

# Resource Allocation and Firm Scope\*

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## Resource Allocation and Firm Scope

**Abstract:** We develop a theory of firm scope based on the benefits and costs of resource allocation within firms. Integrating two firms into one makes it possible to allocate by authority scarce resources that are costly to trade. But to do so efficiently, top management must rely on information that is communicated by self-interested division managers. Two countervailing effects influence the design of optimal incentive contracts: first, intra-firm competition for scarce resources strengthens the incentives of division managers to expend effort on creating profitable projects. Second, a manager who is paid for his own division's performance has an incentive to overstate the quality of his project, which can be corrected only by paying for firm performance as well. We show that the second effect dominates, leading to overall higher costs of providing incentives in an integrated firm, relative to stand-alone firms. The benefits and costs of integration thus arise from the same problem — the aggregation and use of dispersed information. We also show that a hierarchical structure with a purely coordinating role of top management is an optimal solution to the incentive problems in an integrated firm. Only if resources are highly complementary can a decentralized structure in which divisions voluntarily share resources do equally well. Our results lead to several testable predictions concerning integration decisions, wages, and organizational structure.

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# 1 Introduction

A salient feature of modern economies is the prevalence of large multidivisional firms. Their creation and growth are often the result of horizontal or vertical integration of independent businesses. Beginning with Coase (1937), economists have sought to explain these integration decisions, and indeed the very existence of firms, by arguing that managerial authority can create benefits that the market cannot. It is widely believed that integration also creates agency costs; their precise nature and origin, however, tends to be difficult to pin down.

We develop a theory of firm scope based on the idea that integrating two firms into one facilitates the efficient use of resources that are too costly to trade across firm boundaries. This idea has a long tradition. While a firm's top management has many responsibilities, Chandler (1977, p.7) has argued that its role of allocating resources by authority was vital for the growth of modern corporations. Likewise, the strategic management literature emphasizes the ability to use "unique resources and capabilities" in different businesses as the main justification for firms' expansion of scope (see Teece, 1982).

New in our theory is how we explain the costs of integration. We assume that to allocate resources efficiently, top management must rely on information provided by division managers about the projects they create. As we explain below, integration has two effects relative to the case in which each division is run as a stand-alone firm. On one hand, competition among the division managers for resources lowers the cost of inducing effort to create projects. On the other hand, getting each to provide truthful information about his project makes inducing effort more costly for the firm. We show that the second effect dominates. Using resources more efficiently under integration thus necessarily entails higher costs of providing incentives for effort; both the benefit and the cost of integration are the result of aggregating dispersed information.

More specifically, our theory begins with the assumption that production requires some resources that are easier to allocate by managerial authority than through contracts. Otherwise, firms could rely on prices to allocate resources both within and between firms, and there would be no useful role for managerial authority. Many firms' production, however, involves scarce resources (physical assets or specialized high-skilled human resources) that are costly to trade, for instance, because the resources or the terms of their use are difficult to describe. In our model we capture this idea in a simplifying way by assuming that the use of resources is not contractible, while *authority* over resources is.<sup>1</sup> Also for simplicity, the supply of resources is

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<sup>1</sup> See other recent work on authority in organizations (e.g. Aghion et al., 2004, and the survey of Dewatripont, 2001) for similar assumptions. A resource for which this assumption typically does not hold is capital (money). Our theory is therefore strictly speaking not one about internal capital markets; see our discussion in Section 2.

fixed; there is no external market for them.

There are two production units that can be either run as stand-alone firms or as divisions of an integrated firm. Each unit is run by a manager whose job it is to create profitable “projects”. A project’s payoff depends on its quality and the resources invested in it. A project’s quality, in turn, depends on the manager’s effort; to induce effort, firms can offer wage contracts based on the projects’ payoffs. Managers are risk-neutral but protected by limited liability. Each production unit is endowed with a given amount of resources. If run as independent firms, each unit uses that amount, but cannot obtain more resources elsewhere.

When the firms merge, they become divisions of one firm; their resources are pooled and placed under the authority of a CEO who can shift the firm’s resources to divisions with good projects. The quality of each division’s project is the private information of the division manager who created it. Realizing the benefit of integration therefore requires that division managers communicate their information truthfully. But since a project’s payoff is higher the more resources it receives, each manager will have a motive to overstate the quality of his project if his wage is based on his division’s performance.<sup>2</sup>

Pay for individual performance thus gives rise to an endogenous “empire-building” motive, even though managers have no intrinsic preferences for resources. The question is then: Are there incentive contracts under integration that induce both truthful communication about projects and effort to generate good projects in the first place?

The answer is yes, but at additional cost. Inducing both effort and truthful communication is possible by paying managers not only for their own division’s performance, but for that of the other division as well, or equivalently, for firm performance. Doing so ensures that a manager with a bad project will refrain from claiming to have a good one, since he stands to gain from the allocation of the firm’s resources to a better project. Providing “team-based” incentives of this sort, however, makes it more costly to induce managerial effort than under non-integration, an effect we refer to as the *information-rent effect* of integration.

The result that integration leads to higher agency costs is far from obvious because centralized resource allocation also has a positive *competition effect* on effort incentives. Allocating resources based on project quality raises the marginal benefit of effort to create good projects, both because of a complementarity between project quality and resources, and because of competition between

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<sup>2</sup> As Crozier (1965) observed in his field study: “[In making many decisions, higher-level managers] must rely heavily on the information they receive from the section chiefs... The section chiefs, however, ... are running parallel identical units that have to compete for scarce resources. ... Thus they are likely ... to bias the information they give in order to get the maximum of material resources and personal factors with which to run their sections smoothly” (p.45).

the division managers for scarce resources.

We show that even under the most general contracting assumptions, the information-rent effect weakly dominates the competition effect. When contracts contingent on the division managers' messages to the CEO can be written, the two effects cancel each other. In this case, integration is always optimal because it leads to a better resource allocation without any increase in agency costs. In contrast, when communication is cheap talk and the managers' wages can be based only on the divisions' performance (as we assumed in our discussion above), the information-rent effect strictly dominates. There is then a tradeoff between allocating resources optimally and providing effort incentives, and whether integration or non-integration is optimal depends on the relative importance of these objectives.

We strengthen our result by showing that the *CEO hierarchy* considered so far, in which the managers run the divisions and a CEO allocates resources, is in fact the optimal organizational structure of the integrated firm. We compare it with two alternative forms, a *skewed hierarchy*, in which one of the division managers has authority over all resources, and decentralized *horizontal exchange*, where each manager has authority over part of the firm's resources but can voluntarily lend resources to the other division. We show that in terms of dealing with incentive problems, the CEO hierarchy strictly dominates the skewed hierarchy, which suffers from a conflict of interest. The CEO hierarchy also dominates horizontal exchange, unless resources are highly complementary, in which case horizontal exchange can work equally well.

Our paper makes three contributions. First, we provide a new and simple theory of the benefit and cost of integration. In a nutshell, realizing the benefits of allocating resources by managerial authority requires aggregating dispersed information, but establishing truthful communication comes at the cost of muted incentives.<sup>3</sup> Our theory satisfies the postulate that the costs and benefits of integration originate from the same economic environment (see Hart, 1995, and Gibbons, 2005). It also makes minimal assumptions about the underlying agency problem; our model does not rely on conflicting preferences over decisions, influence activities or exogenous empire-building incentives. All we assume is that managers must be given incentives to create investment opportunities, and have private information about them.

Second, our results concerning the optimality of the CEO hierarchy provide an incentive-based rationale for the hierarchical organization of firms as described by Chandler and others.<sup>4</sup>

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<sup>3</sup> At the same time, we provide a micro-foundation for the use of firm-based ("team") incentives in multidivisional firms. Firm-based incentives are often attributed to the existence of positive externalities without specifying their nature. In our model, the positive externality is the gain from using pooled resources efficiently. It can be realized only if managers communicate truthfully to the CEO, and team incentives help to accomplish this goal.

<sup>4</sup>Most alternative explanations are based on team theory, see Crémer (1980), Aoki (1986), Geanakoplos and

In our theory, a hierarchy with a top manager as pure coordinator avoids the conflict of interest that one or both division managers would have if given the double task of running a division and allocating resources. The endogeneity of the CEO hierarchy also suggests that dispersed information, and not the centralization of authority, is the ultimate source of the costs of integration, which further distinguishes our theory from other explanations.

Third, our model generates several empirically testable predictions: (i) A natural measure of the two units' "relatedness" is the complementarity of their resources. It follows that integration is more likely the more closely related the two units are, consistent with intuition and evidence. (ii) Integration is more likely the more variable the production units' profits are. This follows not from a desire to reduce risk, but from the benefit of putting more resources into the most profitable projects. Moreover, conditional on integration, greater variability of division profits is associated with lower overall wages and a lower relative weight on firm-based incentives. (iii) Integration is more likely the better firms can hold division managers accountable for their claims about investment opportunities (in our model, through the use of message-contingent contracts). Also, conditional on integration, accountability for claims about projects is associated with lower overall wages and a lower weight on firm-based incentives. (iv) Horizontal exchange of resources between divisions is more likely to occur the more related the divisions are, and the more variable their profits are. An example where this prediction is borne out is BP's organization of units into "peer groups" according to their similarity, see Roberts (2004, p.187) and Section 6.

