seminal contributions of Hotelling (1929) and Downs (1957) is the most widely used and successful model framework for a formal analysis of political equilibria. Yet, there are some tensions within the model, and between the model and some real-world observations.

First, in the one-dimensional spatial model, there is a strong tendency for candidates to converge to the same, moderate position that appeals to the "median voter" (mitigated only if the candidates care about policy and to the extent that the position of the median is uncertain); furthermore, all voters (including those with extreme preferences) are, in equilibrium, indifferent between the two candidates, as they propose the same policy. Yet, in reality, candidates often run on considerably divergent policy platforms, and voters often intensely favor one candidate over the other.

Second, while the model is formally one-dimensional and continuous, policy in reality is often multidimensional (there are many policy issues) and binary (e.g., candidates are either for withdrawing troops from Iraq or against it).[1] In fact, a widespread casual interpretation of policy in the one-dimensional model is based on multidimensional policies. For example, the statement that "Hillary Clinton used her support for the Iraq war to move towards the political center," implies that her initial position (on other issues, say on health care) is left-of-center, but by adopting a conservative position on a particular issue, she can move to the right on the policy line.

This informal multi-issue interpretation of the one-dimensional model is somewhat problematic regarding the treatment of moderates. For example, suppose we accept the notion that support for state-provided health-care is a liberal position and support for the Iraq war is a conservative position, and suppose that these are the only two positions and that they are equally important. Then both "Hillary" (with positions as described above) and a voter who opposes both state-provided health care and the Iraq war would be considered "moderate" with a position in the center in a one-dimensional model, suggesting that the voter is likely to support her. Yet, the voter could plausibly prefer another candidate with a "more extreme" position, say, someone who supports state-provided health care but opposes the Iraq war.

This paper develops a model that directly treats policy as multidimensional and binary. A multidimensional policy vector describes a candidate's position on a number of different issues.[2] Each candidate is exogenously fixed on some issues. These fixed positions can be interpreted as characteristics of the candidate (such as party affiliation, incumbency, gender, race or experience in previous elected office), or political issues on which a candidate has taken a stand in the past and where a commitment to a different position is not credible and/or not helpful. On the remaining issues, candidates are free to choose any position. By combining fixed and selectable positions, we provide a middle ground between Downsian models, in which candidates are free to choose any

[1]Even more nuanced positions are few in numbers and can usually be expressed by a small number of binary, "yes" or "no" answers. More generally, any information that a candidate can provide to the electorate must be finite and therefore representable in a binary format.

[2]Since positions on the real line (or, more generally, in $\mathbb{R}^n$) can be encoded in binary form, this binary model encompasses as special cases the standard Downsian model and the probabilistic voting model.

position, and the citizen candidate model, in which no commitment is possible.

The most important result for political competition in a Downsian framework is that office-motivated candidates propose policies that appeal to the median voter. In our multi-dimensional model, there is no geometric notion of a median, but our concept of majority-efficiency captures the same fundamental idea of moderation. A policy is *majority-efficient* if a majority of voters prefers the proposed policy to any other policy that a candidate could choose. In some sense, a majority-efficient policy is the most popular policy a candidate can choose, subject to the constraints imposed by his fixed positions. The relevant question is whether candidates choose majority-efficient positions in equilibrium.

We focus on two classes of preferences. Using the first type of "scoring" preferences, our model effectively generalizes the one-dimensional Downsian model to allow for constraints on the positions that candidates can choose. The second type of preferences is not reducible to one dimension: A citizen's utility is a weighted sum of the distances between the candidate's actual position and the voter's ideal position on each issue. This "weighted-issue model" provides a simple and tractable framework to analyze multidimensional policies.

In the (generalized Downsian) scoring model, majority-efficient policies exist, and there is always an equilibrium where both candidates propose such policies. Majority-efficient policies also exist in the weighted-issue model for many distributions of voter preferences, and we characterize sufficient (and generic) conditions for existence. However, the multidimensional nature of the weighted-issue model fundamentally changes the efficiency properties of the model. In equilibrium, a candidate may choose to propose majority-inefficient policies, because adopting minority positions may *increase* his winning probability. Further, candidates' policies may diverge in order to attract different types of constituents, a behavior which some political commentators may refer to as "catering to the base" (see, for example, Edsall (2006)). In the Downsian model, neglecting the median voter in order to choose policies that please extremists reduces a candidate's winning probability, so that platform divergence cannot be rationalized as an electoral success strategy in that model.[3] In contrast, it is easy to generate examples in the weighted-issue model, in which candidates' cater to minority views and where the policies diverge.

The fundamental reason for inefficiency results is the following: When choosing a stand on an issue, candidates consider whether a particular position will win or lose more votes, i.e., they care about *swing voters*. The Downsian model has the special property that a policy position that wins votes from swing voters is also better for a majority of voters than any other position that a candidate might take, but in a more general model, this is usually not true.

A related topic is the issue of divergence. While, in the Downsian model, candidates have a

[3]There are certainly modifications of the Downsian framework in which divergence occurs in equilibrium. For example, policy-motivated candidates (see Calvert (1985)) face a trade-off between proposing a policy that they like and increasing their winning probability through moderation. Similarly, in the citizen-candidate model (see Osborne and Slivinski (1996), Besley and Coate (1997)), candidates enter at non-median positions. However, in each of these models, converging towards the position of the opponent (if possible) would actually increase a candidate's winning probability.

strong incentive to approach their opponent's position (possibly without quite reaching it), the same is not necessarily true in the weighted issue model. There, it may also be the case that both candidates strictly prefer to differentiate from each other in one issue, or that one prefers to differentiate while the other one would like to mimic his opponent.

The result that majority-efficient policies often exist in the weighted-issue model is significant in its own right: Plott (1967) has shown that Condorcet winners exist in a multidimensional Euclidean setting only for a highly non-generic configuration of preferences (in a setting without fixed positions, the concept of a Condorcet winner is equivalent to majority-efficiency). This problem has severely limited the applicability of the Downsian model as a tool to study multidimensional policy. For example, many papers that apply political economy arguments to particular fields essentially analyze policy as if there were only one political issue in which the median voter's opinion is decisive. In contrast, in the binary weighted-issue model, if a majority-efficient position exists for a particular voter distribution, it generically also exists for a slightly perturbed distribution of voters. In many cases, the weighted-issue model provides a tractable framework for the analysis of multidimensional policies.

We also extend our model to analyze the effects of uncertainty on the candidates' policy choice. Generalizing the concept of a valence (i.e., quality) difference between candidates, we characterize conditions that guarantee that candidates choose majority-efficient policies. We also analyze the case of uncertainty about the distribution of voters' *policy* preferences (rather than about valence). Here, we introduce the concept of *competition-efficiency*. The equilibrium is competition-efficient if a social planner could not pick two alternative platforms (one for each candidate) and make a majority of the electorate better off. When voters' policy preferences are uncertain, the equilibrium in the Downsian model is competition-inefficient, because candidates converge too much. Interestingly, in the weighted-issue model, both excessive convergence and excessive divergence in equilibrium are possible.

We present the model in the next section. In Section 3, we introduce the concept of majority-efficiency. Results for the model without uncertainty are in Section 4. Section 5 generalizes the model to allow for valence and preference distribution uncertainty, defines competition-efficiency and contains the results for this framework. We relate our model to previous literature in Section 6. Section 7 concludes. All proofs are in the Appendix.

2 The Model

2.1 Setup

Two candidates, $j = 0, 1$, compete in an election. Candidates are office-motivated and receive utility 1, if elected, and utility 0, otherwise, independent of the implemented policy. Candidate j, if elected, implements a policy described by $a^j = (a_i^j)_{i \in \mathbb{N}}$, where each $a_i^j \in \{0, 1\}$ denotes Candidate j's position on issue i (0 can be interpreted as opposition to a particular proposal, and 1 as support of that proposal).

Candidate j can freely choose a policy on a subset of issues $S^j \subset \mathbb{N}$, while on the remaining issues no commitment is possible. Thus, *Candidate j's type* is given by $a = (a_i^j)_{i \notin S^j}$, while his *platform* is given by $a = (a_i^j)_{i \in S^j}$. Candidate j's policy consists of the combination of his type and platform, so that his *set of feasible policies* is given by $A^j = \{(a_i)_{i \in \mathbb{N}} | a_i = \bar{a}_i^j$ for all $i \notin S^j$ and $a_i \in \{0,1\}$ for $i \in S^j\}$. Each citizen $\theta \in \Theta$ has preferences $u_\theta(a)$, where $a = (a_i)_{i \in \mathbb{N}} \in \{0,1\}^{\mathbb{N}}$ is the implemented policy. Let μ be the distribution of citizens. Note that this is just a frequency distribution which is known to the candidates.[4]

The timing of the game is as follows:

Stage 1 Candidates $j = 1, 2$ simultaneously announce policies $a^j \in A^j$. A mixed strategy by agent j consists of a probability distribution σ^j over A^j.

Stage 2 Each citizen votes for his preferred candidate, or abstains when he is indifferent between both candidates.[5] Candidate j wins if $\mu\left(\{\theta | u_\theta(a^j) > u_\theta(a^{-j})\}\right) > \mu\left(\{\theta | u_\theta(a^j) < u_\theta(a^{-j})\}\right)$. In case of a tie between the candidates, each wins with probability 0.5.

Clearly, mixed strategy equilibria always exist if each S^j is finite.

The assumption that politicians have only a binary choice may, at first glance, seem restrictive, but it is possible to combine several issues in our formal framework to deal with policy questions where candidates can choose between more than just two possible positions. In fact, as mentioned in the introduction any issue that can be described in a finite language is obviously representable in binary form.

2.2 Interpretation and Discussion: Fixed issues

A key feature of our model, made possible by the multidimensional structure, is that candidates can commit to a policy on some issues, while they are fixed to an exogenously given position in other dimensions. Thus, our model combines the commitment assumption of the standard Downsian model (with respect to S^j) with the assumption in citizen candidate models (Osborne and Slivinski (1996), Besley and Coate (1997)) that no commitment to a policy other than the candidate's ideal point is possible. This appears to be a reasonable convex combination of these two central models in the literature. In reality, candidates have commitment power on some issues. If a candidate makes a promise a central campaign theme (say, not to raise taxes, to end a war), then breaking that promise is at least very costly for the candidate, and, counterexamples not withstanding, most candidates keep their central election promises.

However, there are other dimensions in which candidates cannot easily commit to different positions. This is obviously true for characteristics of the candidate that matter for at least some

[4] In section 5 we generalize the model to allow for uncertainty about this distribution, and preference shocks.

[5] If a voter has a strict preference, then it is a weakly dominant strategy to vote for the preferred candidate. If an agent is indifferent, he could in principle vote for any candidate or abstain, but the assumption of abstention is quite natural, and none of the results in this paper depends critically on it.

voters, like gender, race, religious affiliation, experience in previous elected office.[6] These characteristics can be interpreted as "fixed positions", at least in the short run, which is the focus of our analysis.[7] Other fixed positions may correspond to political issues in which a candidate has taken a stand in the past and where a commitment to a different position is not credible and/or not helpful. For example, a candidate who took a strong pro-choice stand in his past legislative voting record may not be able to credibly commit to a pro-life platform, and therefore is essentially fixed to his previous position on the abortion issue.

One important committed issue is party affiliation. When a candidate runs as a Democrat for the U.S. Congress and wins, he is committed to support his fellow Democrats in committee appointments (i.e., even if the candidate chooses to run on a conservative platform, his seat counts for determining whether the Democrats are the majority party in Congress, with the associated privileges for possibly more liberal Democratic party leaders). This may make it difficult for a Democrat to win in very conservative districts, even if he adapted a very conservative platform. For example, in the 2006 elections, many Republican House candidates tried to tie their Democratic opponents to "liberal Nancy Pelosi", the prospective Speaker of the House in case of a Democratic majority. A related case in point is the 2006 Senate race in Rhode Island, which the incumbent, Senator Lincoln Chafee (a relatively liberal Republican) narrowly lost (47% to 53%) in spite of being personally very popular. In exit polls, 63% of voters approved of Chafee's job performance as U.S. Senator and 51% of voters said that Chafee's position on issues was "about right" (and only 25% said that he was too conservative). However, by an overwhelming margin (63%-23%), voters stated that they wanted Democrats rather than Republicans in control of the Senate.[8]

Also, note that most senators from states that usually vote for Democrats in the presidential election are Democrats and vice versa. In a naive Downsian model without constraints on the policy platforms, candidates adopt the position of the median voter in their respective district, and win with equal probability. Hence, while the Downsian model predicts that both Democratic and Republican candidates in conservative districts adopt more conservative positions than in liberal districts, it cannot explain why Republicans win significantly more of the conservative districts than Democrats and vice versa. In contrast, the fixed positions in our framework can generate this result in a natural way.

[6] Even belonging to a "political dynasty" (i.e., being a relative of other politicians) may be interpreted as a fixed characteristic; see Dal Bo, Dal Bo, and Snyder (2006) for a study that explores the importance of this characteristic for electoral success.

[7] Note, however, that if we instead focus on the nominating behavior of parties, then fewer positions should be considered fixed than for any particular candidate. For example, while the party can choose whether to nominate a man or a woman, each candidate has his gender not as a choice variable.

[8] See http://www.cnn.com/ELECTION/2006//pages/results/states/RI/S/01/epolls.0.html for these exit poll results.

3 Majority Efficiency

3.1 Definition

The central result for political competition in a Downsian framework is that candidates propose policies that appeal to the median voter. This median voter result corresponds to a notion that political competition forces candidates to propose "popular" rather than "unpopular" policies in order win elections. "Moving toward the median" (from a non-median initial position) is popular in the Downsian model because it is preferred by majority of the electorate.

In contrast, there is no geometric notion of a "median voter" in our model, because voters have general preferences over multi-dimensional policy vectors. However, our concept of majority-efficiency captures the fundamental idea that a particular policy a is more moderate than some some other policy a' if a is preferred to a' by a majority of voters. We define a policy to be *majority-efficient* if a majority of voters prefers the proposed policy to any other policy that a candidate could choose. In other words, a majority-efficient policy is the most popular policy a candidate can choose, subject to the constraints imposed by his fixed positions. The central question is whether candidates will be moderate, i.e., choose majority-efficient positions in equilibrium.