For the remainder of this introduction, we relate our theory to alternative explanations of the costs of integration. We discuss other related literature in Section 2.

The role of information in our theory distinguishes it from an important literature on influence activities as a constraint to integration (Milgrom and Roberts, 1988, 1990; Meyer, Milgrom and Roberts, 1992; Scharfstein and Stein, 2000). In our model division managers' communication is also aimed at influencing the CEO's decision, but it is at the same time a necessary input into the efficient allocation of resources. We can hence spell out the channel through which influence activities affect the decisions of superiors. This channel is modeled as a game of strategic communication in which the CEO is rational and unbiased but must rely on information provided by managers.

Our theory is also distinct from incentive-system theories of the firm. Holmström and Milgrom (1991, 1994), Holmström and Tirole (1991) and Holmström (1999) argue that firms can

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Milgrom (1991), Bolton and Dewatripont (1994) and Garicano (2000). A few agency-based theories explain hierarchies as solutions to moral hazard in team production (Alchian and Demsetz, 1972; Holmström, 1982; Rayo, 2006). In Hart and Moore (2005), a hierarchy leads to an optimal use of assets (resources) when some agents specialize in how to use particular assets, while others look for ways to combine different assets.

solve multitask problems with systems of organizational rules, asset ownership and incentive schemes. The managers' communication in our model could be seen as a second task. But the incentive-system approach takes as given discrepancies between output(s) and measured performance. No such discrepancies exist in our model; realized output is perfectly measurable. Rather, the organization must solve a team production problem in which managers *strategically* communicate private information. With cheap-talk communication, there is also no interaction between productive effort and communication in the managers' utility functions, in contrast to some of the papers mentioned.

Finally, our paper differs from both property-rights theory and the literature on authority in organizations (Grossman and Hart, 1986, Aghion and Tirole, 1997, Hart and Holmström, 2002, Mailath et al., 2005, Dessein et al., 2005) as we emphasize the role of information problems and suggest a different role for authority. In the papers mentioned, shifting authority over some decision from A to B reduces A's opportunities (e.g. to implement his preferred project) and consequently his incentives (e.g. to discover projects). In our model, there is no divergence of preferences between division managers, CEO, and owner, except that the managers' effort is privately costly. If the CEO had perfect information, his resource allocation decisions would *improve* the managers' incentives. Only when the managers have private information does a new agency problem emerge, which leads to the tradeoff between information aggregation and incentives for effort. Our theory thus explains why "selective intervention" has its costs without relying on a divergence of preferences between CEO and the managers.

The next Section continues the review of related literature. Section 3 sets up the model. The analysis of the benefits and costs of integration then proceeds in several steps in Section 4. In Section 5 we study how the optimal firm scope depends on our model's parameters, and derive a number of predictions. In Section 6, we compare centralized resource allocation and horizontal exchange within the firm. Section 7 concludes.

## 2 Related Literature

**Coordination, incentives and communication:** Several papers in the organizational economics literature investigate the tradeoffs between effort provision, decision making and communication. None of these, however, are concerned with firm scope.

In Levitt and Snyder (1997), a principal can get an agent to communicate bad news by paying a reward for the termination of a project, but this undermines the agent's incentives to

come up with a good project in the first place.<sup>5</sup> A similar logic is at work in our model, except that contracts cannot be based on resource allocations or messages. This leaves team-based performance incentives as the only way to establish truthful communication. The multi-agent structure of our integrated firm thus plays an important role for our result.

Athey and Roberts (2001) show that individual performance contracts can lead to distortions in decisions that have externalities on others. They argue that this problem can be alleviated by introducing a top manager as a pure coordinator; they assume he can obtain all relevant information through monitoring. In our model, in contrast, the CEO must obtain information from the managers, who communicate strategically. Nevertheless, as we show in Section 5, a hierarchy with CEO at the top is always an optimal organizational structure.

Dessein, Garicano and Gertner (2005) investigate a firm's choice between adapting products to market conditions and standardizing them to reduce costs. In their model, too, there is a tradeoff between the incentives to exert effort and to communicate information. There are two main differences. First, in our model, centralization (and integration in the first place) is costly *only* because of dispersed information, whereas otherwise the conditions for centralization are as favorable as they can be: our CEO is unbiased, and with perfect information, his decisions would unambiguously improve the managers' effort incentives. In contrast, in Dessein et al. each agent is biased towards one of the two choices, and hence giving one agent the right to decide reduces the others' effort incentives even with perfect information. Second, we endogenize both firm scope and structure. The two production units can be separate firms or integrated into one. For the integrated firm, we show when and why it is optimal to bring in a third player as coordinator. In Dessein et al., the firm's (three-agent) structure is fixed.

Alonso, Dessein and Matouschek (2006) and Rantakari (2006) investigate the tension between adaptation of two divisions' actions to their environment and coordination between the divisions, and ask when decisions should be centralized or decentralized. Alonso et al. show that decentralization can be optimal when coordination is very important; our Proposition 6 makes a similar point. What is very different in our model, though, is the source of the underlying agency problem and the role of incentive contracts. Alonso et al. do not consider monetary incentives and instead assume that managers are biased towards their own division. In our model, managers have no exogenous bias; their motive to communicate strategically is driven entirely by the presence of incentive contracts. By endogenizing the managers' incentive contracts and

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<sup>5</sup>The same tradeoff was also identified by Povel (1999) and Aghion, Bolton and Fries (1999). Povel, for instance, argues that "soft" bankruptcy procedures (such as chapter 11) make it easier for managers to communicate bad news to creditors. This facilitates early interventions, but also diminishes managers' incentives *ex ante*.

thus the extent of the managers' bias we can derive testable predictions about optimal firm scope and structure that depend only on the firm's production technology

**Internal capital markets:** There are intersections between our theory and the literature on internal capital markets. A fundamental difference, however, is that capital is typically contractible, while the resources in our model are not. Thus, like in other theories of the firm, integration in our model has its roots in contracting limitations. Without that assumption, integration would be either unnecessary or always optimal; leaving little room for a theory of the firm. Indeed, most work on internal capital markets takes multidivisional firms as given and hence does not run into this problem.

Also, most of the literature (except for Inderst and Klein, 2006) assumes that managers are "empire builders" who derive utility directly from the size of their budget or their division. These empire-building tendencies can sometimes be mitigated through incentive contracts. We rely instead on the assumptions of standard incentive theory that agents like money and dislike effort. An empire-building motive emerges endogenously; it is *caused* by incentive contracts that place a large weight on individual performance.

Stein (1997) formalized Williamson's (1975) conjecture that internal capital markets create value by channeling capital to the most productive divisions. We adopt Stein's production technology, but otherwise our assumptions are very different. Stein does not consider effort provision and incentive contracts. Most importantly, he assumes that information is generated through monitoring (see Gertner, Scharfstein and Stein, 1994). In our model information is communicated strategically by division managers; strategic communication is the source of the costs of integration.

More recent work focuses on agency problems at the division level. In Scharfstein and Stein (2000), inefficiencies in an internal capital market result from division managers' influence activities. Stein (2002) and Inderst and Laux (2005) extend the analysis of Stein (1997) by assuming that project qualities are determined by the agents' effort. In both papers, headquarters' winner-picking leads to a positive competition effect on managers' effort, as in our model. But we can show that if managers communicate strategically and can lie about their projects, the associated information rent effect always cancels, and often dominates, the competition effect. Papers that explicitly model division managers' private information include Ozbas (2004), Wulf (2005), and Inderst and Klein (2006).<sup>6</sup> Like most of the literature, these papers are concerned with the

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<sup>6</sup> Private information also plays a prominent role in the related capital-budgeting literature, see Harris and Raviv (1996, 1998) and Bernardo, Cai and Luo (2001, 2004). This literature is generally concerned with the efficiency of investment decisions, but not with divisional competition for scarce resources or with firm scope.

efficiency of investment decisions rather than with firm scope.

### 3 Model

#### 3.1 Setting

**Production units:** There are two ex ante identical production units, 1 and 2. They are either independent firms, or divisions of a single integrated firm headed by a CEO. Our goal is to determine which organizational form will maximize shareholder value and will thus emerge in the market for ownership of the units.

We assume that in both cases, ownership and control are separated: the units' managers have no wealth of their own to finance the necessary assets, and investors lack the necessary managerial skills (cf. Stein 1997).<sup>7</sup> This assumption is not essential but facilitates comparison of the two cases.