We first define majority preferences over policies. The definition accounts for the fact that some voters may be indifferent between a and a'. We cannot solely require that at least 50% of voters find a at least as good as a', because, if some citizens are indifferent, it can be the case that also more than 50% of citizens find a' at least as good as a. Thus, our definition compares the number of citizens who find a at least as good as a' with the number of citizens who find a' at least as good as a.

Definition 1

1. *Let $a, a' \in A$. Then a is* ***majority preferred*** *to a', denoted by $a \succeq a'$, if and only if $\mu(\{\theta | u_\theta(a) \geq u_\theta(a')\}) \geq \mu(\{\theta | u_\theta(a') \geq u_\theta(a)\})$.*

2. *a is* ***strictly majority preferred*** *to a', denoted by $a \succ a'$, if $a \succeq a'$ but not $a' \succeq a$.*

We are now ready to introduce the definition of majority-efficiency.

Definition 2 *Candidate j's policy $a^* \in A^j$ is* ***majority-efficient*** *if and only if $a^* \succeq a$ for all $a \in A^j$.*

Note that a policy is majority-efficient if and only if it is a Condorcet winner *relative to the set of policies that a candidate can choose.* In general, the smaller the set of feasible policies is, the more likely it is that a majority-efficient policy in such a set exists. In fact, it is straightforward to find examples in which majority-efficient policies exist, but there is no Condorcet winner in the traditional sense (where all positions can be chosen). Thus, recognizing that candidates are in practice fixed on many positions, and restricting the comparison set to a candidate's feasible policies, makes majority-efficiency a much more applicable concept than that of a Condorcet winner.

3.2 Majority-efficiency as a normative concept

Majority-efficiency can also be interpreted as a normative concept. Pareto optimality is the standard normative concept in economics, but it has little bite in political settings, because, very often, most policies are Pareto optimal. For example, in the Downsian model, all policies located between the two most extreme voters' bliss points are Pareto optima. Similarly, in our more general model, a policy is guaranteed to be a Pareto optimum if it is the most preferred one for one voter. However, we usually do not consider policies that are supported only by a small minority as efficient.

While Pareto efficiency is the appropriate efficiency concept for measuring the performance of economic exchange institutions, majority-efficiency appears to be the more appropriate concept for measuring the efficiency of democracy. The institution of a market is characterized by voluntary exchange; any transaction must lead to a Pareto improvement for the set of individuals who participate in it, otherwise at least one individual can veto the exchange. Pareto efficiency then asks whether the *entirety of all trades* that occur within the market institution lead to an allocation that exhausts all potential efficiency gains, in the sense that it does not allow further improvements for the set of all individuals. In contrast, democracy is characterized by a "dictatorship of the majority", in the sense that a majority of people can impose a policy on all citizens. In two candidate elections, the majority-preferred candidate wins, and thus the election leads to an outcome that is majority-preferred relative to the other candidate and his platform. The relevant question for the institutional efficiency of democracy is then whether the *entirety of the political process* (including the platform choice by candidates) leads to a majority-efficient solution.[9]

The major alternative concept to Pareto efficiency used in the literature is utilitarianism. A policy is considered efficient if it maximizes the sum of the utilities of all voters. Typically, there is only one such policy. The concept thus generates a sharp benchmark against which we can measure the outcome of the political system. However, in order for the concept to make sense, the cardinal value of individual utilities must be meaningful, and we must be willing to trade-off the utility of different individuals in a particular way.

[9]Note that we can see majority-efficiency and Pareto efficiency as the two extreme endpoints of a general concept that we can call α-efficiency, defined as follows: A policy a is α-efficient if the percentage of people who prefer a' to a (among all people with a strict preference) is not more than $1-\alpha$, for all other policies a'. Clearly, as long as the α defined this way is positive for a policy (or $\geq 1/N$, in a finite electorate with N voters), then the policy is Pareto efficient. Majority efficiency corresponds to $\alpha = 1/2$. The higher α, the more stringent is the requirement that no other policy is preferred by a fraction $1-\alpha$ of voters, so it is clear that the set of majority-efficient policies is always a subset of the set of Pareto optimal policies. More generally, the higher α, the fewer policies are α-efficient, and the more likely it is that no α-efficient policy exists. In contrast, for α sufficiently low, an α-efficient policy exists in almost all applications. A sufficient condition for the existence of a Pareto optimum is that there is at least one voter who has a bliss policy (a policy that he strictly prefers to all other policies) – that policy is a Pareto optimum. Even a Pareto optimum is not guaranteed to exist in all settings. For example, suppose that all individuals prefer policy $(0, 0, \ldots)$ to policy $(1, 1, \ldots)$, and for all other policies, all voters prefer a policy a to a policy a', if policy a starts with a longer string of '1's before having the first 0. In this economy, no α-efficient policy exists for any α. For example, note that the result of Caplin and Nalebuff (1988) can be interpreted as follows: In a multidimensional Euclidean voting model, under certain assumptions on the distribution of preferences (and with no fixed policies), $\frac{1}{e}$-efficient policies exist (where $e = 2.71\ldots$ is Euler's number).

Proponents of utilitarianism have argued that the right policy was that which would cause "the greatest happiness of the greatest number" of people, a quote usually ascribed to Jeremy Bentham. The obvious problem in formalising this theory is that choosing a policy that maximizes utility for all people is usually not possible. Utilitarianism has been interpreted as equivalent to maximizing the sum of voters' utilities, but that is not the only possible interpretation. The concept of majority-efficiency is quite close to the original quote in that it requires that a policy achieve a "greater happiness for the greater number of people" than any other policy.

4 Results

In Section 4.1, we provide some results concerning the relation between majority-efficiency and equilibrium that hold for general preferences of voters. In general, majority-efficient positions (or a pure strategy equilibrium) may not exist. Therefore, we analyze two classes of preferences. In Section 4.2 we consider preferences that effectively reduce the policy space to one dimension. In contrast, in Section 4.3, we consider the truly multidimensional weighted-issue model.

4.1 Results for General Preferences

Our first result obtains in a setting in which both candidates have exactly the same set of feasible policies. This means that fixed positions do not matter, either because they are irrelevant for all voters, or because they are the same for both candidates. Obviously, the assumption that both candidates share the same characteristics and fixed policy positions is not very realistic, but this is an important benchmark: To our knowledge, all previous models in which candidates can choose a position, abstract from fixed positions.

Theorem 1 *Suppose that $A^0 = A^1$:*

1. *Then (a^0, a^1) is a pure strategy equilibrium if and only if both a^0 and a^1 are majority-efficient.*
2. *Suppose a majority-efficient policy exists. If (σ^0, σ^1) is a mixed strategy equilibrium, then almost every policy in the support of σ^0 and σ^1 is majority-efficient.*

Theorem 1 should be interpreted as highlighting the role of fixed positions for all majority-inefficiency results in this paper, and indeed, for any such result in other frameworks. One interesting issue is, for example, whether there are any models, with general preferences for voters, in which office-motivated candidates have a strict incentive to differentiate from their opponent for electoral gain.[10] Theorem 1, point 1, can be interpreted as showing the crucial role of fixed positions for any such model. If there are no differences between the candidates' fixed positions, then a pure strategy equilibrium with differentiation exists only if there are two (or more) majority-efficient policies.[11] Moreover, even if this is the case (say, there are two majority-efficient policies, a and b),

[10]We are grateful to Ernesto Dal Bo for raising this question.

[11]This result generalizes Ledyard (1984), who shows in a Downsian model where voters have voting costs that the two candidates converge to the bliss point of the median voter.

there are also equilibria in which both candidates choose the same policy (that is, (a,a) and (b,b) are equilibria), and in all equilibria, candidates are indifferent between playing a and b, so there is never a strict incentive for candidates to differentiate. The same is obviously true for symmetric mixed strategy equilibria.

Our second general result obtains in a setting where the two candidates may have different fixed positions. If a candidate has the ability to choose a policy that is majority-preferred to any other policy that either candidate can choose, then choosing this policy guarantees a victory. If both candidates have access to this policy, then the unique equilibrium is that both candidates choose it. If only one candidate has access, then this candidate can force a victory by adopting it; however, if he is sufficiently better than his opponent in the fixed positions, then he might also win for sure with other policies, so there may be majority-inefficient equilibria.

Theorem 2 *Suppose there exists $a^* \in A^j$ such that $a^* \succ a$ for all $a \in A^0 \cup A^1$. Then there exists a pure strategy equilibrium in which Candidate j plays a^*. Furthermore, if $a^* \in A^0 \cap A^1$, then (a^*, a^*) is the unique equilibrium. Otherwise, if $a^* \notin A^{-j}$, then Candidate j wins with probability 1 in any equilibrium.*

The setting of Theorem 2 is one in which one candidate is superior, in the sense that he cannot be beaten (at least strictly) by the other candidate. However, in a framework with general preferences, it can easily be the case that each candidate has a majority-efficient policy (relative to *his own* set of feasible policies), but no candidate has access to a policy that is majority-preferred *to all* policies that his opponent could choose. This is, in particular, true if there are some fixed attributes in which a majority of voters prefers Candidate 0, and other attributes in which the majority prefers Candidate 1. (We will provide very natural examples for such a scenario in Section 4.4 below).

4.2 Scoring Preferences

While Theorems 1 and 2 hold for general preferences, we need to impose more structure on citizens' preferences to prove existence of majority-efficient policies. In this section, we analyze preferences similar to the standard one-dimensional Downsian model, and in Section 4.3, we analyze a truly multidimensional model that we call the *weighted-issue model*.

In the standard one-dimensional Hotelling-Downsian model, policies and voter bliss points can be expressed as a number in the interval $[0,1]$. In our *scoring model*, a candidate's policy a is first translated into an overall, one-dimensional score $x = f(a)$ that can be interpreted as a measure of how liberal or how conservative the candidate is. All citizens agree on the scoring function (i.e., for any two policy vectors proposed by the candidates, they agree on which one is more conservative), but they have different ideal points, depending on their type θ.

For example, if $f(a) = \frac{1-\lambda}{\lambda} \sum_{i=1}^{\infty} a_i(\lambda)^i$, then the score is a weighted sum of the a politician's position on each issue. (The factor $(1-\lambda)/\lambda$ normalizes the sum such that the score is in $[0,1]$ for all policies a). This method resembles the calculation of legislative scores by lobby groups such as the ACLU and NRA for members of the U.S. congress. Note that two candidates can have the same policy score, even though their policy vectors differ. For example, for $\lambda = 0.5$, policy $(0, 1, 1, 1, \ldots)$ and policy $(1, 0, 0, 0, \ldots)$ both result in a score of 0.5.[12] For citizens with scoring preferences, only the weighted percentage of liberal and conservative positions matters, but not the position on specific issues.

Formally, the utility of type $\theta \in \mathbb{R}$ is given by

$$u_\theta(a) = v(\theta, f(a)); \tag{1}$$

where each $v\colon \mathbb{R}^2 \to \mathbb{R}$. Assume furthermore that $\frac{\partial^2 v(\theta,x)}{\partial\theta\partial x} > 0$, which implies that higher θ-types (i.e., more conservative citizens) prefer a candidate with a higher score (i.e, a more conservative policy score).

Theorem 3 shows that both candidates adopt majority-efficient policies in the scoring model, even if candidates' choice of policies is restricted due to fixed issues.

Theorem 3 *Suppose that citizens' utilities are of the form $u_\theta(a) = v(\theta, f(a))$, where f is real valued and continuous. Suppose that $\frac{\partial^2 v(\theta,x)}{\partial\theta\partial x} > 0$ and that there exists a unique median voter θ_m, i.e., $\mu(\{\theta \leq \theta_m\}) \geq 0.5$ and $\mu(\{\theta \geq \theta_m\}) \geq 0.5$. Then*

1. *Policy a^{j*} is majority-efficient if and only if a^{j*} solves $\max_{a^j \in A^j} u_{\theta_m}(a^j)$.*
2. *(a^{0*}, a^{1*}) is an equilibrium.*
3. *If there exists an equilibrium in which one candidate i chooses a non-majority-efficient position, then one of the candidates wins with probability 1. The election outcome of such an equilibrium is the same as in (a^{0*}, a^{1*}).*

In the scoring model, the median voter's most preferred positions are majority-efficient. If, as in the standard Downsian model, both candidates can implement the median voter's most preferred policy, then the median voter is indifferent between the two candidates, and they win with equal probability. With fixed positions, a majority-efficient equilibrium continues to exist. If, however, one of the candidates is sufficiently inferior then additional equilibria arise that are inefficient. In particular, if the inferior candidate cannot win, then his actions are irrelevant and he can choose any policy. The superior candidate can select any policy that is not "too far" from the majority efficient policy, i.e., any policy that the median voter prefers to the inferior candidate's best policy. Thus, the quality of the inferior candidate puts a limit on the extent of a possible inefficiency in the worst equilibrium.

While Theorem 3 provides a reassuring efficiency result, the interpretation of the scoring model in a multi-issue context is problematic. First, in order to appeal to the median voter, a left-wing candidate (i.e., a candidate who is exogenously committed to several zeros) would mostly choose conservative positions (i.e., 1) on pledgeable issues, and vice versa for a right-wing candidate. Thus, on flexible issues, the model predicts that liberal candidates takes more conservative positions in

[12] While for $\lambda = 0.5$ these are the only two sequences that lead to an effective policy of 0.5, there is an infinite number of such sequences for $\lambda > 0.5$.

equilibrium than their conservative opponents. While candidates may occasionally "moderate" themselves in such a way, there also appears to be the opposite behavior, which the Downsian model cannot explain.

Second, the majority-efficiency result implies that every position taken by a candidate on pledgeable issues is popular with the majority of the electorate.[13] In reality, it appears that candidates sometimes take minority positions on some issues and do well. Since majority-efficiency is a very robust result in the Downsian model, it may appear that a model with more complicated features (say, regarding the motivations of politicians and the information of the electorate) is needed to explain such a behavior.[14] However, we will see in the next sections that the majority-efficiency result is a specific feature of the Downsian model, and that the weighted-issue model provides a simple explanation of why office motivated candidates may choose majority-inefficient positions *in order* to maximize their electoral success probability.