**Integration:** We define an integrated firm as an entity in which the allocation of decision rights or the incentive contracts of managers differs from the case of independent firms. This definition includes the "CEO hierarchy" that we describe below, as well as alternative structures considered in section 6. These structures typically require common ownership of the two units. However, our definition rules out mere common ownership of the two production units, if otherwise the units operate completely autonomously. Our definition is consistent with the common use of the terms "integration" or "merger", but differs from the definition of a firm in property-rights theory by the common ownership of assets. This change in definition is necessary because of the separation of ownership and control: if owners are not in control, then a change in ownership alone is immaterial to how the two units are run, and does not imply "integration" in any useful sense.

**Resources:** Each production unit, run as an independent firm, is endowed with one unit of resources  $K = 1$ . The resources are specific to the firm; they cannot be obtained in an external market. Resources can be complex physical assets about which it is difficult to write contracts. More intuitively, we think of strategic, organizational or procedural capabilities of the firm's personnel that are embedded in the routines of the organization. Hence, the rents that the resources generate accrue to the firm, not to individuals in the firm. When firms merge, they can pool their resources. Resources can then be allocated to different uses for different product

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<sup>7</sup> If the CEO of an integrated firm were also its owner, nothing would change in our model. If independent firms were run by owner-managers, the analysis would be slightly different but the results still largely the same; see Section 5 below.

lines or divisions. This notion of resources is not widely used in the economics literature (for an exception, see Matsusaka 2001), but is very similar to both Chandler’s (1990) notion of “organizational capabilities” and to the concept of “unique resources and capabilities” in the strategic management literature.<sup>8</sup>

**Contracting:** We assume that one cannot write contracts about the precise use of resources. This rules out bilateral spot contracting between the units, as well as compensation contracts for managers that are conditioned on the use of the resources.<sup>9</sup> What can be contracted upon, however, is the *authority* over how to allocate resources. Our assumptions follow a literature that assumes that authority over actions is contractible while actions themselves are not (see Aghion et al., 2004, and the survey of Dewatripont, 2001), which has proven to be a simple and tractable way to model frictions in the marketplace. In contrast to most of this literature, however, incentive contracts are a crucial component of our theory; more on these below.

**Managerial effort, projects, payoffs:** Each production unit is run by a manager, whose job is to create profitable investment opportunities, or “projects”. Once a project has been created, it requires resources to be carried out; the payoff depends on the quality of the project and the resources invested.

Specifically, unit  $i$ ’s manager chooses between high ( $e_i = 1$ ) or low effort ( $e_i = 0$ ), his choice is unobservable. The manager’s cost of low effort is zero; the cost of high effort is  $c > 0$ . High effort generates a good project (type  $\theta_i = G$ ) with probability  $p$ , and a bad one (type  $\theta_i = B$ ) with probability  $1 - p$ . Low effort generates a good project with probability  $q$ , and a bad one with probability  $1 - q$ , with  $q < p$ . Let  $\boldsymbol{\theta} = (\theta_1, \theta_2)$ .

Resources and projects translate into expected payoffs as in Stein (1997), except that we introduce some noise. The resource investment in any project can be either 1 or 2; a zero investment has a zero return. If an amount of  $k_i \in \{1, 2\}$  is invested in a bad project in unit  $i$ , the resulting *expected* payoff  $z_i$  is  $y_{k_i}$  with  $y_2 > y_1 > 0$ . A good project has an expected payoff of  $\varphi y_{k_i}$  for  $\varphi > 1$ :

$$z_i(k_i, \theta_i) = \begin{cases} \varphi y_{k_i} & \text{if } \theta_i = \text{“G”} \\ y_{k_i} & \text{if } \theta_i = \text{“B”} \end{cases} \quad \text{for } k_i = 1, 2 \quad (1)$$

We assume that the production process is noisy; a “full support” property ensures that no

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<sup>8</sup>See e.g. Penrose (1959), Teece (1982), Wernerfelt (1984) and Barney (1991).

<sup>9</sup>The exact cause of contracting difficulties can take different forms. For example, suppose firm A has a unique capability in say, engineering. Even if one could attribute this capability to a particular department within the firm (as argued above, capabilities of individuals would not count as firm-specific), the firm would be reluctant to rent the department to another firm B for fear of leakage of know-how or ideas. On the other hand, the usefulness of firm A’s capability in firm B’s business may well be a reason for firms A and B to merge.

direct inferences can be made about  $k_i$  or  $\theta_i$  from the observed  $\tilde{z}_i$ . Specifically, the *actual* payoff for each unit, denoted  $\tilde{z}_i$ , is either  $\mu$  or 0; let  $\tilde{\mathbf{z}} = (\tilde{z}_1, \tilde{z}_2)$ . The probability of the event  $\tilde{z}_i = \mu$  is given by  $z_i(k_i, \theta_i)/\mu$ , where  $\mu$  is assumed to be large enough so that all  $z_i/\mu$  are less than 1. It follows that  $[z_i(k_i, \theta_i)/\mu]\mu + [1 - z_i(k_i, \theta_i)/\mu]0 = z_i(k_i, \theta_i)$  is indeed the expected payoff.

We make two assumptions:

**Assumption 1**  $1 < y_1 < y_2 < 2$ .

**Assumption 2**  $\varphi y_2 > (1 + \varphi)y_1$ .

These assumptions imply that it is optimal to invest one unit of resources in a bad project, and two in a good one; this follows because Assumption 2 and  $y_1 > 1$  imply  $\varphi y_2 - 2 > \varphi y_1 - 1$ . In particular, both types of projects have a positive NPV. Further, Assumption 1 implies that for each project type there are decreasing returns to resources invested (since  $2y_1 > y_2$ ). Hence, given two equally good or bad projects, it is better to invest 1 in each project instead of 2 in one of them.<sup>10</sup> Assumption 2, in turn, implies that project payoffs are supermodular in project quality and resources: if there are both a good and a bad project to invest in, it is optimal to invest 2 units in the good project rather than 1 in each, which is essential for any benefit from integration to exist.

Summarizing, the assumptions imply that the efficient way to allocate two units of resources is given by

$$k^*(\boldsymbol{\theta}) = \begin{cases} k_1 = k_2 = 1 & \text{if } \theta_1 = \theta_2 = \text{“G” or if } \theta_1 = \theta_2 = \text{“B”} \\ k_1 = 2, k_2 = 0 & \text{if } \theta_1 = \text{“G” and } \theta_2 = \text{“B”} \\ k_1 = 0, k_2 = 2 & \text{if } \theta_1 = \text{“B” and } \theta_2 = \text{“G”} \end{cases} \quad (2)$$

**Managers’ preferences:** Unit  $i$ ’s manager is risk-neutral but protected by limited liability. The manager’s utility is given by  $U_i(w_i, e_i) = w_i - ce_i$ , where  $w_i$  is the monetary wage and  $ce_i \in \{0, c\}$  is the disutility of effort. This also means that the managers are not empire builders; they do not derive utility directly from their resource allocation or payoff. As we will see, however, empire-building motives can emerge endogenously from the design of incentives. We assume the managers’ reservation wages are low enough such that in equilibrium, the managers’ participation constraints are not binding.

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<sup>10</sup>For the same reason, if the projects are unknown but have the same expected quality, then it is best to invest the resources equally instead of putting all in one of the production units. This means that a potential gain from integrating the two production units exists only if whoever allocates resources is informed about the project qualities; there is no gain from allocating resources randomly.

**Contracts:** For simplicity, we restrict the analysis to contracts that are symmetric for both managers. Managers’ wages can be contingent on both production units’ realized payoffs  $\tilde{z}_1$  and  $\tilde{z}_2$ . There is nothing else wages can be based on. Resources are not contractible. Moreover, following Crawford and Sobel (1982) and Dessein (2002), in the bulk of the paper, we assume that communication from a manager to his boss (the owner or CEO of his firm) is cheap talk. That is, a manager’s messages do not cost the manager anything, nor can any contracts be conditioned on them. However, in order to make clear the important role of this assumption, we examine message-contingent contracts as a benchmark case in Section 4.3.

In its most general formulation, performance-based contracts specify a wage for every possible combination of the divisions’ payoffs. With two possible payoffs for each unit, there are four possible outcomes overall. So each manager’s contract can be described by a quadruple of wages. We have completely analyzed this general case. Its results are presented in Appendix B and occasionally we will refer to these results in the main text.

However, the main results are the same and the exposition is greatly facilitated if we restrict attention to wage contracts that are separable in the divisions’ payoffs, i.e. contracts of the form:

$$\tilde{w}_i(\tilde{z}_1, \tilde{z}_2) = \alpha + \beta\tilde{z}_i + \gamma\tilde{z}_j \text{ for } i = 1, 2 \text{ and } j \neq i.$$

Expected wages are then given by  $w_i(z_1, z_2) = \alpha + \beta z_i + \gamma z_j$ .<sup>11</sup> With a non-binding participation constraint, the limited-liability constraint must be binding when contracts are optimal. Since the  $\tilde{z}_i$  can be zero,  $\alpha$  should be set to zero, and since the  $\tilde{z}_i$  can be positive,  $\beta$  and  $\gamma$  must be nonnegative given that  $\alpha = 0$ . Optimal contracts are then completely characterized by the parameters  $\beta$  and  $\gamma$ . Most of the results discussed in the text are derived for such contracts that are additively separable in the divisions’ payoffs.

### 3.2 Independent Firms

When the two units are independent firms, each firm  $i$  can only use its own resources  $K = 1$ ; hence its investment  $k_i$  is constrained by  $k_i \leq 1$ . The timing of events is as follows:

1. The owner of firm  $i$  offers her manager a wage contract, which he accepts or rejects.
2. Manager  $i$  exerts effort  $e_i \in \{0, 1\}$ . The manager thereupon learns the profitability of his project  $\theta_i \in \{G, B\}$ , which is his private information.

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<sup>11</sup>We refer to  $\beta$  and  $\gamma$  as “bonuses” since the  $\tilde{z}_i$  are binary, but technically  $\beta$  and  $\gamma$  are *shares* of the units’ payoffs, akin to piece rates.

3. Manager  $i$  invests  $k_i = 1$  in his project.
4. The payoff  $\tilde{z}_i$  is realized, and the manager is paid  $\beta\tilde{z}_i$ .

Notice that it is weakly optimal for firm  $i$ 's owner to give the manager authority over resources, since investing at least 1 unit of resources is always optimal and the manager will do so if given nonnegative incentives based on  $\tilde{z}_i$ . To induce low effort, the owner can simply pay the manager his reservation wage. To induce high effort, the owner can reward the manager for high output. Finally, there is no reason for firm  $i$  to base its manager's wage on  $\tilde{z}_j$  as well as on  $\tilde{z}_i$  (although we allowed for this), since  $\tilde{z}_j$  contains no information about manager  $i$ 's effort. It therefore suffices to pay manager  $i$  a bonus  $\beta \geq 0$  (expressed as share of the payoff) if  $\tilde{z}_i = \mu$ .<sup>12</sup> For each firm  $i$ , the optimal contract that induces its manager to exert high effort solves the problem

$$\begin{aligned}
& \max_{\beta} && (1 - \beta)E_{\theta_i}[z_i(1, \theta_i)|e_i = 1] \quad \text{s.t.} \\
(\text{IC-}e_i) & && \beta E_{\theta_i}[z_i(1, \theta_i)|e_i = 1] - c \geq \beta E_{\theta_i}[z_i(1, \theta_i)|e_i = 0], \\
(\text{LL}) & && \beta \geq 0.
\end{aligned} \tag{3}$$

### 3.3 Integrated Firm

In the integrated firm, the units' resources are pooled, and there is only a joint resource constraint  $k_1 + k_2 \leq 2$ ,  $k_i \in \{0, 1, 2\}$  for investment in the two divisions. Since contracting on resources is infeasible, someone must have the authority to allocate the firm's resources.

We first assume that this is a manager at the top, the CEO; we examine alternative structures in Section 6. We abstract from any agency problems that may arise at the CEO level; it therefore makes no difference whether the CEO is the owner herself, or a hired agent.<sup>13</sup> The purpose of this simplification is to understand what the limits to integration are even if top management pursues value-maximizing actions.

The game under integration differs from non-integration in what happens at stage 3 of the timing:

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<sup>12</sup> Once again, the manager's actual wage is  $\beta\tilde{z}_i \in \{0, \beta\mu\}$ ; the expected wage then is  $\beta z_i$ .

<sup>13</sup>Suppose a hired CEO must exert effort to evaluate information obtained from division managers and to allocate resources, and that this effort is observable. The owner can then simply pay the CEO wage  $\bar{w}_0$  for his effort, plus a very small share of total profits to give him an incentive to allocate resources optimally. As there is no interesting interaction between  $\bar{w}_0$  and the tradeoff between coordination and incentives that is our main concern, we can simplify further by setting  $\bar{w}_0 = 0$ .

- 3a. The division managers simultaneously send costless and unverifiable messages  $\hat{\theta}_i$  about their projects to the CEO. Let  $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ .
- 3b. The CEO allocates resources to the two divisions, subject to the constraint  $k_1 + k_2 \leq 2$  and  $k_i \in \{0, 1, 2\}$ .

Thus, the core problem in the integrated firm is that the division managers have private information about their projects, which the CEO needs to learn to allocate resources efficiently. Aside from the managers' possible incentive to lie, a further complication is that the CEO cannot commit himself in advance to any allocation rule, since resources are not contractible. Instead, he allocates resources ex post so as to maximize the firm's profit net of wages. This means that the CEO's response to the managers' messages becomes part of the contracting problem.

If the owner wants the managers to exert low effort, she can simply pay them their reservation wage — just as under non-integration. This also induces truthtelling because the managers then have no reason to misrepresent their projects. However, if the firm wants to implement high effort, several incentive constraints must be satisfied to ensure both effort and truthtelling, i.e., for a separating equilibrium to exist.<sup>14</sup> Below, we state the owner's optimization problem for this case, for an unspecified set of feasible contracts  $C$ . In subsequent sections, we solve this problem for different assumptions about  $C$ . For any contract  $\zeta \in C$ , denote by  $\bar{w}_i(\theta, \hat{\theta}, \zeta)$  manager  $i$ 's expected wage at stage 3a of the game if his project is of type  $\theta$  and he reports it to be of type  $\hat{\theta}$ , under the assumptions that manager  $j$  exerts high effort and reports his type truthfully, and that the CEO allocates resources according to the efficient rule  $k^*$ . We will occasionally suppress the argument  $\zeta$  where no confusion can arise. The firm's optimal contract inducing high effort, managers' truthtelling, and an efficient allocation of resources by the CEO, then solves the following problem:

$$\max_{\zeta, e_1, e_2, \hat{\theta}_1, \hat{\theta}_2, k_1, k_2} E_{\theta} \{ [z_1(k_1, \theta_1) + z_2(k_2, \theta_2)] - \bar{w}_1(\theta_1, \hat{\theta}_1, \zeta) - \bar{w}_2(\theta_2, \hat{\theta}_2, \zeta) \mid e_1 = e_2 = 1 \} \quad \text{s.t.}$$

$$\text{(IC-e)} \quad p\bar{w}_i(G, G, \zeta) + (1-p)\bar{w}_i(B, B, \zeta) - c \geq q\bar{w}_i(G, G, \zeta) + (1-q)\bar{w}_i(B, B, \zeta) \quad \text{for } i = 1, 2$$

$$\text{(IC-G)} \quad \bar{w}_i(G, G, \zeta) \geq \bar{w}_i(G, B, \zeta) \quad \text{for } i = 1, 2$$

$$\text{(IC-B)} \quad \bar{w}_i(B, B, \zeta) \geq \bar{w}_i(B, G, \zeta) \quad \text{for } i = 1, 2$$

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<sup>14</sup> The utility functions of managers with good and with bad projects are identical, which would make separation infeasible in most cheap-talk games. Here, separation is possible because the managers differ in their ability to generate profits for the firm. See Fingleton and Raith (2005) for a model of delegated bargaining that exhibits the same properties.

$$\begin{aligned}
(\text{RA}) \quad \mathbf{k} &= \mathbf{k}^*(\hat{\boldsymbol{\theta}}) = \arg \max_{k'_1, k'_2} [z_1(k'_1, \theta_1) + z_2(k'_2, \theta_2) - \mathbb{E}[\tilde{w}_1(\tilde{\mathbf{z}}|k'_1, \theta_1), \zeta] - \mathbb{E}[\tilde{w}_2(\tilde{\mathbf{z}}|k'_2, \theta_2), \zeta]] \\
&\text{s.t. } k_1 + k_2 \leq 2, k_i \in \{0, 1, 2\} \\
(\text{LL}) \quad \tilde{w}_i(\tilde{\mathbf{z}}|k_i, \theta_i, \zeta) &\geq 0 \quad \text{for } i = 1, 2.
\end{aligned} \tag{4}$$

Condition (IC-e) is the standard effort incentive constraint of manager  $i$ . The ancillary assumptions that agent  $j \neq i$  also exerts high effort, that both managers report truthfully, and that resources are allocated according to  $\mathbf{k}^*$ , are embodied in the definition of  $\tilde{w}_i$  above. Conditions (IC-G) and (IC-B) ensure that manager  $i$  reports his project type truthfully, depending on his project type. Finally, (RA) states as condition that the CEO allocates resources to maximize the firm's net profit, and in doing so implements the efficient allocation rule  $\mathbf{k}^*$ .

## 4 Benefits and Costs of Integration

### 4.1 Independent Firms

Determining each firm's optimal incentive contract for its manager is straightforward:

**Lemma 1** *With independent firms, the optimal contract for each division manager that leads to high effort is given by*

$$\beta^{ni} = \frac{c}{(p-q)(\varphi-1)y_1} \quad \text{and} \quad \gamma^{ni} = 0. \tag{5}$$

For all proofs, see Appendix A.