Third, if candidates successfully achieve convergence in the Downsian model, all voters are approximately indifferent between them. This is true even if candidates propose different policies on the issues, because, for voters with scoring preferences, only the proportion of liberal and conservative positions count. In contrast, if voters care about positions on different issues in a meaningful way (as in the weighted-issue model of the next section), then very few voters, if any, are indifferent between candidates in equilibrium. Note, for example, that 71% of respondents in a 2004 Gallup poll indicated that they had a strong preference for their candidate in the 2004 U.S. presidential election.[15]

4.3 Majority Efficiency in the Weighted-Issue Model

In the *weighted-issue model*, each citizen has a preferred position (0 or 1) on each issue, and utility is linearly-additive across issues. A citizen compares the two candidates by looking at how often they agree with him on issues. We allow for issues to have different weights for a voter, and for the weights to vary among individuals. The weighted-issue model is a natural and quite general framework to capture preferences of individuals in a multidimensional policy setting. It is similar to a multidimensional Euclidean model, but the binary nature of policy makes it much more tractable.

Formally, in the weighted-issue model, a citizens's type is given by $\theta \in \prod_{i=0}^{\infty} \Theta_i$, where Θ_0 is an arbitrary set and $\Theta_i = \{0, 1\}$ for $i \geq 1$. The utility function of citizen θ is

$$u_\theta(a) = -\sum_{i=1}^{\infty} \lambda_{i,\theta_0} |\theta_i - a_i|, \tag{2}$$

[13] This is true even if the two candidates take opposite positions on some issue. With scoring preferences, the majority of the electorate would like an otherwise left-wing candidate to take the conservative position, and an otherwise right-wing candidate to take the liberal position on the same issue.

[14] For example, Kartik and McAfee (2006) motivate their model with an example of John McCain, who took the (unpopular) position of opposing ethanol subsidies before the 2000 Iowa caucuses. They then build a signaling model in which some politicians have "character", which compels them to take unpopular positions, but where the electorate ceteris paribus prefers candidates with "character".

[15] See, e.g., Bernhardt, Krasa, and Polborn (2006) for further documentation of polarized preferences in U.S. presidential elections.

where all $\lambda_{i,\theta_0} \geq 0$ and $\sum_{i=1}^{\infty} \lambda_{i,\theta_0} < \infty$. Parameter θ_0 determines the weights $(\lambda_{i,\theta_0})_{i\in\mathbb{N}}$ that a citizen puts on the different issues. For $i \geq 1$, $\theta_i \in \{0, 1\}$ denotes a citizen's preferred position on policy i. Clearly, this framework also accommodates settings with finitely many issues by fixing $\lambda_{i,\theta_0} = 0$ for all but a finite number of issues.

Theorem 4 and its corollaries provide conditions under which majority efficient policies exist in the weighted-issue model. In all results of this section, we assume that the distribution of issue weights in the utility function (2) is independent from the distribution over ideal points. More formally, we assume that the following condition is satisfied:

Independence Condition. *Let μ be the distribution of citizen characteristics. Then the marginal distribution over weights, μ_{Θ_0}, is independent of the marginal distribution over ideal points, $\mu_{\prod_{i=1}^{\infty} \Theta_i}$.*

Theorem 4 *Suppose that all voters have preferences of the form given by (2), and that μ and μ' are distributions of voter types that satisfy the independence condition. Let $a^j \in A^j$ be a majority-efficient policy for μ. Then a^j is also majority-efficient for μ' if there exists $k \in \mathbb{N}$ such that $\mu'\left(\{a_k^j\} \times C_{-k}\right) \geq \mu\left(\{a_k^j\} \times C_{-k}\right)$ for all measurable sets $C_{-k} \subset \Theta_{-k}$.*

Intuitively, Theorem 4 shows that, if we start from a setting in which a majority efficient policy a^j exists and then increase the number of citizens who prefer the majority efficient policy on some issue k (while leaving the preference distribution on the other issues unaffected), then a^j remains majority efficient. Theorem 4 suggests that the existence of majority-efficient policies is relatively robust in the weighted issue model, in the sense that the distribution of citizens can be changed in a generic way without affecting the existence of a majority efficient policy. This robustness result contrasts sharply with the multidimensional Euclidean model of Plott (1967), in which each voter is indifferent between all policies that have the same distance from his bliss point. In that model, a majority efficient policy corresponds to a Condorcet winner (as there are no fixed positions). Plott shows that a Condorcet winner exists if and only if the distribution of voter ideal points is radially symmetric around one voter's ideal point (i.e., that voter is the "median in all directions"). This existence condition is highly non-generic: Starting from a radially symmetric distribution and changing the ideal point of only one voter usually destroys radial symmetry. This is true even if, in the spirit of our Theorem 4, we move that voter's ideal point closer to the previous median, as long as we don't move him exactly on the straight line that connects the median with the voter's previous ideal point. In contrast, in the weighted-issue model, we can change an initial distribution that admits a majority-efficient policy in a large number of ways and preserve existence.[16]

We now elaborate on this point through a series of corollaries and examples to provide a number of existence results. First, repeated application of Theorem 4 yields the following corollary.

[16] Also, the assumption in Theorem 4 that the weight of issues in the citizens' utility functions is distributed independently of their preferred position simplifies the proof, but is by no means a necessary condition for the existence of a majority-efficient position. That is, starting from a situation in which a majority-efficient position exists, we can introduce some correlation between issue weights and preferred policy positions without (generically) affecting existence.

Corollary 1 *Suppose that preferences are given by (2). Let μ^k, $k \in \mathbb{N}$ be distributions on Θ satisfying the independence condition, with $\lim_{k\to\infty} \mu^k_{\Theta_{-0}}(C) = \mu_{\Theta_{-0}}(C)$ for all closed subsets C of Θ_{-0}.*[17] *Suppose that $\mu^{k+1}\left(\{a^j_{i_k} \times C_{-i_k}\}\right) \geq \mu^k\left(\{a^j_{i_k} \times C_{-i_k}\}\right)$, for all measurable subsets $C_{-i_k} \subset \Theta_{-i_k}$. Suppose that a is majority-efficient for μ^1. Then a is also majority-efficient of μ.*

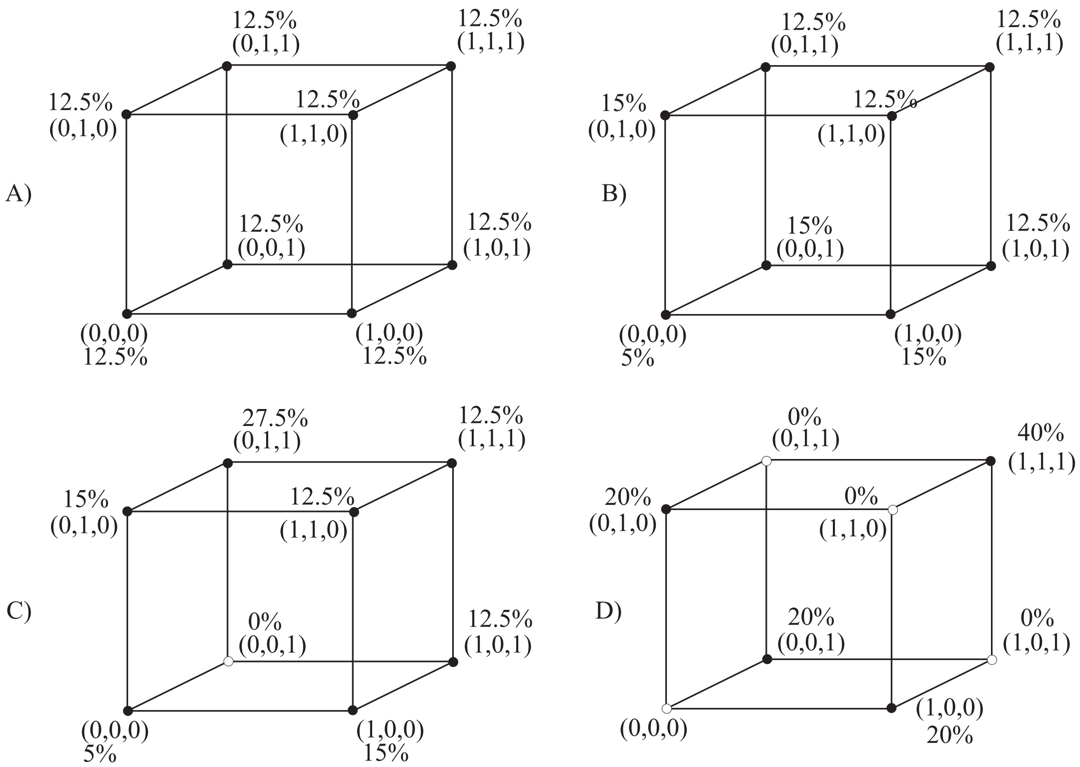

Figure 1: Existence and Non-Existence of Majority-efficient Policies

Example 1 Suppose there are three issues that enter utility with the same weight, i.e., $u_\theta(a) = -\sum_{i=1}^{3} |a_i - \theta_i|$, and the candidate can choose positions on all of them. The distribution of voter types is given in Figure 1. Note that a citizen's disutility is given by the distance, measured along the edges of the cube, between the voter's location and the proposed policy.

In panel (A), where voter preferences are uniformly distributed, it is obvious that any policy, for example $(1,1,1)$, is majority-efficient. Starting from Panel (A), we shift a mass of 2.5% from $(0,0,0)$ to each of the three neighboring vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$, respectively. By Corollary 1, $(1,1,1)$ remains majority-efficient. Panel (C) is generated from panel (B) by shifting the mass of voters at $(0,0,1)$ to $(0,1,1)$. Thus, the weight of $\{0,1\}\times\{1\}\times\{0,1\}$ increases, while the percentage of citizens with characteristics in $\{1\} \times \{0,1\}^2$ and $\{0,1\}^2 \times \{1\}$ is unchanged. Hence, $(1,1,1)$ remains majority-efficient in panel (C).

[17]Consider the product topology on Θ_{-0}. The basis for the topology is given by all cylinder sets $\{\theta_1\} \times \cdots \times \{\theta_i\} \times \{0,1\}^{\mathbb{N}}$.

Generally, any distribution that "first-order stochastically dominates" the uniform one (in the sense that it can be generated by starting from the uniform distribution and shifting voter mass "upwards") has $(1,1,1)$ as majority-efficient policy.

In contrast, while for each issue a majority of citizens prefers position 1 in panel (D), this preference distribution cannot be generated through a series of "upward" mass shifts, starting from a uniform distribution. (We cannot get 20% weight on each of the nodes adjoining $(0,0,0)$ just from shifting upwards the 1/8 mass on $(0,0,0)$ in panel (A)). No majority-efficient policy exists in panel (D): Start with $(1,1,1)$. 60% of citizens prefer $(0,0,0)$. However, $(0,1,0)$ is majority preferred to $(0,0,0)$ (because it is closer to the ideal points of citizens at $(0,1,0)$ and $(1,1,1)$), but $(0,1,1)$ is majority preferred to $(0,1,0)$. Finally, $(0,1,1)$ is dominated by $(1,1,1)$. By symmetry, all other policies are also not majority-efficient. ■

Corollary 2 below shows that, if there are only two pledgeable issues and if all citizens have the same issue weights in their respective utility functions, then majority-efficient policies always exist. Thus, Corollary 2 shows that panel (D) in the above example presents the simplest situation in which non-existence of majority-efficient policies can occur.

Corollary 2 *Suppose that preferences are given by (2) and that μ satisfied the independence assumption. Suppose that $|S^j| \leq 2$ (i.e., all but two policies are fixed). Then a majority-efficient policy exists.*

Again, the contrast to Plott (1967) is sharp, because in that model, majority-efficient positions almost never exist.[18] Corollary 2 also has a practical significance. A number of interesting examples can already be generated with one or two flexible issues, and in all these applications, existence is guaranteed.

The next corollary shows that, if the distribution of preferred positions is independent across issues, then majority-efficient policies always exist. As stated in Example 1, all policies are majority-efficient if the preference distribution is uniform. Increasing the likelihood that a citizen prefers one outcome, say 1, on issue i to p_i means that we raise $\mu(\{1\} \times \Theta_{-i})$ from 0.5 to $p_i > 0.5$. Thus, a policy a which sets $a_i = 1$ remains majority-efficient. Using corollary 1, this argument can be repeated for all issues i, thereby proving the following result.

Corollary 3 *Suppose that preferences are given by (2) and that all marginal distributions μ_{Θ_i}, $i \geq 0$ are independent. Then policy a^j is majority efficient if and only if $a^j_i = 1$ when $\mu_{\Theta_i}(\theta_i) > 0.5$ and $a^j_i = 0$ when $\mu_{\Theta_i}(\theta_i) < 0.5$ for all $i \in S^j$.*

[18] A result similar to Corollary 2, but based on very different assumptions, is derived by Bade (2006). She shows, in a two-dimensional Euclidean model, that an equilibrium of the game between candidates exists (located at the median in each dimension), if candidates are uncertain about the shape of voters' indifference curves and are uncertainty-averse (rather than expected-utility maximizers).

4.4 Adoption of Minority Positions in the Weighted-Issue Model

We have shown above that majority-efficient positions for candidates often exist in the weighted-issue model. In this section, we analyze whether candidates choose these majority-efficient positions in equilibrium and find that this is often not the case. Fundamentally, the reason for inefficiency results is the following: When choosing a stand on an issue, candidates consider whether a particular position will win or lose more votes. In other words, they care about *swing voters.* The scoring model has the special property that a policy position that wins votes from swing voters is also always good for a majority of voters, but this is not true in general.