We already discussed the optimality of  $\gamma^{ni} = 0$ . The optimal bonus for own output,  $\beta^{ni}$ , is increasing in the cost of effort  $c$ , and decreasing in the marginal effectiveness of effort in generating a good project  $(p-q)$  and the difference in the marginal profitabilities between a good and a bad project  $((\varphi-1)y_1)$ . Separability of the wage function is no restriction under non-integration; the optimal contract is the same in the more general case.

As usual, for any interesting agency problem to exist, we need to assume that under first-best conditions, high effort is optimal, which is the case if  $c < (p-q)(\varphi-1)y_1$ . However, if under second-best conditions low effort is optimal, then integration is always optimal because an integrated firm can induce low effort *and truthtelling* for the same flat wage, and thus attain a higher profit because of a better resource allocation. It follows that the comparison between integration and non-integration is interesting only if with independent firms, high effort is optimal under *second-best* conditions. For a reservation wage of zero, the relevant condition is stated in the next result:

**Lemma 2** *With a zero reservation wage, the contract of Lemma 1 is optimal if*

$$\frac{(p-q)^2(\varphi-1)^2y_1}{p(\varphi-1)+1} > c. \quad (6)$$

## 4.2 The Competition Effect of Integration on Incentives

Suppose that the CEO can perfectly observe the project types  $\theta_i$ , while effort remains unobservable. We then obtain the following result; see Stein (2002), Inderst and Laux (2005) or Marino and Zbojnik (2004) for related results:

**Proposition 1** *In an integrated firm in which the CEO has perfect information about  $\theta$ , the optimal separable contract for each division manager is given by*

$$\beta^{pi} = \frac{c}{(p-q)\{(1-p)[\varphi(y_2-y_1)-y_1] + \varphi y_1\}} \quad \text{and} \quad \gamma^{pi} = 0,$$

where  $\beta^{pi} < \beta^{ni}$ .

Notice first that with perfect information, it is again optimal to provide individual incentives only, i.e.  $\beta > 0$  and  $\gamma = 0$ . The divisions are now linked through the allocation of the pooled resources, but manager  $i$ 's effort has a *negative* effect on  $z_j$ : the higher  $e_i$ , the more likely it is that division  $i$  will find a good project, which leads to a lower expected resource investment in division  $j$  and hence a lower expected output  $z_j$ . Paying manager  $i$  a reward  $\gamma > 0$  for division  $j$ 's output would therefore only reduce  $i$ 's incentive to exert effort, whereas  $\gamma < 0$  is not feasible because of limited liability.

The more important part of Proposition 1 is that  $\beta^{pi} < \beta^{ni}$ , which means that compared with independent firms, integration with perfect information leads to *better* incentives for managers in the sense that effort is less costly to induce. Intuitively, while the resource investment always equals 1 in an independent firm, under integration a good project receives expected resources larger than 1, and a bad project resources of less than 1. Since managers are rewarded for division performance, creating a good project that warrants a large investment becomes more valuable to the manager. This effect would be present even without competition between the managers; that is, it would be present even if the same manager ran both divisions.

There is, however, a more subtle effect that is caused by competition for scarce resources. When one manager exerts more effort, this raises his division's expected payoff but it lowers the other division's payoff as explained above. Running two divisions is hence a case of "conflicting tasks" in the sense of Dewatripont and Tirole (1999), and the firm benefits from separating the

tasks into two jobs. In the following we refer to both effects jointly as the *competition* effect of centralized resource allocation on managerial incentives.<sup>15</sup>

### 4.3 Competition vs. Information-Rent Effects: a General Result

Now assume that the information about the divisions' projects is the managers' private information, and that wage contracts can be written contingent on the units' payoffs and on any messages sent by the two managers. Formally, the owner's optimization problem is essentially the same as (4), where  $C$  is the set of all payoff- and message-contingent contracts.

For an independent firm, the contract of Lemma 1 remains optimal; that is, a message-contingent contract cannot improve upon a purely performance-based contract. The reason is that even a bad project is worth carrying out because its NPV is positive; the manager's information is therefore not decision-relevant. Likewise, reducing the manager's income risk by using a message-contingent contract serves no purpose since the manager is risk-neutral.

In the integrated firm, it is a priori unclear whether centralized resource allocation with privately informed division managers leads to higher or lower wage costs for the firm. There are two countervailing effects. One is the competition effect we just discussed, which makes inducing effort less costly compared with non-integration. However, there is also an *information rent effect* that raises wage costs. It arises because managers must be paid a reward for reporting bad projects, which undermines the incentives for effort (see Levitt and Snyder, 1997).

Part (a) of the next result shows that in our model, the information-rent effect always weakly dominates for any contract that induces truth-telling. Part (b) shows that with message-contingent contracts, the two effects exactly cancel each other. In that case, integration is always optimal since the benefit of pooling resources comes at no additional cost.

**Proposition 2** (a) *Any contract that induces truth-telling must lead to a wage bill at least as high as that under non-integration.*

(b) *Consider the contract  $(w^1, w_B)$  given by*

$$w^1 = \frac{c\mu}{(p-q)(\varphi-1)[py_1 + (1-p)y_2]} \quad \text{and} \quad w_B = \frac{c}{(p-q)(\varphi-1)},$$

where  $w_B$  is the payment to a manager who reports a bad project, and  $w^1$  is the payment to manager who reports a good project and whose unit has a high payoff (if the unit's payoff is

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<sup>15</sup>More precisely, one can compute the optimal bonus  $\beta^{sm}$  for a single manager running both divisions (and exerting effort twice). One can then show that  $\beta^{sm} < \beta^{ni}$ , which captures the pure effect of a better resource allocation on incentives for effort. Furthermore, we have  $\beta^{pi} < \beta^{sm}$ , which captures the additional gain for the firm from having two competing managers. Thanks to Ricardo Alonso for his insights here.

zero, the manager gets zero). If  $w^1 \leq \mu(1 - \frac{y_1}{\varphi(y_2 - y_1)})$ , then  $(w^1, w_B)$  leads to high effort and an efficient resource allocation. (Otherwise, it is not a feasible solution of (4) as the CEO would misallocate resources to save the firm payments of  $w^1$ ). The resulting expected wage payments are the same as under non-integration, which because of part (a) means it is an optimal contract.

What plays a key role in part (a) of Proposition 2 is the strong incentive for a manager with a bad project to claim that his project is good. Specifically, we show that the expected wage of a manager who falsely claims to have a good project,  $\bar{w}_1(B, G)$ , is always at least  $1/\varphi$  times the expected wage of a manager who indeed has a good project, i.e.  $\bar{w}_1(G, G)$ . The incentive constraints for truthtelling and effort provision then jointly imply that the firm must pay the managers at least as much under integration with truthful upward communication as they receive in non-integrated firms.

As the proof shows, whether the wage bill under integration is the same as or greater than under non-integration hinges on whether the condition  $\bar{w}_1(G, G) \leq \varphi \bar{w}_1(B, G)$  holds with equality or not. It holds with equality for the contract stated in the proposition. The inequality is strict when the set of feasible contracts is more restricted. As we will see, whenever getting a manager with a bad project to report truthfully require paying him a bonus for the other division's payoff, the above condition holds strictly (see the proof of Proposition 2), and the resulting wage bill is strictly higher than under non-integration.

Proposition 2 thus sheds light on the interaction of two opposite effects identified in the literature: we know from Stein (2002) and others that competition for resources can have a positive effect on incentives, and we know from Levitt and Snyder (1997) and Dessein et al. (2006) that inducing truthful communication can have a negative effect on incentives. Proposition 2 shows that the latter always weakly dominates the former in our framework.

#### 4.4 Integration With Strategic Information Transmission

When the managers' communication is cheap talk, only performance-based contracts are feasible. The managers' bonuses for each division's payoff must then be structured in a way to ensure that managers report their projects truthfully:

**Lemma 3** *For any contract  $(\beta, \gamma)$ , and assuming the CEO believes that the division managers' reports are truthful and allocates the firm's resources according to  $k^*$ , a manager with a bad project has an incentive to report his type truthfully if and only if*

$$\frac{\gamma}{\beta} \geq \frac{p(2y_1 - y_2) + y_2 - y_1}{y_1 + p[\varphi(y_2 - y_1) - y_1]}, \quad (7)$$

and a manager with a good project has an incentive to report his type truthfully if and only if

$$\frac{\gamma}{\beta} \leq \frac{\varphi[y_2 - y_1 + p(2y_1 - y_2)]}{(1-p)y_1 + p\varphi(y_2 - y_1)}. \quad (8)$$

The right-hand side of (7) is between 0 and 1, and the right-hand side of (8) is greater than 1.