As a practical example for the optimality of adopting minority positions (i.e., majority-inefficient policies), consider the policy towards (illegal) immigration in the 2006 U.S. congressional elections. A majority of the electorate appears to favor tougher measures against illegal immigrants. For example, Propositions 102 and 300, which banned illegal immigrants from receiving punitive damages in civil lawsuits and from certain public services, passed in Arizona with a large majority of more than 70%, respectively. Yet, an anti-immigrant platform appears to have backfired for Republican candidates, by losing more Hispanic votes than gaining white votes.[19]

Another model in the literature that provides an explanation for this phenomenon is the probabilistic voting model pioneered by Lindbeck and Weibull (1987) (henceforth PVM; see Persson and Tabellini (2000) for a review of the various developments of this literature). In the PVM, voters are divided in different groups according to their utility from policies (that are choice variables for candidates), a common utility shock (like valence) to all voters in a group, and an idiosyncratic "ideology" shock to individual voters. In equilibrium, both candidates choose the same policy platform, which maximizes a weighted sum of utility of different groups in society. The weight of a group in the candidates' objective function is higher than its population share, if voters in the group are more "movable", i.e. if they are relatively likely to switch to a candidate who offers them a more favorable policy position. Thus, interpreting the above example in the PVM framework, one would conclude that, while most voters had a mild preference for an anti-immigration policy, those voters who had the opposite preference cared sufficiently more strongly about this issue to make adoption of an anti-immigrant platform an electoral mistake for a candidate.

Example 2 is based on a analogous effect. Note, however, that the weighted-issue framework is a lot simpler than the PVM: In the PVM, there are a number of exogenous random shocks, and members of a group with the same interests about issues must be sufficiently differentiated "ideologically" (i.e., in a dimension that cannot be addressed by the candidates) for an equilibrium to exist. In contrast, Example 2 is completely deterministic.

Example 2 There are two issues, where the first issue (e.g., party affiliation) is fixed for both candidates. There are four types, characterized by a pair (θ_1, θ_2), $\theta_i \in \{0, 1\}$. Types $(0, 0)$ and $(1, 0)$ have utility functions $u_\theta(a) = -0.5|\theta_1 - a_1| - 0.2|\theta_2 - a_2|$. Types $(0, 1)$ and $(1, 1)$ have utility

[19]While it is hard to measure the electoral effects of a specific issue precisely, this argument is made, for example, in Blumenthal (2006) and Kirkpatrick (2006). Also, two prominent members of the anti-immigrant wing of the Republican Party in Arizona (J.D. Hayworth and Randy Graf) lost their races in previously Republican-held districts.

functions $u_\theta(a) = -0.5|\theta_1 - a_1| - |\theta_2 - a_2|$. Types are stochastically independently distributed. Specifically, suppose that $\theta_2 = 1$ for 20% and $\theta_1 = 1$ for $p \in (37.5\%, 62.5\%)$.

Since a majority of citizens prefers $a_2 = 0$, any majority-efficient policy must have $a_2 = 0$. However, $(0, 0)$ and $(1, 0)$ are not equilibrium policies. First, suppose that $p \leq 0.5$, i.e., in equilibrium Candidate 1 wins at most with probability 0.5. If Candidate 1 deviates and offers $(1, 1)$, then types $(0, 1)$ will now vote for Candidate 1. Thus, Candidate 1 receives a vote proportion of $p + (1 - p)0.2 > 0.5$, so that he wins with probability 1, a contradiction. Similarly, if $p \geq 0.5$ it follows that Candidate 0 wins if he chooses $a_2 = 1$. Note that the example is robust, i.e., small perturbations of the distribution of citizens and of preferences do not change the result. ■

In the PVM, the only reason for the adoption of majority-inefficient positions by candidates is that the minority cares more intensely about an issue than the majority. In contrast, Example 3 shows that, in the weighted-issue model, minority positions may be adopted in equilibrium even if the issue weights are the same for all citizens, and preferred policy positions on different issues are independently distributed. Thus, Example 3 warns against interpreting the adoption of minority positions as evidence that the minority cares more intensely about a particular issue. Also, in Example 3, one candidate wants to choose a different policy from his opponent, while the other one wants to mimic his opponent. In contrast, candidates' platforms always converge in the PVM.

Example 3 There are three issues. Candidate 0's and 1's position on the first two issues are fixed to $(0, 1)$ and $(1, 0)$, respectively. They can choose a position on issue 3. A voter's type is given by $(\theta_1, \theta_2, \theta_3)$, with $\mu(\theta_1 = 1) = 0.4$, $\mu(\theta_2 = 1) = 0.2$ and $\mu(\theta_3 = 1) = 0.4$. The realizations of the θ_i are independent. All citizens' utility functions are $-|a_1 - \theta_1| - 0.8|a_2 - \theta_2| - 0.5|a_3 - \theta_3|$.

Citizen's type	Percent of citizens	Net Benefit from candidate 0, when $a_3^0 = a_3^1$.	Net Benefit from candidate 0, when $a_3^0 = 0, a_3^1 = 1$.	Net Benefit from candidate 0, when $a_3^0 = 1, a_3^1 = 0$.
$(0, 0, 0)$	28.8	0.2	0.7	−0.3
$(0, 0, 1)$	19.2	0.2	−0.3	0.7
$(0, 1, 0)$	7.2	1.8	2.3	1.3
$(0, 1, 1)$	4.8	1.8	1.3	2.3
$(1, 0, 0)$	19.2	−1.8	−1.3	−2.3
$(1, 0, 1)$	12.8	−1.8	−2.3	−1.3
$(1, 1, 0)$	4.8	−0.2	0.3	−0.7
$(1, 1, 1)$	3.2	−0.2	−0.7	0.3
Candidate 0's vote share		60	45.6	34.4

Table 1: Citizen Preferences and Vote Shares

Note that Candidate 0 is the "stronger" candidate in the following sense: If both candidates choose the same policy on issue 3, then Candidate 0 has the support of all types with $\theta_1 = 0$, which constitute 60% of the population. However, Candidate 1 can win by offering a different policy on issue 3. To see this, consider Table 1, which shows the net-payoffs of citizens from choosing Candidate 0 over Candidate 1. Clearly, a citizen votes for Candidate 0 if and only if this net-payoff is positive.

Candidate 0's vote share in the bottom line of the table indicates that Candidate 0 wins whenever both candidates choose the same policy, while Candidate 1 wins whenever they choose different policies on issue 3. Therefore, there is no equilibrium in which candidates always choose the majority-efficient policy $a_3^j = 0$. Structurally, candidates play a "matching pennies game", so that each of them randomizes, in equilibrium, with equal probability over the two policies. Hence, the majority-efficient policy is only implemented with probability 1/2. ■

To gain intuition for Example 3, note that citizens with preferences $(0, 1)$ on the first two issues always vote for Candidate 0, and those with preference $(1, 0)$ always vote for Candidate 1. The potential swing voters are citizens with preference $(0, 0)$ or $(1, 1)$ on the first two issues, but there are many more with preference $(0, 0)$ than with $(1, 1)$ (48% vs. 8%). If both candidates choose 0 on the third issue, then all $(0, 0)$ types vote for Candidate 0, who therefore wins. If, instead, Candidate 1 differentiates from his competitor and picks the less popular position (1) on the third policy, he wins 40% of the $(0, 0)$ types (19.2% of the electorate), while losing 60% of the $(1, 1)$ types to his competitor (a loss of only 4.8% of the electorate). The net gain of 14.4% of the electorate is enough for Candidate 1 to swing the election in his favor.

Note that swing voters in Example 3 have the same preference distribution over issue 3 as the general population, and also put the same weight on this issue as the rest of the population. Thus, in Example 3, the success of adopting a minority position has nothing to do with differential preference intensity. Rather, if the distribution of swing voters is asymmetric (as it arises here quite naturally from the other positions of the candidates, even without assuming any correlation), then the "weaker" candidate, who has fewer swing voters to defend, benefits if he opens a controversy on an additional issue in which the candidates differ.

Note that such a differentiation result cannot be obtained in the Downsian model, because of its one-dimensional structure: There is only one group of swing voters (i.e., those at or close to the median of the distribution), and thus, candidates have to deliver a policy that is popular with this one group. Furthermore, in the Downsian model, a policy that the median voter likes is also preferred by a majority of the population. This leads to a presumption that candidates should pick popular positions in order to maximize their probability of winning,[20] but this argument logically only applies in the Downsian framework.

Finally, it is instructive to compare Example 3 with Aragones and Palfrey (2002) who analyze a one-dimensional Downsian model with office-motivated candidates, an uncertain position of the median voter and one candidate (say, the Democrat) who has a small valence advantage (e.g., is more competent, which is appreciated by all citizens). In order to have a chance of winning, the Republican candidate has to differentiate himself from the Democrat, while the Democrat

[20]See, for example, Kartik and McAfee (2006) for such an argument.

wins if both candidates adopt a position close to each other. No pure strategy equilibrium exists, and depending on the platforms chosen by the candidates in the mixed strategy equilibrium, the Democrat gets the votes of either all left-wing voters, or all right-wing voters, or all voters (the latter happens when the two candidates choose a position that is sufficiently close to each other). Furthermore, the Republican can win only if he chooses the ex-post "more moderate" position (i.e., is closer to the realized median voter).

In comparison, in Example 3, Candidate 1 also has an incentive to differentiate, while Candidate 0 has an incentive to mimic his opponent. In contrast to Aragones and Palfrey (2002), however, there is no uncertainty about the voters' preference distribution in Example 3. More fundamentally, the nature of the advantage of one of the candidates is different: The Democrat in Aragones and Palfrey (2002) is strictly preferred by *all* voters if the candidates choose the same policy position. In contrast, there are two fixed issues with diverse preferences in our example, so that Candidate 0 is preferred by *a majority* if the candidates choose the same position on the third issue, but different voter types differ in their preferred position on the third issue. Depending on the two platforms, a candidate can attract and lose several different "swing voter" groups and not just shift a "cutoff voter" as in a one-dimensional model. Also, because candidates are differentiated in their fixed position, each candidate has "core supporters", that is, voters who always support him, independent of the platform choice of the candidates on the pledgeable issues; this property of our model appears empirically more plausible than the complete flexibility of voter-candidate assignments in the Downsian model. Lastly, the weaker candidate in our model (considering the preferences only over the fixed issues) can even win by adopting the "more extremist" (i.e., minority-preferred) position on the third issue, which is not possible in Aragones and Palfrey's model.

5 Uncertainty, Majority-Efficieny, and Competition-Efficiency

We now generalize the model to incorporate uncertainty about voters' preferences. In Section 5.1, we analyze the effects of *valence* uncertainty. This uncertainty is resolved after the candidates select policies but before citizens vote.[21] Valence is a quality of the candidate that all voters value and that does not affect any voter's preferences over a candidate's *policies*. We provide a general definition of valence and characterize conditions for the economy that guarantee that candidates choose majority-efficient platforms in a model with valence uncertainty.

In Section 5.2, we analyze the impact of uncertainty about the distribution of voter preferences on the efficiency of the candidates' platform choices. To do this, we first need to generalize our definition of majority-efficiency to take into account that the majority-preference depends on the state of the world. We also define *competition-efficiency*, a concept that asks whether a social planner would be able to make a majority better off (in expectation), if he were able to choose both

[21] If uncertainty is resolved only after the election, we can consider this as a model without uncertainty from the candidates' point of view. Certainly, *individual preferences* over policies must then take into account the likelihood of different states of the world. However, a pair of platforms chosen by the politicians translates deterministically into vote shares.

candidates' platforms. We show that, in a weighted issue model with type uncertainty, candidates can opposing platforms that are not competition efficient. In Section 5.3, we analyze type and valence uncertainty in the scoring model.

5.1 Valence Uncertainty

An important model in the literature that introduces some multidimensionality into the standard Downsian framework is a one-dimensional policy model in which voters care also about a candidate's "valence". Valence is interpreted as a characteristic of the candidate that all voters appreciate (like competence or honesty). While most models in the literature simply assume that valence enters as a linearly additive shock into all voters' utility functions, the following definition is somewhat more general. It says that valence does not affect preferences over a candidate's policies.

Definition 3 *Let $u_\theta(a^i, \kappa^i)$ be the utility of voter type θ from Candidate i's proposed policy a^i, if the state of the world is $\kappa = (\kappa^1, \kappa^2) \in \mathbb{R}^2$. The state κ is a* ***valence shock*** *if and only if $u_\theta(a^i, \kappa^i) \geq u_\theta(\hat{a}^i, \kappa^i) \Longleftrightarrow u_\theta(a^i, \hat{\kappa}^i) \geq u_\theta(\hat{a}^i, \hat{\kappa}^i)$, for all $\kappa^i, \hat{\kappa}^i \in \mathbb{R}$, for all policies $a^i \in A^i$ and for all citizens θ.*

Under what conditions do candidates in a valence model choose majority-efficient policies? Clearly, this will not be the case in general, because majority-inefficient equilibria arise in the weighted issue model, even without valence shocks.[22] It turns out that a sufficient condition for candidates to choose a majority-efficient policy is that a society is "polarized" over policy, i.e., citizens can be ordered from left to right such that each policy change is beneficial either to all citizens θ that are further to the left (or further to the right) than some citizen θ_c.

Formally, let $\Theta^i(a^0, a^1, \kappa) := \{\theta | u_\theta(a^i, \kappa^i) \geq u_\theta(a^{-i}, \kappa^{-i})\}$ be the set of voter types who find Candidate i to be at least as good as his opponent. Let $\Psi^i(a^i, \hat{a}^i, \kappa) := \{\theta | u_\theta(a^i, \kappa) \geq u_\theta(\hat{a}^i, \kappa)\}$ be the set of voters who weakly prefer that candidate 1 implements policy a^i rather than policy $\hat{a}^i$.

Definition 4 *A society of voters is* ***polarized*** *if there exists a linear order "$\leq$" of voter types such that the following two conditions hold:*

1. *$\Theta^i(a^0, a^1, \kappa)$ is an upper or lower interval (i.e., of the form $\{\theta | \theta \leq \theta_c\}$ or $\{\theta | \theta \geq \theta_c\}$),[23] or empty, for all policies $(a^0, a^1) \in A^0 \times A^1$ and all $\kappa \in \mathbb{R}^2$.*

2. *$\Psi^i(a^i, \hat{a}^i, \kappa)$, $i = 0, 1$ is either empty or an upper or a lower interval for all policies $a^i \in A^i$ and all $\kappa \in \mathbb{R}^2$.*

For example, a scoring model with linear valence shocks satisfies Definition 4.

Remark 1 *Suppose that citizens' utility is of the form $u_\theta(a^i, \kappa^i) = v(\theta, f(a)) + \kappa^i$ and that $\frac{\partial^2 v(\theta,x)}{\partial\theta\partial x} > 0$. Then the society of voters is polarized.*

[22] For example, introducing a small linearly-additive valence shock in Example 2 will not change the equilibrium platforms chosen by the candidates.

[23] The cutoff θ_c can depend on the policies a and a' and on the realization of the shock κ.

We now show our main result in this section, namely that candidates choose majority-efficient platforms if the society is polarized and κ is a valence shock.

Theorem 5 *Suppose that the society is polarized and that κ is a valence shock. Let Θ be endowed with a topology such that $u_\theta(a)$ is continuous in a and θ, and suppose there exist a unique median voter. Then*

1. *Majority efficient policies m^0 and m^1 exist.*
2. *(m^0, m^1) is an equilibrium.*

5.2 Type and Preference Uncertainty

We now introduce two sources of uncertainty into our model, which are in fact formally equivalent: First, we can allow for the possibility that voter preferences are uncertain; second while the preference ranking of each type may be known, the number of voters of each type could be random random. A simple example of the latter case would be a Downsian model with uncertainty about the position of the median voter. In this framework, we first need to generalize our definition of majority-efficiency to take into account that the majority-preference depends on the state of the world.

Definition 5 *Let $\omega \in \Omega$ parameterize the model uncertainty and let μ_Ω be the distribution of ω. Let $\succeq_\omega$ denote majority preference in state ω. Candidate j's policy $a^* \in A^j$ is* ***ex-ante majority-efficient*** *if and only if $\mu_\Omega(\{\omega | a^* \succeq_\omega a\}) \geq \mu_\Omega(\{\omega | a \succeq_\omega a^*\})$ for all $a \in A$.*

Ex-ante majority-efficiency requires that there is no other of the candidate's feasible policies that is more likely to be preferred by a majority of agents. Our preferred way of thinking about type uncertainty is as follows: Each voter actually knows his policy preferences and does not receive a "shock" between platform choice and election; however, politicians are uncertain about how many voters have which preferences. In this case, using a utilitarian approach based on some form of "expected utility" would integrate over the different incarnations of voters that the candidates consider possible, but not aggregate utilities of actually existing voters. In contrast, our definition just looks at which policy is more likely to be majority-efficient.[24]

Next, we introduce our normative concept of competition-efficiency. Rather than looking at the behavior of one candidate (like majority-efficiency), competition-efficiency deals with the two platforms proposed by the candidates. It asks whether a social planner could choose the two platforms of the candidates in a way that makes a majority of the electorate better off than the platforms chosen in equilibrium by the candidates. If the distribution of voter preference types is

[24] An alternative way of defining efficiency with uncertainty would be to base the definition on the voters' "expected utility" (say, a policy is ex-ante majority-efficient if there is no other policy that gives a majority a higher expected utility). This definition would be reasonable if there is some fixed set of voters with known preferences, and the state of the world is some random event that influences preferences over policies and is realized after politicians commit to their platform, but before the election. For uncertainty resolved after the election, see Footnote 21.

known, competition-efficiency is very much related to majority-efficiency: If both candidates choose majority-efficient positions in equilibrium, then the pair of platforms is competition-efficient.[25] However, if there is uncertainty about the preference distribution, then the two concepts are not necessarily related. The reason is that the social planner has *two* instruments (the two platforms) for an optimal response to the uncertain preference distribution, and may find it optimal to choose platforms that are not ex-ante majority-efficient so as to diversify the choices that are available for the realized electorate. The concept of competition-efficiency therefore allows us, for example, to analyze whether equilibrium candidate convergence is excessive.

We now define competition-efficiency formally. Suppose the two candidates propose policies a^0 and a^1. Let $P(a^0, a^1, \omega)$ be the probability that Candidate 0 wins. Formally,

$$P(a, a', \omega) = \begin{cases} 1 & \text{if } a \succ_\omega a'; \\ 0 & \text{if } a' \succ_\omega a; \\ 0.5 & \text{otherwise.} \end{cases} \tag{3}$$

Using function P, we can map each pair of policy platforms proposed by the candidates into the policy implemented in equilibrium, and hence into utility allocations to voters.

Let $Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega)$ denote the percentage of citizens who prefer the policy that results if candidates choose platforms (a^0, a^1) rather than $(\hat{a}^0, \hat{a}^1)$. Let $\mu_{\Theta|\omega}$ be the distribution over Θ in state ω. Then,

$$\begin{aligned} Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega) = {} & P(a^0, a^1, \omega)P(\hat{a}^0, \hat{a}^1, \omega)\mu_{\Theta|\omega}\left(\{\theta | u_\theta(a^0, \omega) \geq u_\theta(\hat{a}^0, \omega)\}\right) \\ & + (1 - P(a^0, a^1, \omega))P(\hat{a}^0, \hat{a}^1, \omega)\mu_{\Theta|\omega}\left(\{\theta | u_\theta(a^1, \omega) \geq u_\theta(\hat{a}^0, \omega)\}\right) \\ & + P(a^0, a^1, \omega)(1 - P(\hat{a}^0, \hat{a}^1, \omega))\mu_{\Theta|\omega}\left(\{\theta | u_\theta(a^0, \omega) \geq u_\theta(\hat{a}^1, \omega)\}\right) \\ & + (1 - P(a^0, a^1, \omega))(1 - P(\hat{a}^0, \hat{a}^1, \omega))\mu_{\Theta|\omega}\left(\{\theta | u_\theta(a^1, \omega) \geq u_\theta(\hat{a}^1, \omega)\}\right). \end{aligned}$$

Definition 6

1. *A pair of policies* (a^0, a^1) *is* ***competition-efficient in state*** $\boldsymbol{\omega}$ *if and only if*

$$Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega) \geq Q(\hat{a}^0, \hat{a}^1, a^0, a^1, \omega)$$

   for all $(\hat{a}^0, \hat{a}^1) \in (A^0, A^1)$*, i.e. a majority of citizens prefer the policy that is implemented given platforms* (a^0, a^1) *to the policy implemented given any other pair of platforms,* $(\hat{a}^0, \hat{a}^1)$*.*

2. *A pair of policies* (a^0, a^1) *is* ***ex-ante competition-efficient*** *if and only if*

$$\begin{aligned} & \mu_\Omega(\{\omega | Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega) \geq Q(\hat{a}^0, \hat{a}^1, a^0, a^1, \omega)\}) \\ & \qquad \geq \mu_\Omega(\{\omega | Q(\hat{a}^0, \hat{a}^1, a^0, a^1, \omega) \geq Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega)\}) \end{aligned}$$

[25] To see why this must be true, suppose, to the contrary, that a social planner is able to improve upon a situation in which both candidates choose a majority-efficient platform and, say, Candidate 0 wins. If in the social planner's plan, the same Candidate 0 wins, then no improvement for a majority is feasible (as Candidate 0 has, by assumption, a majority-efficient platform). If Candidate 1 wins in the social planner's plan and this makes a majority better off, then the original situation cannot be an equilibrium, as Candidate 1 could have chosen this platform and have won.

for all $(\hat{a}^0, \hat{a}^1) \in (A^0, A^1)$*, i.e. it is more likely than not that a majority of citizens prefer the implemented policy given platforms* (a^0, a^1) *to the policy given any other pair of platforms,* $(\hat{a}^0, \hat{a}^1)$*.*

Example 4 below demonstrates two points for weighted issue models with type uncertainty. First, it is possible that *both* candidates wish to differentiate from each other and offer divergent policies on the same pledgeable issue. This result differs from Example 3 where only one candidate wants to differentiate while the other candidate would like to match his opponent's action. As a consequence, we have a pure strategy equilibrium with policy divergence in Example 4 whereas the equilibrium in Example 3 involved mixed strategies. Second, the equilibrium need not be competition-efficient. It is interesting to note that, in this example, the social planner would choose the same policy for the candidates, while, in equilibrium, they choose to differentiate. In this sense, the equilibrium exhibits "excessive differentiation" by candidates. In contrast, as we will see in Section 5.3, the equilibrium in a scoring model with type uncertainty generally exhibits "excessive sameness".

Example 4 There are three issues. Candidate 0 is fixed to 0 while Candidate 1 is fixed to 1 on the first two issues. On the third issue, candidates can choose any position. There are two states, ω_1, ω_2, where ω_1 occurs with probability $p \in (0, 1)$. There are three types of citizens:[26]

1. 40% are of type $(0, 0, 0)$ and have utility functions $-0.8|\theta_1 - a_1| - 0.5|\theta_2 - a_2| - 1.2|\theta_3 - a_3|$.
2. 20% are of type $(1, 0, 0)$. In state ω_1, utility is $-0.8|\theta_1 - a_1| - 0.5|\theta_2 - a_2| - 0.2|\theta_3 - a_3|$. In state ω_2, utility is $-0.5|\theta_1 - a_1| - 0.8|\theta_2 - a_2| - 1.2|\theta_3 - a_3|$.
3. 40% are of type $(1, 0, 1)$ and have utility functions $-0.8|\theta_1 - a_1| - 0.5|\theta_2 - a_2| - 1.2|\theta_3 - a_3|$.

We now determine how citizens rank the four feasible policy vectors $(0, 0, 0)$, $(0, 0, 1)$, $(1, 1, 0)$ and $(1, 1, 1)$. First, note that type $(0, 0, 0)$ always votes for Candidate 0 independently of the choice of a_3. The ranking of citizens of type $(1, 0, 0)$ depends on the state: In state ω_1 the ranking is $(1, 1, 0) \succ (1, 1, 1) \succ (0, 0, 0) \succ (0, 0, 1)$. In state ω_2, the ranking is $(0, 0, 0) \succ (1, 1, 0) \succ (0, 0, 1) \succ (1, 1, 1)$. Finally, citizens of type $(1, 0, 1)$ rank policies as follows: $(1, 1, 1) \succ (0, 0, 1) \succ (1, 1, 0) \succ (0, 0, 0)$.

We now show that $(0, 0, 0)$ and $(1, 1, 1)$ is the unique Nash equilibrium. Given these strategies, Candidate 0 wins in state ω_2 and loses in state ω_1. If Candidate 0 deviates to $(0, 0, 1)$ then he always loses. If, instead, Candidate 1 deviates to $(1, 1, 0)$ then the candidate's winning probability does not change. However, $(0, 0, 0)$, $(1, 1, 0)$ is not an equilibrium, because, by choosing $a_3 = 1$, Candidate 0 can get the votes of type $(1, 0, 1)$. These arguments also immediately imply that there is no mixed strategy equilibrium.

Clearly, $(1, 1, 0)$ is Candidate 1's majority efficient policy rather than the equilibrium policy $(1, 1, 1)$. We next show that $(0, 0, 0)$, $(1, 1, 0)$ is competition-efficient in both states. Note that $(0, 0, 0)$, $(1, 1, 0)$ dominate both $(0, 0, 0)$, $(1, 1, 1)$ and $(0, 0, 1)$, $(1, 1, 1)$, because the winner is the

[26]For simplicity of presentation we focus on the only three types that matter. This example can be easily extended to all 8 possible types $(\theta_1, \theta_2, \theta_3)$, $\theta_i \in \{0, 1\}$.

same in both states, but with positive probability the majority preferred policy $a_3 = 0$ is replaced by $a_3 = 1$. For the pair of platforms $(0, 0, 1)$ and $(1, 1, 0)$ Candidate 0 always wins. Again, in state ω_2, the identity of the winning candidate is the same and the majority prefers $a_3 = 0$. In state ω_1, the outcome changes from $(1, 1, 0)$ to $(0, 0, 1)$, which makes both type $(0, 0, 0)$ and $(1, 0, 0)$ worse off. Since $(0, 0, 0), (1, 1, 1)$ is ex-ante competition-efficient, it immediately follows that the equilibrium is not competition-efficient.

Finally, note that the example is robust with respect to small perturbations of preferences and changes to the distribution of types. First, since preferences over policies are strict, any small perturbation of preferences does not change the ranking over policies. Secondly, Example 4 goes through for any distribution in which no type alone has a majority. ■

The intuition for Example 4 is that Candidate 0 has a very solid base of supporters (type $(0, 0, 0)$), who vote for him independent of his policy on issue 3. In contrast, the base of support for Candidate 1 is less solid. For example, type $(1, 0, 1)$ is willing to switch to Candidate 0 if Candidate 0 offers $a_3 = 1$ and Candidate 1 does not. This forces Candidate 1 to appease his base by selecting $a_3 = 1$. The "gamble" that Candidate 1 must take in order to win is that type $(1, 0, 0)$ does not care too much about issue 3, which is true with probability p.

There is some anecdotal evidence that "catering to the base" is sometimes a strategy that *increases* a candidate's winning probability. For example, in a memo written after the 2000 U.S. presidential election, Bush campaign strategist Matthew Dowd recommends to abandon the focus on swing voters, because there are only very few truly independent voters that could be won over by adopting more moderate positions.[27] While intuitive, such an argument is incompatible with the standard Downsian model in which the median voter is decisive for the election outcome, no matter how many voters "sit on the fence" (i.e., no matter how small the density of types is in the neighborhood of the median). An argument based on the Downsian model is neatly summarized in a comment by Suellentrop (2004), written two days before the 2004 elections: "The secret of Bill Clinton's campaigns and of George W. Bush's election in 2000 was the much-maligned politics of small differences: Find the smallest possible majority (well, of electoral votes, for both men) that gets you to the White House. In political science, something called the median voter theorem dictates that in a two-party system, both parties will rush to the center looking for that lone voter – the median voter – who has 50.1 percent of the public to the right (or left) of him. Win that person's vote, and you've won the election." In contrast, Suellentrop's anticipated that Bush's political strategist Karl Rove made a fatal mistake in the 2004 election, "Bush's campaign — and his presidency — have appealed almost entirely to the base of the Republican Party. [. . .] Rove has tried to use the Bush campaign to disprove the politics of the median voter. It was as big a gamble as any of the big bets President Bush has placed over the past four years." Assuming that Rove and Bush indeed abandoned majority-efficient policies in order to cater to the Republican base, the election results have validated this strategy. As Example 4 shows, such a success is entirely consistent with the equilibrium behavior in a weighted-issue model, but hard to rationalize in a

[27] For a detailed account of the memo and its alleged implementation by the Bush administration, see Edsall (2006).