Condition (7) implies, in particular, that individual incentives alone (i.e.  $\beta > 0$  and  $\gamma = 0$ ) can never elicit truthful reports, since a manager with a bad project always has an incentive to claim that his project is good, in order to receive resources. In this sense, the provision of performance incentives endogenously generates “empire-building” behavior, even though the managers do not derive any intrinsic utility from the resources they receive.<sup>16</sup> Combining the truth-telling constraints of Lemma 3 with the managers’ effort incentive constraints leads to the main result of this section:

**Proposition 3** *In an integrated firm, the optimal contract for each division manager that leads to high effort, truthful reports about projects, and an efficient resource allocation, is given by*

$$\beta^{int} = \frac{c}{(p-q)(\varphi-1)[(1-p)y_2 + py_1]} \quad \text{and} \quad (9)$$

$$\gamma^{int} = \frac{c[p(2y_1 - y_2) + y_2 - y_1]}{(p-q)(\varphi-1)[(1-p)y_2 + py_1][p\varphi(y_2 - y_1) + (1-p)y_1]}, \quad (10)$$

where  $\beta^{int} \in (\beta^{pi}, \beta^{ni})$  and  $\gamma^{int} > 0$ . The expected wage per agent is strictly higher than under non-integration.

Although a manager with a bad project stands to benefit from resources invested in his division, he also knows that the firm’s resources can be more profitably invested in the other division than in his own. The key is then to let the manager participate in this gain. While the firm cannot reward the manager directly based on the report of a bad project or the allocation of resources, it can reward him indirectly in the form of incentive pay based on the other division’s output (or equivalently, the firm’s profit).

The downside is that rewarding manager  $i$  for division  $j$ ’s good performance raises the wage costs for the firm. Worse, since manager  $i$ ’s effort has a negative effect on  $z_j$  (see Section 4.2), rewarding him for  $j$ ’s good performance *reduces* his incentives to exert effort. Thus, as  $\gamma$  is raised from zero to satisfy the manager’s truth-telling constraint,  $\beta$  must be raised (starting from  $\beta^{pi}$ ) as well to maintain the manager’s incentive to exert high effort. Although the resulting optimal  $\beta^{int}$  is still lower than the bonus  $\beta^{ni}$  required under non-integration, having to pay  $\gamma^{int} > 0$

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<sup>16</sup> This extreme case of a tradeoff between effort and truth-telling incentives follows from the non-contractibility of resources, which explains the contrast with a result of Levitt and Snyder (1997).

for the other division’s good performance leads to an expected wage bill for the firm that is strictly higher than under non-integration. In other words, the competition effect of centralized coordination on the managers’ incentives is outweighed by the information rent effect.

Propositions 1 and 3 and Lemma 3 are illustrated in Figure 1. Lemma 3 characterizes a cone (shaded in light gray) in which  $\gamma/\beta$  must lie to induce truthful reports about project types. The effort-incentive constraint (IC-e) in (4) defines feasible combinations of  $\beta$  and  $\gamma$  that induce high effort (shaded in medium gray), conditional on truthtelling by both managers. The dashed line represents one of the firm’s isoprofit curves, which have a slope of  $-1$  (assuming high effort and truthtelling at each point); the lower the curve, the higher the profit. The isocurves for expected wages look the same but are ordered in the opposite direction.

Under non-integration, the optimal contract is given by  $(\beta^{ni}, \gamma^{ni})$ ; the incentive constraint for effort (not depicted) is a vertical line through that point. Under integration with perfect information, the effort incentive constraint changes to the line IC-e depicted, as a result of the competition effect. The profit-maximizing contract in this case is  $(\beta^{pi}, \gamma^{pi})$ . With strategic communication, the contract must lie in the dark-shaded area in order to satisfy both effort and truthtelling constraints. The profit-maximizing point in that area is  $(\beta^{int}, \gamma^{int})$ , which is associated with a higher wage bill than the contract  $(\beta^{ni}, \gamma^{ni})$  under non-integration.

**Results with general contracts.** The most general (non-separable) contracts can be characterized by  $(\beta, \gamma, \delta)$ . Manager  $i$  gets paid  $\beta$  (as before, as a share of  $\mu$ ) if only  $\tilde{z}_i = \mu$ ,  $\gamma$  if only  $\tilde{z}_j = \mu$ , and  $\delta$  if both units have a high payoff (as before, it is optimal to pay zero if neither does). New incentive constraints come into play: with separable contracts, the CEO’s incentive to allocate resources efficiently never poses a binding constraint because the firm’s net profit is simply  $1 - \beta - \gamma$  times expected total payoff. With general contracts that is no longer the case. For instance, if  $\delta$  is too large, the CEO might be tempted to allocate all resources to one division — leaving the other with no investment to make — even when doing so is inefficient, simply to prevent an outcome in which both units have high payoff. Likewise, if  $\beta$  is large but  $\delta$  small, the CEO might be tempted to allocate the resources equally even when the projects are not equally good, to influence the outcome towards one where both divisions have a high payoff rather than only one.

The resulting solutions for an optimal contract are either interior solutions with  $\beta, \gamma, \delta > 0$ , or have one of the variables set to zero. There are two main cases to distinguish. As is explained in greater detail in Appendix B, if  $p < 1/(1 + \varphi)$ , then  $\delta$  has a positive effect on the truthtelling constraint of a manager with a bad project. It is then possible to induce truthtelling by paying a large enough  $\delta$ , whereas high effort can be induced with  $\beta > 0$ . The optimal  $\gamma$  in this case is

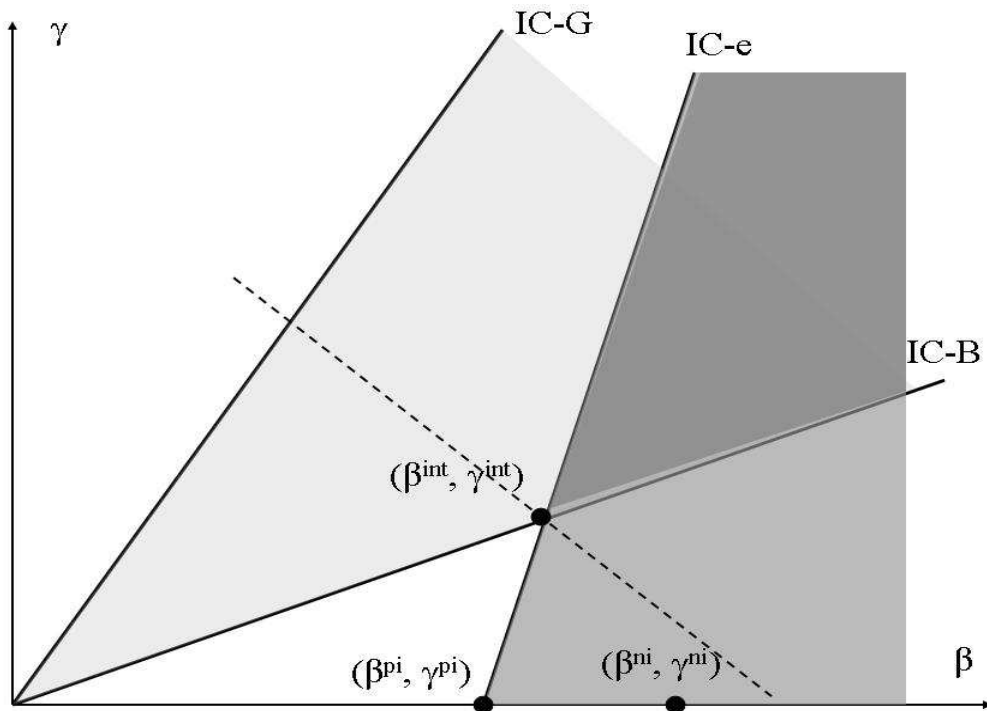


Figure 1: Truthtelling constraints ( $IC-B$ ) and ( $IC-G$ ) and effort incentive constraints ( $IC-e$ ) as functions of  $\beta$  and  $\gamma$ .

zero; it is therefore not necessary to pay a manager whose unit does not produce. Consequently, the managers' information can be elicited without additional cost, implying that integration is always optimal, cf. the discussion in Section 4.3. If  $p > 1/(1 + \varphi)$ , in contrast, then  $\delta$  has a negative effect on the truthtelling constraint of a manager with a bad project. In this case, inducing truthtelling requires setting  $\gamma > 0$ , and the resulting wage bill is strictly higher than under non-integration, like in Proposition 3.

To conclude, inducing high effort and truthtelling in general requires higher wages than are needed in independent firms, which may or may not be worth paying from the perspective of the firm's owner. That is, even when incentives for high effort are optimal in independent firms, a constant wage that induces low effort may be optimal in an integrated firm. In a model with continuous instead of binary effort, the owner's response to a higher cost of inducing effort under integration would be to offer a lower reward for a high payoff ("lower-powered incentives") in any case. <sup>17</sup>

<sup>17</sup> We have studied a variant of our model in which effort is continuous. For the model to be tractable, we then need to assume that the project qualities are exogenous rather than determined by effort, and that payoffs depend

## 5 Optimal Firm Scope

We assume that the organizational form most likely to emerge as the equilibrium in a market for ownership of the production units is the one that maximizes total firm value. In our model – in which there is no debt – this is simply shareholder value. With separate ownership and control, the profit of the owner of an independent firm is the expected payoff minus the manager’s wage; if the owner ran the firm herself, the relevant profit would be the total surplus created. For the integrated firm, the separation of ownership and control does not matter for the calculation of profit, since we disregard the CEO’s compensation. Thus, whether the CEO is the owner herself or an agent, the relevant profit is the firm’s expected total payoff minus the division managers’ wages.<sup>18</sup>

Recall that in our theory integration is defined not just by the joint ownership of the units, but by an organizational structure or by incentive contracts that allow the firm’s resources to be shifted across divisions.<sup>19</sup> Recall also from Section 4.1 that non-integration with low effort and constant wages is strictly dominated by integration with low effort. This leaves three solutions that can be optimal, and the Proposition that follows states how the optimal solution depends on the model’s parameters:

1. Integration with high effort and truth-telling according to Proposition 3,
2. Integration with low effort ( $\beta = \gamma = 0$ ) and truth-telling,
3. Non-integration with high effort according to Lemma 1.