Downsian model.[28]

5.3 Type Uncertainty in the Scoring Model

Again, let $\omega \in \Omega$ parameterize the uncertainty about the distribution of θ, and let μ_Ω be the distribution of ω. Let $\theta_m(\omega)$ be the median voter in state ω — we assume for simplicity that there is a unique median voter.

The key determinant of equilibrium policies in the scoring model with uncertainty is the position of the *ex-ante median voter* θ_m defined such that the realized median voter $\theta_m(\omega) \geq \theta_m$ with probability 0.5 and $\theta_m(\omega) \leq \theta_m$ with probability 0.5, i.e., $\mu_\Omega(\{\omega|\theta_m(\omega) \leq \theta_m\}) = \mu_\Omega(\{\omega|\theta_m(\omega) \geq \theta_m\}) = 0.5$.

Theorem 6 *Suppose that citizens' utility is of the form $u_\theta(a) = v(\theta, f(a))$, where f is real valued and continuous. Let $\frac{\partial^2 v(\theta,x)}{\partial\theta\partial x} > 0$. Let θ_m denote the unique ex-ante median voter and let x_m be the most preferred policy score of θ_m, i.e., x_m solves $\max_{x\in\mathbb{R}} v(\theta, x)$. Then*

1. *Policy a^{j*} is ex-ante majority-efficient if and only if a^{j*} solves $\max_{a^j\in A^j} u_{\theta_m}(a^j)$.*
2. *If $f(a^{0*}) = f(a^{1*})$ then (a^{0*}, a^{1*}) is an equilibrium, and all pure strategy equilibria are ex-ante majority efficient.*
3. *If $f(a^{0*}) < x_m < f(a^{1*})$ then (a^{0*}, a^{1*}) is an equilibrium.*

Theorem 6 mirrors Theorem 3 in the characterization of majority-efficiency, however, the conditions for existence of majority-efficient equilibria are stronger in the case of type uncertainty. The following example shows that there are robust cases in which only ex-ante majority-inefficient equilibria exist.

Example 5 Let $f(a) = \sum_{i=1}^{\infty} 2^{-i}a_i$, and $v(\theta, x) = -(\theta - x)^2$. Suppose that candidate 0 is fixed to 0 on the first two positions, while candidate 1 is fixed to 1 on the first position. Thus, candidate 0 can obtain all policy scores in $[0, 1/4]$, while candidate 1 can obtain all policy scores in $[1/2, 1]$. Suppose that the median voter is distributed in $[0, 1]$ with a strictly positive density function, and that the ex-ante median is at $\theta_m > 0.5$. Then in equilibrium policy scores are 1/4 and 1/2, respectively. Note that candidate 1's ex-ante majority efficient policy is $x_m = \theta_m > 0.5$. However, by moving closer to candidate 0's position, he can increase his probability of winning. ■

We now show that, if there is policy convergence, then in general there is too much convergence from an efficiency perspective.

[28] In Downsian models with policy-motivated candidates, or in the citizen-candidate model, policy divergence occurs in equilibrium. However, in these models divergence has a cost for candidates: If a candidate would (or could) move closer to his opponent's position, his winning probability would increase. Candidates are willing to sustain this cost of divergence because they care about the policy that they can implement if elected. However, assuming that campaign consultants are primarily motivated by winning (as their reputation and income depends on it), they should always urge their clients to be more moderate.

Theorem 7 *Suppose that citizens' utility is of the form* $u_\theta(a) = v(\theta, f(a))$*, where* f *is real valued and continuous. Let* $\frac{\partial^2 v(\theta,x)}{\partial\theta\partial x} > 0$*. Let* θ_m *denote the unique ex-ante median voter. Let* x_m *be* θ_m*'s most preferred policy score, i.e.,* $x_m \in \arg\max_{x\in\mathbb{R}} v(\theta_m, x)$*. Suppose that* $x_m \in f(A^0) \cap f(A^1)$*. Then the equilibrium is competition-efficient if and only if* $\theta_m = \theta_m(\omega)$ *a.e., (i.e., there is no uncertainty about the median voter).*

Intuitively, if one candidate offered a policy that is most preferred by some voter θ' with, for example, $\theta' < \theta_m$ then efficiency of the electoral outcome is always strictly improved when the realized median voter $\theta_m(\omega) \leq \theta'$. In other words, if the position of the median voter is not known, then efficiency can be increased if candidates offer different platforms. However, for strategic reasons, offering differing platforms is not in the interest of the candidates in a scoring model.

As Myerson (1993) points out, whether political competition induces too much convergence is not obvious in the simple Downsian model: "Many authors seem to accept Hotelling's view that convergence of candidates is an undesirable outcome, because this 'excessive sameness' gives voters no real choice. This view ignores some crucial differences between the economic and political interpretations of Hotelling's game. In the economic interpretation, when two shops are locating on Main Street, minimization of the consumers' total transportation cost requires separation of the two shops. In the political interpretation, however, every voter's utility is derived from the policy position of the winning candidate (rather than the policy position of the one for whom he votes), and so voters get no intrinsic utility from a diversity of options in the selection. Thus, candidate convergence in equilibrium does not necessarily cause any welfare loss." For example, while voters effectively have no choice in a horizontal differentiation framework without uncertainty (because both candidates offer the median voter's preferred policy), this equilibrium is still competition-efficient. However, when there is non-trivial uncertainty about the state of the world in the scoring model, equilibria featuring complete candidate convergence are competition-inefficient, because some distance between candidates' platforms would be valuable for a majority of voters.

6 Related Literature

We depart from most of the existing literature by using a binary description of the policy space.[29] An advantage of the binary multidimensional model relative to the multidimensional Euclidean model (Plott (1967), McKelvey (1976)) is that our model is relatively tractable, as the set of preferences for which an equilibrium or a majority-efficient position exists has positive measure, and even if no pure strategy equilibrium exists, the mixed strategy equilibrium is straightforward to compute.

Apart from tractability, we also believe that the binary description of policy is quite natural, because most political campaigns are focused on relatively few clearly defined issues, where the politician can only be on record as being in favor or against the position. For example, in the 2006

[29]While such a framework has been used in contract theory (e.g., Krasa and Williams (2006)), our model is, to our knowledge, the first use of this approach in political economy.

US midterm elections, the key issues of "whether or not to impose a timetable for the withdrawal of troops from Iraq"; "whether to support stem cell research"; or "whether to support a constitutional ban on gay marriage"; all can be answered by yes or no. In contrast, the standard assumption of a continuous policy space (say, the interval $[0, 1]$), essentially implies that there are extremely many, very finely distinguished different positions to which candidates can commit. In reality, very nuanced differences may be hard to communicate to voters (whose private incentives to understand fine differences between candidates' platforms are small anyway, because the chance to be pivotal for the election outcome is small for every voter in large electorates).

An alternative approach to ours of introducing multidimensionality of policies in a tractable form is the probabilistic voting model (PVM; see Lindbeck and Weibull (1987), Lindbeck and Weibull (1993), Coughlin (1992)). The reason why candidates may choose non-median policies in the PVM is that some voters may be more "movable" through a favorable policy and, in equilibrium, candidates cater more strongly to the views of such swing voters. Such an effect can also arise in our model. More generally though, the range of possible effects is larger in our model. For example, we show that majority-inefficient positions can also arise if all voters have the same issue weights in their respective utility functions. Also, while both candidates converge to the same position in the PVM, this need not be the case in our model: Candidates may find it attractive to cater to their hardcore supporters and hence choose substantially different policies from their opponent (so that very few voters are indifferent between candidates). Furthermore, "ideological" preferences enter only as random utility shocks to voters in the PVM, but are explicitly modeled as arising from fixed positions in our general model. We also show the crucial role of different fixed positions for the result that candidates may choose different positions in pledgeable issues.

With respect to the ability of candidates to commit to a policy, our model combines the Downsian model and the citizen-candidate model. In the literature on the standard one-dimensional model pioneered by Hotelling (1929) and Downs (1957),[30] candidates are free to choose their position. In the citizen candidate literature pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997), candidates are policy motivated and cannot commit to any other position than their ideal one.[31] Our model combines some dimensions on which candidates have no choice, either because these dimensions capture some innate characteristics of candidates, or because their preferences on some questions are well known and not credibly changeable, with other dimensions in which candidates are free to choose a position.

A large literature exists that tries to explain, within the Downsian model, the empirical observation that candidates often propose considerably divergent policies. Candidates may diverge even though this decreases their winning probability, because they care about the implemented policy and have non-median preferences (see, e.g., Wittman (1983), Calvert (1985), Roemer (1994), Martinelli (2001), Gul and Pesendorfer (2006)). Other papers obtain policy divergence with office-motivated candidates, but assume incomplete information among voters about candidates charac-

[30]For a survey of this literature, see Osborne (1995).

[31]While the citizen candidate model can, in principle, handle multiple policy dimensions, most papers in this literature only look at a standard one-dimensional framework.

teristics (Callander (2003), Kartik and McAfee (2006)) or among candidates about the position of the median voter (Castanheira (2003), Bernhardt, Duggan, and Squintani (2006)).[32] In contrast to all previous papers, policy divergence is possible in our weighted-issue model in a full information environment, and, unlike in models with policy-motivated candidates, divergence increases a candidate's probability of winning.

7 Conclusion

In this paper, we have developed a multidimensional binary model of political competition and introduced the concepts of majority-efficiency and competition-efficiency. We believe that both the binary model and the efficiency concepts, jointly or separately, will be useful for future research.[33]

The binary model, and in particular the weighted-issue model, provide an intuitive and tractable framework for the analysis of multidimensional policy choice. For some dimensions, a candidate's position is fixed, like in the citizen-candidate model, while for other issues, candidates can commit to positions that maximize their electoral success probability, like in the Downsian model. The combination of these two central models is both realistic and makes it possible to analyze questions that cannot be adequately addressed in either the Downsian or the citizen candidate model. In particular, we can analyze how different fixed positions influence the candidates' position choice in pledgeable issues. For example, in our model, a phenomenon like two candidates both "catering to their respective base" (rather than converging on a common, "moderate", position) arise naturally. Also, our model shows that candidates may not only adopt minority positions if the minority cares more intensely about an issue than the majority. This result contrasts sharply with the probabilistic voting model.

To highlight the fact that our results are generated by multidimensionality of the policy space and fixed positions, we consider the framework with two office motivated candidates, which generates policy convergence in the Downsian setting. In contrast, in the weighted-issue model, equilibrium policies need not be majority-efficient. This shows that two standard results of the Downsian model — policy convergence of candidates, and movement of candidates "into the middle", i.e., in a direction that is preferred by a majority of the electorate — are actually generated by the one-dimensional structure of the Downsian model.

While we keep the assumptions of two, office-motivated candidates to facilitate a comparison with the standard Downsian model, it is certainly interesting to analyze the effects of different assumptions (such as, e.g., policy-motivated candidates, more than two candidates, runoff rule, proportional representation) in the binary policy model. We leave these questions to future research.

[32] A third branch of literature, which is less directly related to this paper, explains policy divergence as entry deterrence by two dominant parties (e.g., Palfrey (1984), Callander (2005)), or in a framework where candidates have costs of deviating from the national party platform (Eyster and Kittsteiner (2007)).

[33] In political economy models, majority-efficiency and competition-efficiency have the advantage of providing a much tighter benchmark than Pareto-efficiency, while being based only on ordinal preferences and not requiring summation of the utilities of different voters (such as utilitarianism). The notions of majority-efficiency and competition-efficiency can clearly be applied in any political economy model, not just in the binary policy model.

8 Appendix

Proof of Theorem 1. Suppose that (a^0, a^1) is an equilibrium. Then, each candidate wins with probability 0.5. (Suppose, to the contrary, that Candidate 0 (say), always loses. However, because $A^0 = A^1$, he could improve by choosing $\tilde{a}^0 = a^1$, a contradiction). Let $\hat{a}$ be an arbitrary feasible policy. If $\hat{a} \succ^* a^0$, then Candidate 1 wins (with probability 1) if he offers policy $\hat{a}$. Since a^0 is an equilibrium strategy, $a^0 \succeq^* \hat{a}$ for all $\hat{a}$. Similarly, $a^1 \succeq^* \hat{a}$ for all $\hat{a}$. Hence, both a^0 and a^1 are majority-efficient.

Now suppose that a^0 and a^1 are majority-efficient. We have to show that (a^0, a^1) is an equilibrium. Since $a^0 \succeq^* a^1$ and $a^1 \succeq^* a^0$ (by majority-efficiency), each candidate gets 50% of the votes and thus wins with probability 0.5. Furthermore, by majority-efficiency of a^0 and a^1, $a^0 \succeq^* \hat{a}$ and $a^1 \succeq^* \hat{a}$, for all $\hat{a}$. Hence, there is no profitable deviation, so that (a^0, a^1) is an equilibrium.

Now consider a mixed strategy equilibrium (σ^0, σ^1). Each candidate must win with probability 0.5 (otherwise, the candidate who wins with the lower probability could deviate to the strategy of his opponent, thereby increasing his winning probability to 0.5). Furthermore, in order for mixing to be optimal, almost every policy in the support of σ^j must give agent j a winning probability of 0.5. Now, assume by way of contradiction that the support of σ^j contains a set B with $\mu(B) > 0$, such that no policy in B is majority-efficient. Then policies in B only win if Candidate $-j$ also selects a non-majority-efficient policy. Because the winning probability must be 0.5, this implies that the opponent uses a non-majority-efficient strategy with strictly positive probability. Let $\tilde{a}^j$ be a majority-efficient policy. Suppose that Candidate j uses the alternative strategy $\tilde{\sigma}^j$ which uses $\tilde{a}^j$ whenever a policy in B is selected under σ^j and corresponds to σ^j, otherwise. Then $\tilde{a}^j$ wins whenever the opponent selects a non-majority-efficient policy and ties whenever the opponent uses a majority-efficient policy. Thus, Candidate j's winning probability strictly increases, a contradiction. Hence, almost every policy in the support of σ^j is majority-efficient. ■

Proof of Theorem 2. Without loss of generality, let $a^* \in A^0$. First, suppose that $a^* \notin A^1$. Then $(\hat{a}, a^1)$, is an equilibrium for any $a^1 \in A^1$ (since $a^* \succ a$ for all $a \in A^0 \cup A^1$, Candidate 0 wins with probability 1, and Candidate 1 has no available policy that can beat or tie with a^*).

Now suppose that $a^* \in A^1$. Then (a^*, a^*) is the unique equilibrium: Since $a^* \succ a$ for all $a \in A^0 \cup A^1$, any deviation leads to a loss. Furthermore, since a^* wins against any other policy with probability 1, no other strategy profile can be an equilibrium. ■

The following lemma shows that, for any two policies a and a', the set of voters who prefer a to a' is an upper or lower interval of the type space.

Lemma 1 *Let* $\frac{\partial^2 v(\theta,x)}{\partial\theta\partial x} > 0$. *If* $v(\tilde{\theta}, f(a)) \geq v(\tilde{\theta}, f(a'))$, *then either* $v(\theta, f(a)) \geq v(\theta, f(a'))$ *for all* $\theta \geq \tilde{\theta}$, *or* $v(\theta, f(a)) \geq v(\theta, f(a'))$ *for all* $\theta \leq \tilde{\theta}$.

Proof of Lemma 1. Assume first that $f(a') = x_1 > f(a) = x_0$. By assumption, $\frac{\partial}{\partial\theta}v(\theta,x)$ is increasing in x. Note that $\int_{\tilde{\theta}}^{\theta}\left[\frac{\partial v(t,x_1)}{\partial t} - \frac{\partial v(t,x_0)}{\partial t}\right]dt = v(\theta,x_1) - v(\theta,x_0) - [v(\tilde{\theta},x_1) - v(\tilde{\theta},x_0)]$, which we can rearrange for

$$v(\theta,x_1) - v(\theta,x_0) = [v(\tilde{\theta},x_1) - v(\tilde{\theta},x_0)] + \int_{\tilde{\theta}}^{\theta}\left[\frac{\partial v(t,x_1)}{\partial t} - \frac{\partial v(t,x_0)}{\partial t}\right]dt. \quad (4)$$

Hence, if the term in square brackets on the right-hand side is positive, then the left hand side is positive for all $\theta > \tilde{\theta}$. If $f(a') = x_1 < f(a) = x_0$, then a symmetric argument establishes that, if $v(\tilde{\theta},x_1) - v(\tilde{\theta},x_0) > 0$, then $v(\theta,x_1) - v(\theta,x_0)$ for all $\theta < \tilde{\theta}$. ■

Proof of Theorem 3. Lemma 1 immediately implies statement 1, because the set of voters preferring any other policy to the median's favorite policy must be an upper or lower interval not containing the median, hence not a majority. The reverse implication in statement 1 is also obvious.

For statement 2, note that Lemma 1 implies that a candidate wins if and only if his policy is preferred by type θ_m to his opponent's policy. Hence, proposing a^{j*} is clearly a weakly dominant strategy for Candidate j, so that (a^{0*}, a^{1*}) is an equilibrium.

For statement 3, weak dominance of a^{j*} implies that the election outcome must be the same in any equilibrium. To prove that one candidate wins with probability 1 in any equilibrium involving majority-inefficient strategies, suppose (without loss of generality) that (a^0, a^1) is an equilibrium and that a^1 is not majority-efficient. Suppose that, contrary to the claim, both candidates tie in (a^0, a^1); then Candidate 1 could deviate to a^{1*} and win with probability 1. ■

Proof of Theorem 4. Let X_i be defined by

$$X_i(\theta) = \begin{cases} 1 & \text{if } |\theta_i - a_i| = 0; \\ 0 & \text{if } |\theta_i - a_i| = 1. \end{cases}$$

Let a' be an arbitrary alternative policy. Let D be the set of issues for which $a_i \neq a'_i$. Then policy a is at least as good as policy a' for agent θ if and only if $\sum_{i\in D}\lambda_{i,\theta_0}X_i(\theta) \geq \sum_{i\in D}\lambda_{i,\theta_0}(1 - X_i(\theta))$, which is equivalent to

$$\sum_{i\in D}\lambda_{i,\theta_0}X_i(\theta) \geq 0.5\sum_{i\in D}\lambda_{i,\theta_0}. \quad (5)$$

Let $B = \{0,1\}$. For each $i \in \mathbb{N}$, let $Y_i \colon \Theta \times B \to \{0,1\}$ be defined by $Y_i(\theta,b) = X_i(\theta)$. We next define $T_k \colon \Theta \times B \to \{0,1\}$ by

$$T_k(\theta,b) = \begin{cases} a_k^j & \text{if } \theta_k = a_k^j, \text{ or } b = 1; \\ |1 - a_k^j| & \text{otherwise.} \end{cases}$$

Let $T_i(\theta,b) = \theta$ for all $i \neq k$. Then define $T \colon \Theta \times B \to \Theta$ by $T = (T_i)_{i\in\mathbb{N}}$. Note that

$$X_i(T(\theta,b)) \geq Y_i(\theta,b), \quad (6)$$

for all $(\theta, b) \in \Theta \times B$ and for all $i \in \mathbb{N}$.

We now construct a probability measure ν on $\Theta \times B$ such that

$$\mu'(C) = \nu(T^{-1}(C)), \text{ and } \nu(C \times B) = \mu(C), \tag{7}$$

for all measurable subsets C of Θ.

Recall that $a_k^j \in \{0,1\}$. For all measurable sets $C_{-k} \subset \Theta_{-k}$ we define the marginal distribution $\mu(\cdot|C_{-k})$ on B by

$$\mu(\{1\}|C_{-k}) = \frac{\mu'\left(\{a_k^j\} \times C_{-k}\right) - \mu\left(\{a_k^j\} \times C_{-k}\right)}{1 - \mu\left(\{a_k^j\} \times C_{-k}\right)}. \tag{8}$$

Let $\mu(\{0\}|C_{-k}) = 1 - \mu(\{1\}|C_{-k})$. By assumption $\mu'\left(\{a_k^j\} \times \Theta_{-k}\right) \geq \mu\left(\{a_k^j\} \times \Theta_{-k}\right)$, and therefore (8) provides a well defined probability. Now let $\nu(\{\theta_k\} \times C_{-k} \times \{b\}) = \mu(\{\theta_k\} \times C_{-k})\mu(\{b\}|C_{-k})$ for all measurable subsets C_{-k} of Θ_{-k}, for all $\theta_k \in \{0,1\}$ and for all $b \in B$. Then the definition of ν extends to all measurable subsets of $\Theta \times B$.

Let $\hat{a}_k^j$ be the other element in $\{0,1\}$, i.e., $\hat{a}_k^j \neq a_k^j$. The definition of ν immediately implies that $\nu(C \times B) = \mu(C)$. Next, note that $T^{-1}(\{a_k^j\} \times C_{-k}) = \big(\{a_k^j\} \times C_{-k} \times B\big) \cup \big(\{\hat{a}_k^j\} \times C_{-k} \times \{1\}\big)$. Hence,

$$\begin{aligned}
\nu(T^{-1}(\{a_k^j\} \times C_{-k})) &= \nu\big(\{a_k^j\} \times C_{-k} \times B\big) + \nu\big(\{\hat{a}_k^j\} \times C_{-k} \times \{1\}\big) \\
&= \mu\big(\{a_k^j\} \times C_{-k}\big) + \mu\big(\{\hat{a}_k^j\} \times C_{-k}\big)\mu(\{1\}|C_{-k}) = \mu'\big(\{a_k^j\} \times C_{-k}\big),
\end{aligned}$$

where the last equality follows from (8). Thus, (7) is satisfied.

Conditions(6) and (7) imply

$$\begin{aligned}
\mu'_{\Theta_{-0}}\left(\left\{\theta \,\middle|\, \sum_{i\in D} \lambda_{i,\theta_0} X_i(\theta) \geq 0.5 \sum_{i\in D} \lambda_{i,\theta_0}\right\}\right) &= \nu_{\Theta_{-0}}\left(T^{-1}\left\{\theta \,\middle|\, \sum_{i\in D} \lambda_{i,\theta_0} X_i(\theta) \geq 0.5 \sum_{i\in D} \lambda_{i,\theta_0}\right\}\right) \\
&= \nu_{\Theta_{-0}}\left(\left\{(\theta,b) \,\middle|\, \sum_{i\in D} \lambda_{i,\theta_0} X_i(T(\theta,b)) \geq 0.5 \sum_{i\in D} \lambda_{i,\theta_0}\right\}\right) \\
&\geq \nu_{\Theta_{-0}}\left(\left\{(\theta,b) \,\middle|\, \sum_{i\in D} \lambda_{i,\theta_0} Y_i(\theta,b) \geq 0.5 \sum_{i\in D} \lambda_{i,\theta_0}\right\}\right) \\
&= \mu_{\Theta_{-0}}\left(\left\{\theta \,\middle|\, \sum_{i\in D} \lambda_{i,\theta_0} X_i(\theta) \geq 0.5 \sum_{i\in D} \lambda_{i,\theta_0}\right\}\right)
\end{aligned} \tag{9}$$

Thus, the percentage of citizens who prefer a^j to the alternative policy increases when we move from μ to μ'. Similarly, (6) and (7) imply

$$\mu_{\Theta_{-0}}\left(\left\{\theta \,\middle|\, \sum_{i\in D} \lambda_{i,\theta_0} X_i(\theta) \leq 0.5 \sum_{i\in D} \lambda_{i,\theta_0}\right\}\right) \geq \mu'_{\Theta_{-0}}\left(\left\{\theta \,\middle|\, \sum_{i\in D} \lambda_{i,\theta_0} X_i(\theta) \leq 0.5 \sum_{i\in D} \lambda_{i,\theta_0}\right\}\right) \tag{10}$$

Recall that a being preferred to a' by citizen θ is equivalent to condition (5) being satisfied. Thus, (9) and (10) imply

$$\mu'_{\Theta_{-0}}(\{\theta|u_{\theta,\theta_0}(a) \geq u_{\theta,\theta_0}(a')\}) \geq \mu_{\Theta_{-0}}(\{\theta|u_{\theta,\theta_0}(a) \geq u_{\theta,\theta_0}(a')\}) \\ \geq \mu_{\Theta_{-0}}(\{\theta|u_{\theta,\theta_0}(a') \geq u_{\theta,\theta_0}(a)\}) \geq \mu'_{\Theta_{-0}}(\{\theta|u_{\theta,\theta_0}(a') \geq u_{\theta,\theta_0}(a)\}). \tag{11}$$

Integrating both sides of (11) with respect to θ_0 implies that a is majority preferred to a'. Since a' was arbitrary, this proves that a is majority-efficient under μ'. ■

Proof of Corollary 1. Let a' be an arbitrary policy. Since, $\sum_{i=1}^{\infty} \lambda_{i,\theta_0} < \infty$ it follows that $(\theta_{-0}, a) \mapsto u_\theta(a)$ is continuous in the product topology for fixed θ_0. Applying Theorem 4 inductively proves that

$$\mu^k_{\Theta_{-0}}(\{\theta|u_\theta(a) \geq u_\theta(a')\}) \geq \mu^k_{\Theta_{-0}}(\{\theta|u_\theta(a') \geq u_\theta(a)\}). \tag{12}$$

for all $k \in \mathbb{N}$. Because $u_\theta(a)$ is continuous, it follows that $\{\theta|u_\theta(a) \geq u_\theta(a')\}$ and $\{\theta|u_\theta(a') \geq u_\theta(a)\}$ are closed sets. Taking the limit in (12) for $k \to \infty$ yields $\mu_{\Theta_{-0}}(\{\theta|u_\theta(a) \geq u_\theta(a')\}) \geq \mu_{\Theta_{-0}}(\{\theta|u_\theta(a') \geq u_\theta(a)\})$. Integrating over Θ_0 implies that a is majority efficient for μ. ■

Proof of Corollary 2. If $|S| = 1$, the result is immediate. Thus, without loss of generality, let $S^j = \{1, 2\}$, i.e., the first two positions can be chosen freely. Let $a = \mu(\{(0,0)\} \times \{0,1\}^{\mathbb{N}})$, $b = \mu(\{(1,0)\} \times \{0,1\}^{\mathbb{N}})$, $c = \mu(\{(0,1)\} \times \{0,1\}^{\mathbb{N}})$, and $d = \mu(\{(1,1)\} \times \{0,1\}^{\mathbb{N}})$. Clearly, $a+b+c+d = 1$. Further, without loss of generality, let assume that $c + d \geq a + b$ and $b + d \geq a + c$. This immediately implies that $d \geq a$. Now consider a distribution $\tilde{\mu}$ with $a = \tilde{\mu}(\{(0,0)\} \times \{0,1\}^{\mathbb{N}}) = \tilde{\mu}(\{(1,1)\} \times \{0,1\}^{\mathbb{N}})$, and $0.5 - a = \tilde{\mu}(\{(0,1)\} \times \{0,1\}^{\mathbb{N}}) = \tilde{\mu}(\{(1,0)\} \times \{0,1\}^{\mathbb{N}})$. Because of symmetry of $\tilde{\mu}$, any policy is majority efficient, for example a policy with $a_1 = a_2 = 1$. In order to get distribution μ from $\tilde{\mu}$ we must move weight from $\{(0,1)\} \times \{0,1\}^{\mathbb{N}}$ and $\{(1,0)\} \times \{0,1\}^{\mathbb{N}}$ to $\{(1,1)\} \times \{0,1\}^{\mathbb{N}}$. This is possible as long as $0.5 - a \geq c$ and $0.5 - a \geq b$. Suppose by way of contradiction that $0.5 - a < c$. Then $a + c > 0.5$. Because $a+b+c+d = 1$ this implies $a+c > b+d$, a contradiction. Similarly, $0.5 - a < b$ contradicts $c + d \geq a + b$. ■

Proof of Corollary 3. See text. ■

Proof of Remark 1. Consider the natural linear ordering of Θ (given by the standard "$\geq$"). Without loss of generality suppose that $x^i = f(a^i) > f(a^{-i}) = x^{-i}$. Thus, for $\theta_1 > \theta_0$, we have

$$u_{\theta_1}(a^i, \kappa^i) - u_{\theta_1}(a^{-i}, \kappa^{-i}) - [u_{\theta_0}(a^i, \kappa^i) - u_{\theta_0}(a^{-i}, \kappa^{-i})] = \\ \int_{\theta_0}^{\theta_1} \left[\frac{\partial u_\theta(a^i, \kappa^i)}{\partial \theta} - \frac{\partial u_\theta(a^{-i}, \kappa^{-i})}{\partial \theta} \right] d\theta = \int_{\theta_0}^{\theta_1} \left[\frac{\partial v(\theta, x^i)}{\partial \theta} - \frac{\partial v(\theta, x^{-i})}{\partial \theta} \right] d\theta > 0,$$

because the assumption that $\frac{\partial^2 v(\theta,x)}{\partial\theta\partial x} > 0$ implies that $\left[\frac{\partial v(\theta,x^i)}{\partial\theta} - \frac{\partial v(\theta,x^{-i})}{\partial\theta}\right] > 0$ for all θ. Thus, if citizen $\theta_0 \in \Theta^i(a^0, a^1, \kappa)$ (i.e., $[u_{\theta_0}(a^i, \kappa^i) - u_{\theta_0}(a^{-i}, \kappa^{-i})] > 0$ in the first line), then, for the inequality to hold, it must be true that $u_{\theta_1}(a^i, \kappa^i) - u_{\theta_1}(a^{-i}, \kappa^{-i}) > 0$ so that $\theta_1 \in \Theta^i(a^0, a^1, \kappa)$.