**Proposition 4** *(a) For any given  $y_1$ ,  $y_2$ ,  $\varphi$ ,  $q$ , and  $\mu$ , there exists  $p_0 < 1$  such that non-integration with high effort dominates integration with high effort for all  $p > p^0$ . Moreover,*

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*additively* on project quality and effort, similar to Dessein et al. (2005). In that modified model, our conjecture above indeed holds; the reward for a high payoff is always lower under integration than under non-integration. But although this result is appealing, overall the modified model leads to less rich implications than our main model because of the assumed exogeneity of project qualities.

<sup>18</sup> The rents that the managers receive are not to be included in the firm value. If they were — amounting to some kind of *stakeholder* value calculation —, integration would always be optimal since total surplus can only increase when resource allocation is improved while effort is held at a high level. But that would also amount to assuming away agency problems, since the tradeoff between efficiency and rent extraction is the essence of any agency problem when agents have limited liability.

<sup>19</sup> If, in contrast, there are unmodeled reasons for the two units to be integrated, then the tradeoffs emphasized in this paper imply a choice between centralized decision-making with lower effort incentives, and decentralized decisions with higher effort incentives.

for any given  $y_1$ ,  $y_2$ ,  $\varphi$ ,  $q$ , and  $\mu$ , if  $\varphi$  is large enough, then non-integration with high effort dominates integration with low effort.

(b) For any given  $y_1$ ,  $y_2$ ,  $p$ ,  $q$ , and  $\mu$ , there exists  $\varphi_0 > 1$  such that integration with high effort is optimal for all  $\varphi > \varphi_0$ .

(c) For any given  $y_1$ ,  $\varphi$ ,  $p$ ,  $q$ , and  $\mu$ , if integration with high or low effort dominates non-integration for some  $y_2 \in (y_1, 2)$ , then the same is true for all  $y_2' > y_2$ .

Proposition 4 is illustrated in Figure 2.<sup>20</sup>

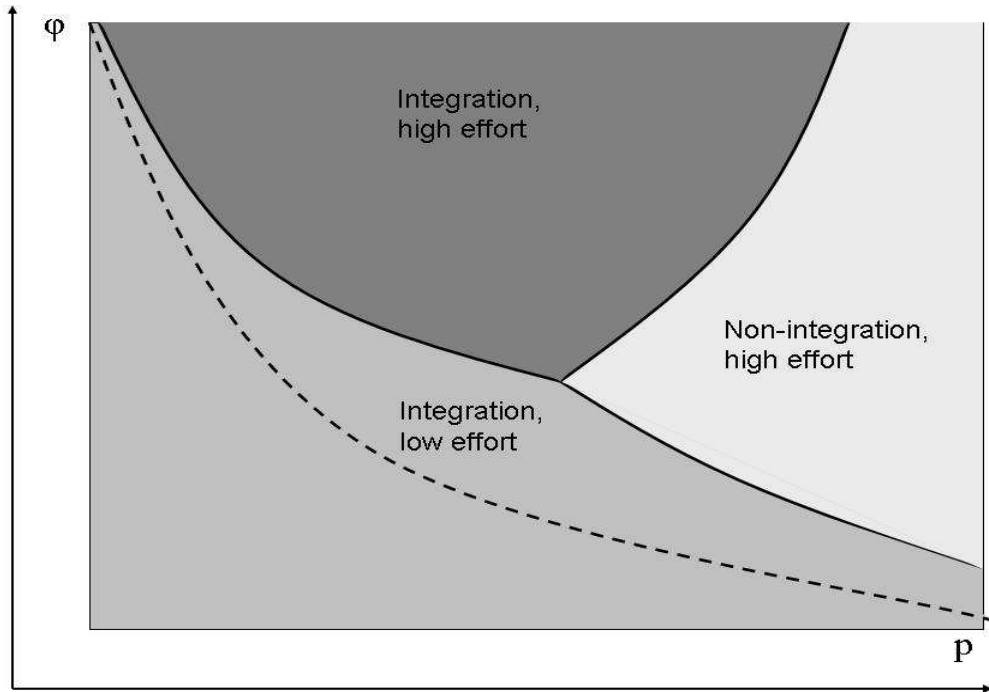


Figure 2: *Optimal organizational form as function of  $p$  and  $\varphi$ . Light gray: non-integration; medium gray: integration with zero wages (low effort); dark gray: integration with team incentives (high effort)*

The dashed line is the set of  $(p, \varphi)$  pairs for which condition (6) holds with equality, that is, where high and low effort lead to equal profits under non-integration. It is downward-sloping because high effort is better the larger the resulting probability of having a good project ( $p$ ), or its value ( $\varphi$ ).

<sup>20</sup> The figure is based on plot obtained by fixing  $q = 0.2$ ,  $y_1 = 1.01$ ,  $y_2 = 1.9$ , and  $c = 0.2$ , and letting  $p$  vary between .5 and 1, and  $\varphi$  between 1.5 and 3.

Below the dashed line, low effort is optimal under non-integration; as we discussed, it follows that integration with low effort is strictly optimal. By continuity, the same holds over some range of  $p$  and  $\varphi$  above the dashed line, where *high* effort is optimal under non-integration.

As  $p$  increases, eventually non-integration with high effort becomes optimal, see part (a) of Proposition 4 and the light-gray area in Figure 2. This is intuitive in comparison with integration with low effort, since a higher  $p$  means a greater value of providing incentives for effort. But compared with integration and high effort, too, non-integration must eventually dominate for  $p$  sufficiently large: the higher  $p$  (approaching 1), the more likely it becomes that both managers have a good project, in which case there is no need to shift resources to one division. Thus, as  $p$  grows, the benefit of integration shrinks to zero. The costs of integration, meanwhile, remain positive because  $\gamma/\beta$  must remain strictly positive in order to induce truth-telling, and the bonus  $\gamma$  must be paid whenever the “other” division has a high payoff.<sup>21</sup>

As  $\varphi$  increases, eventually integration with high effort becomes optimal, as stated in part (b) of Proposition 4 and depicted by the dark-gray area in Figure 2. Intuitively, integration with high effort eventually dominates integration with low effort, because a greater value of having a good project raises the value of providing incentives to create good projects. Also, integration with high effort eventually dominates non-integration (with high effort). To see why, observe that the integrated firm’s ability to move all resources to one division is relevant only when that division has a good project and the other a bad one, in which case the benefit of moving resources is proportional to  $\varphi y_2 - (1 + \varphi)y_1$ , which is positive according to Assumption 2, and increasing in  $\varphi$ .<sup>22</sup>

**Predictions:** Since  $\varphi$  measures by how much good and bad projects differ in profitability,  $\varphi$  can be considered a measure of the variability of the divisions’ payoffs. We thus obtain the first prediction: the more variable profits in two related production units are, the more likely they are to be integrated, because a greater variability means a greater benefit from shifting resources to the most profitable projects. Moreover, it is straightforward to establish, using (9), that  $\beta^{int}$ ,  $\gamma^{int}$  and  $\gamma^{int}/\beta^{int}$  are all decreasing in  $\varphi$ . We thus obtain the prediction that conditional on integration, both total wages and the relative weight on firm-based incentives are decreasing in

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<sup>21</sup> Figure 2 suggests stronger predictions than are expressed in Proposition 4: whether non-integration or integration is optimal depends *monotonically* on  $p$  and  $\varphi$ , whereas parts (a) and (b) of the proposition only express limit results. The stronger predictions are consistent with all of our numerical simulations, but we have been unable to prove them generally.