An analogous proof shows that Ψ^0 and Ψ^1 are upper or lower intervals, or empty. ■

Proof of Theorem 5. Let $m^i \in \arg\max_{a^i \in A^i} u_{\theta_m}(a^i, \kappa^i)$. Note that a maximum exists because utility is continuous in a^i and A^i is compact, and, by Definition 3, the maximum is independent of κ^i. We show that m^i is majority efficient. For all $a^i \in A^i$, $u_{\theta_m}(m^i, \kappa^i) \geq u_{\theta_m}(a^i, \kappa^i)$. By condition 2 of Definition 4, $\Psi^i(m^i, a^i, \kappa^i)$ is either empty or an upper and lower interval. Since $\theta_m \in \Psi^i(m^i, a^i, \kappa^i)$ it follows that $\Psi^i(m^i, a^i, \kappa^i)$ is an interval containing θ_m. Thus, $\mu(\Psi^i(m^i, a^i, \kappa^i)) \geq 0.5$, so that m^i is majority efficient.

Next, suppose that $\tilde{a}^i \notin \arg\max_{a^i \in A^i} u_\theta(a^i, \kappa^i)$, so that $u_{\theta_m}(m^i, \kappa^i) > u_{\theta_m}(\tilde{a}^i, \kappa^i)$. Thus, continuity of utility with respect to θ implies that there exists a neighborhood U_{θ_m} of θ_m such that $U_{\theta_m} \subset \Psi(m^i, \tilde{a}^i, \kappa^i)$. Condition 2 of Definition 4 implies that $\Psi(m^i, \tilde{a}^i, \kappa^i)$ is an upper or lower interval. Because this interval contains an open set containing θ_m (the unique median) it follows that $\mu(\Psi(m^i, \tilde{a}^i, \kappa^i)) > 0.5$, i.e., $\tilde{a}^i$ is not majority efficient.

It remains to prove that (m^0, m^1) is an equilibrium. Let $a^0 \in A^0$ be an arbitrary alternative policy that is not majority efficient. Then Candidate 0 wins if

$$\mu(\{\theta | u_\theta(a^0, \kappa^0) \geq u_\theta(m^1, \kappa^1)\}) > \mu(\{\theta | u_\theta(m^1, \kappa^1) \geq u_\theta(a^0, \kappa^0)\}), \tag{13}$$

and ties if the equality holds in (13). By condition 1 of Definition 4, there exists $\theta_p \in \Theta$ such that $\{\theta | u_\theta(a^0, \kappa^0) \geq u_\theta(m^1, \kappa^1)\}$ is either an upper interval $\{\theta | \theta \geq \theta_p\}$ or a lower interval $\{\theta | \theta \leq \theta_p\}$. Without loss of generality, suppose that the latter is the case. Then (13) implies that $\theta_m < \theta_p$. Since a^0 is not majority efficient it follows that $u_{\theta_m}(m^0, \kappa^0) > u_{\theta_m}(a^0, \kappa^0) \geq u_{\theta_m}(m^1, \kappa^1)$. Thus, there exists an open neighborhood U_{θ_m} of θ_m with $U_{\theta_m} \subset \Theta^0(m^0, m^1, \kappa)$. By condition 1 of Definition 4, $\Theta^0(m^0, m^1, \kappa)$ is an upper or lower interval containing θ_m, and thus $\mu(\Theta^0(m^0, m^1, \kappa)) > 0.5$, i.e., Candidate 0 wins also with m^0. If a^0 ties, i.e., the equality holds in (13), then candidate 0 wins with m^0. Since this argument applies for any κ, the winning probability does not increase if Candidate 0 deviates to a^0. An analogous argument applies for Candidate 1, so that (m^0, m^1) is an equilibrium. ■

Proof of Theorem 6.

Proof of Statement 1. Suppose that a^{j*} solves $\max_{a^j \in A^j} u_{\theta_m}(a^j)$. If $f(a^j) \neq f(a^{j*})$ then θ_m is strictly better off with a^j. By continuity of v, there exists a neighborhood of θ_m such that all citizens θ in the neighborhood are strictly better off with a^{j*}. Lemma 1 implies that the set of all citizens θ who prefer a^{j*} to another policy $a^j \in A^j$ is an upper or lower interval. Since this interval contains a neighborhood of θ_m and θ_m is the unique ex-ante median voter, it follows that $\mu_\Omega(\{\omega | u_{\theta(\omega)}(a^{j*}) > u_{\theta(\omega)}(a^j)\}) > 0.5$, i.e., a^{j*} is ex-ante majority efficient. Conversely, if a^{j*} is ex-ante majority efficient, then $u_{\theta_m}(a^{j*}) \geq u_{\theta_m}(a^j)$ for all $a^j \in A^j$. Otherwise, if $u_{\theta_m}(a^{j*}) < u_{\theta_m}(a^j)$ a majority of citizens would prefer a^j to a^{j*} in a majority of states. Thus, a^{j*} solves $\max_{a^j \in A^j} u_{\theta_m}(a^j)$.

Proof of Statement 2. If candidates choose (a^{0*}, a^{1*}) then each candidate wins with 50% probability. Without loss of generality suppose that candidate 0 deviates to a^0. Statement 1 implies that θ_m strictly prefers a^{1*} to a^0. Thus, repeating the argument from above, candidate 0 now wins with a probability of strictly less than 50%. Hence the deviation is not optimal. Conversely, if candidate 0 chooses a strategy a^0 that is not ex-ante majority efficient, then candidate 1 wins with probability strictly greater than 50% by selecting a^{*1} and hence also by selecting the best response to a^0. Instead, candidate 0's winning probability is at least 50% if he chooses a^{0*}.

Proof of Statement 3. Without loss of generality suppose that candidate 0 deviates to a^0. Lemma 1 implies that the set of citizens θ that prefer a^0 to a^{1*} is an interval. However, since a^{0*} solves $\max_{\hat{a}^0 \in A^0} u_{\theta_m}(\hat{a}^0)$ and $f(a^{0*}) < x_m$, this interval is strictly smaller than the interval of citizens that prefer a^{0*} to a^{1*}. ■

Proof of Theorem 7. Theorem 6 implies that in equilibrium $f(a^j) = x_m$ for $j = 0, 1$. Suppose that $\theta_m(\omega) < \theta_m$ with positive probability. Let $\hat{x}$ be marginally smaller than x_m, and consider the alternative pair of platforms $(x_m, \hat{x})$. Then, in states with $\theta_m(\omega) < \theta_m$, a majority of voters prefers $\hat{x}$, while in states with $\theta_m(\omega) \geq \theta_m$, the outcome remains the same. Hence, the equilibrium is not competition efficient.

Conversely, suppose that $\theta_m(\omega) = \theta_m$ for a.e. ω, but that the equilibrium (a^0, a^1) is not competition-efficient. Let $\hat{a}^j \in A^j$, $j = 0, 1$ be arbitrary. If $f(\hat{a}^1) = f(\hat{a}^2)$ then citizens are indifferent between the two policies. Furthermore, if $x_m \neq f(\hat{a}^j)$ then a majority of citizens in state ω prefer (a^1, a^2) to $(\hat{a}^1, \hat{a}^2)$, i.e.,

$$Q(a^1, a^2, \hat{a}^1, \hat{a}^2, \omega) \geq Q(\hat{a}^1, \hat{a}^2, a^1, a^2, \omega). \tag{14}$$

If $f(\hat{a}^1) \neq f(\hat{a}^2)$ then Candidate j wins if $u_{\theta_m(\omega)}(\hat{a}^j) \geq u_{\theta_m(\omega)}(\hat{a}^{-j})$. Without loss of generality suppose that $f(\hat{a}^j) > x_m$. Then there exists $\tilde{\theta} > \theta_m$ such that all agents $\theta < \tilde{\theta}$ strictly prefer a^j. Since, $\theta_m(\omega) = \theta_m$, for a.e. ω, this means that a majority of citizens prefers a^j to $\hat{a}^j$, i.e., (14) holds. Thus, (a^1, a^2) is competition-efficient. ■

References

Aragones, E. and T. R. Palfrey (2002). Mixed equilibrium in a Downsian model with a favored candidate. *Journal of Economic Theory 103*(1), 131–161.

Bade, S. (2006). Electoral competition with uncertainty averse parties. Mimeo, Penn State.

Bernhardt, D., J. Duggan, and F. Squintani (2006). Private polling in elections and voter welfare. mimeo, University of Illinois.

Bernhardt, D., S. Krasa, and M. Polborn (2006). Political polarization and the electoral consequences of media bias. University of Illinois.

Besley, T. and S. Coate (1997). An economic model of representative democracy. *Quarterly Journal of Economics 112*, 85–114.

Blumenthal, S. (2006). Generation Dem. Salon.com, November 30, 2006; available at http://www.salon.com/opinion/blumenthal/2006/11/30/2006_election_trends/print.html.

Callander, S. (2003). Political motivations. mimeo, Northwestern.

Callander, S. (2005). Electoral competition in heterogeneous districts. *Journal of Political Economy 113*, 1116–1145.

Calvert, R. L. (1985). Robustness of the multidimensional voting model: Candidate motivations, uncertainty, and convergence. *American Journal of Political Science 29*, 69–95.

Caplin, A. and B. Nalebuff (1988). On 64%-majority rule. *Econometrica 56*, 787–814.

Castanheira, M. (2003). Why vote for losers? *Journal of the European Economic Association 1*(5), 1207–1238.

Coughlin, P. (1992). *Probabilistic Voting Theory.* Cambridge, Cambridge University Press.

Dal Bo, E., P. Dal Bo, and J. Snyder (2006). Political dynasties. mimeo, UC Berkeley.

Downs, A. (1957). *An economic theory of democracy.* Harper & Row.

Edsall, T. B. (2006). *Building Red America.* Basic Books.

Eyster, E. and T. Kittsteiner (2007). Party platforms in electoral competition with heterogeneous constituencies. Forthcoming, Theoretical Economics.

Gul, F. and W. Pesendorfer (2006). Partisan politics and aggregation failure with ignorant voters. Princeton.

Hotelling, H. (1929). Stability in competition. *Economic Journal 39*, 41–57.

Kartik, N. and P. McAfee (2006). Signaling character in electoral competition. Forthcoming, American Economic Review.

Kirkpatrick, D. D. (2006). G.O.P. risking hispanic votes on immigration. New York Times, March 30, 2006.

Krasa, S. and S. R. Williams (2006). Limited observability as a constraint in contract design. forthcoming *Journal of Economic Theory.*

Ledyard, J. (1984). Pure theory of two-candidate competition. *Public Choice 44*, 7–41.

Lindbeck, A. and J. Weibull (1987). Balanced-budget redistribution as the outcome of political competition. *Public Choice 52*, 273–297.

Lindbeck, A. and J. Weibull (1993). A model of political equilibrium in a representative democracy. *Journal of Public Economics 51*, 195–209.

Martinelli, C. (2001). Elections with privately informed parties and voters. *Public Choice 108*(1), 147–167.

McKelvey, R. (1976). Intransitivities in multidimensional voting models, with some implications for agenda control. *Journal of Economic Theory 12*, 472–482.

Myerson, R. (1993). Review of: Advances in the spatial theory of voting. *Journal of Economic Literature 31*, 237–240.

Osborne, M. (1995). Spatial models of political competition under plurality rule: A survey of some explanations of the number of candidates and the positions they take. *Canadian Journal of Economics 27*, 261–301.

Osborne, M. and A. Slivinski (1996). A model of political competition with citizen candidates. *Quarterly Journal of Economics 111*, 65–96.

Palfrey, T. R. (1984). Spatial equilibrium with entry. *Review of Economic Studies 51*, 139–156.

Persson, T. and G. Tabellini (2000). *Political Economy: Explaining Economic Policy*. MIT Press.

Plott, C. (1967). A notion of equilibrium and its possibility under majority rule. *American Economic Review 57*, 787–806.

Roemer, J. E. (1994). A theory of policy differentiation in single issue electoral politics. *Social Choice and Welfare 11*(4), 355–380.

Suellentrop, C. (2004). The vanishing nonvoter. Slate, http://www.slate.com/id/2108924/.

Wittman, D. (1983). Candidate motivation: A synthesis of alternative theories. *American Political Science Review 77*, 142–157.