<sup>22</sup> With continuous instead of binary effort, the high- and low-effort regions for integration would merge into one region, in which incentives are always weaker than under non-integration but still vary with  $p$  and  $\varphi$  for the same reasons as described above.

the variability of division profits.<sup>23</sup>

Not depicted in Figure 2 is part (c) of Proposition 4: integration is more likely the larger  $y_2$ . This is intuitive since  $y_2$  is the expected payoff of investing all resources in one project, which is possible only under integration. With homogeneous resources,  $y_2$  can be simply thought of as a measure of scale economies. Suppose, however, that firm's resources are not homogeneous, and that instead each unit of resources is partly specific to its original production unit. Then  $y_2$  can be interpreted as a measure of "relatedness" of the two divisions, since it measures the value of using one division's resources in the other division. We thus obtain the intuitive prediction that integration is more likely the more related the two production units are. This prediction is consistent with a large literature, beginning with Montgomery and Wernerfelt (1988), that documents that more widely diversified firms tend to be less highly valued.<sup>24</sup>

Also, recall from Propositions 2 and 3 that integration with high effort entails additional agency costs (reflected in a larger wage bill) if communication is cheap talk, whereas there is no additional cost if message-contingent contracts are feasible. We can thus infer as a prediction that integration is more likely the easier it is to implement contracts and procedures that enable firms to hold managers accountable for their claims about their investment opportunities. The actual use of message-contingent contracts may be rare, but such contracts are approximated when managers whose claims about investment opportunities are not substantiated by results suffer a reputation loss — in other words, when talk is not cheap.

We would like to point out that the separation of ownership and control is not an important part of our story. Suppose that under non-integration both units are run by owner-managers. Then integration is optimal if the resulting profit exceeds the owner-managers' total profits, which in this case is expected output minus the cost of effort. There are two possible arrangements. One is that upon integration, the previous owner-managers become the division managers as employees of the new owner. Somewhat paradoxically, in this case integration is always optimal. This is because the owner-managers can be paid off with the rents they receive as agents of the integrated firm (if necessary, by adding a salary to the total wage). The owner then has no agency costs of integration to bear, and can pocket the full difference in total

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<sup>23</sup> More precisely, suppose manager  $i$ 's contract takes the form  $w = \beta' \tilde{z}_i + \gamma'(\tilde{z}_i + \tilde{z}_j)$ , where the second term is a payment based on total firm profits. Then the optimal contract parameters are  $\gamma' = \gamma$  and  $\beta' = \beta - \gamma$  with  $\beta$  and  $\gamma$  given by (9), and the relative weight on firm-based incentives is  $\gamma'/\beta' = 1/(\beta/\gamma - 1)$ , which is decreasing in  $\varphi$  whenever  $\gamma/\beta$  is.

<sup>24</sup> Stein (1997) discusses why measuring the relatedness of the divisions simply by the correlation of project qualities would be unsatisfactory. Instead, he assumes that in a more focused firm headquarters is better able to monitor the divisions' project qualities.

surplus between integration and non-integration as profit. A second arrangement is where the owner of the integrated firm hires new managers. In this case, she must pay the new managers' wages on top of paying off the previous owner-managers. Integration is then less likely to be profitable, but can still be optimal as long as the benefit from centralized resource allocation is large enough; replicating the analysis above for this case leads to qualitatively the same results as are depicted in Figure 2.

## 6 Optimality of a Hierarchical Structure

We have assumed that the integrated firm is a pyramidal hierarchy in which a CEO has authority over resources. But why would such a structure be optimal? Is a CEO — a third player — really needed? In this section, we compare the CEO hierarchy with two other structures that do not require a CEO: one is a *skewed hierarchy* in which one of the division managers allocates the firm's resources in addition to the task of running a division. The other is a structure with *horizontal exchange* where each manager retains authority over his division's resources, but may lend his resources to the other division.<sup>25</sup>

We have been ignoring the agents' participation constraints, and will continue to do so. It is rather clear how they would affect the owner's choice of organizational structure: If the division managers in the CEO hierarchy earn substantial rents, then the owner might be able to leave the resource allocation to them (and do without CEO) without paying them any more than before. If the managers' rents are very small, the owner might have to pay one or both of them more under a decentralized structure to compensate them for any increase in their workload. This could, however, be done with a salary component; there would hence be no interesting interaction with incentives.

**Skewed hierarchy:** We allow for the managers' contracts to be different since their jobs are now different. The modified timing of events is as follows:

1. The firm's owner offers each manager  $i$  a contract  $(\beta_i, \gamma_i)$ , which he accepts or rejects.
2. The managers simultaneously exert effort  $e_i \in \{0, 1\}$ . Each manager then learns the profitability of his project  $\theta_i \in \{G, B\}$ , which is his private information.
3. Manager 2 sends a costless and unverifiable message  $\hat{\theta}_2$  about  $\theta_2$  to manager 1.
4. Manager 1 allocates resources subject to the constraint  $k_1 + k_2 \leq 2$  and  $k_i \in \{0, 1, 2\}$ .

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<sup>25</sup> We would like to thank Niko Matouschek for suggesting this second solution.

5. The payoffs  $z_1$  and  $z_2$  are realized, and the managers are compensated.

As for the CEO hierarchy, we look at contracts that lead to high effort and an efficient resource allocation. For manager 2, nothing changes: provided that manager 1 exerts high effort and that resources are allocated efficiently, his effort and truth-telling incentive constraints are the same as before; hence the optimal wage contract for manager 2 is given by Proposition 3. Manager 1's effort incentive constraint remains unchanged, too, provided that manager 2 exerts high effort and reports truthfully, and that manager 1 himself allocates resources efficiently at stage 4 of the game.

The only difference is that manager 1's truth-telling constraint is replaced with incentive constraints that induce him to allocate resources efficiently. Two of these constraints may bind in equilibrium: one ensures that manager 1 distributes the resources equally if both divisions' projects are equally good or bad, instead of allocating all of it to his own division. The other condition ensures that he allocates all resources to division 2 if it has a good project and division 1 a bad one. Both conditions lead to lower bounds on  $\gamma/\beta$ . We can then show the following.

**Proposition 5** *Assume that the owner of an integrated firm wants the managers to exert high effort, and wants resources to be allocated efficiently. Then the incentive constraints in the skewed hierarchy are unambiguously more restrictive than those in the CEO hierarchy.*

The result is the same with general contracts; see Appendix B. The intuition rests on an equivalence between lying to the CEO in one structure and misallocating resources in the other. To make this clear, suppose that in the skewed hierarchy, manager 1 has a bad project and learns that manager 2 has a good project. Manager 1 will allocate resources efficiently if he is better off giving all resources to division 2 rather than dividing the resources equally. The equivalent situation in the CEO hierarchy is a manager 1 with a bad project who — contrary to our assumptions — happens to know that manager 2 has a good project. Manager 1 will then report his type truthfully if he is better off admitting to a bad project than claiming to have a good project, anticipating that the CEO allocates all resources to division 2 in the first case and divides them equally in the second.

Since the outcomes in each structure are the same, the relevant constraints for manager 1 are the same too; the same is true when manager 2 has a bad project. The important difference is that in the CEO hierarchy, manager 1 reports his type *without* knowing manager 2's, which means his truth-telling constraint is a weighted average of the constraints for each type of manager 2. In contrast, in the skewed hierarchy, manager 1 allocates resources *after* learning 2's project

type, which means that each constraint must be satisfied.<sup>26</sup>

The logic of Proposition 5 is reminiscent of Dewatripont and Tirole (1999). The tasks of running a division and of allocating the firm's resources are not directly opposed (like those of prosecution and defense in Dewatripont and Tirole), but are sufficiently misaligned to warrant separation into two jobs. Unlike in Dewatripont and Tirole, however, here the interaction with a third agent (manager 2) plays a critical role.

**Horizontal exchange:** Here, each manager retains authority of his division's resources. The timing in this case differs from the previous one only in stages 3 and 4:

3. Each manager  $i$  sends a costless and unverifiable message  $\hat{\theta}_i$  about  $\theta_i$  to manager  $j \neq i$ .
4. Each manager  $i$  can either use division  $i$ 's resources in his own division, or lend them to division  $j \neq i$ .

It is important to recall that we assume that the use of resources is not contractible. This rules out bilateral trade involving direct transfer payments. It does not, however, rule out the option for a manager to voluntarily provide the resources under his authority to the other division. It is efficient for (say) manager 1 to lend his resources to division 2 if and only if division 1 has a bad project and division 2 a good one. Getting manager 1 to lend his resources voluntarily to division 2 requires a wage contract that allows him to participate in division 2's performance, as is also required in the CEO hierarchy.<sup>27</sup> The conditions for inducing high effort and an efficient resource allocation are thus very similar to those of the CEO hierarchy, and the next result states when they differ:

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<sup>26</sup>We believe that Proposition 5 is not specific to our binary model setup but quite general. First, the arguments above should apply to more general type spaces, since what drives the result is the difference in the timing between the two organizational structures rather than the binary project types. Second, the binary nature of effort matters insofar as Proposition 5 assumes that the owner wants both managers to exert high effort. In a model with continuous effort (see also Footnote 17), we would expect the owner to offer weaker incentives to manager 1 than to manager 2, precisely because his resource allocation incentive constraints are more difficult to satisfy than the corresponding truth-telling constraint in the firm with CEO (which is the same constraint that determines manager 2's optimal contract). But the latter comparison is what matters for Proposition 5; the precise characteristics of the resulting optimal contract for manager 1 is secondary.

<sup>27</sup> Moreover, implementing an incentive contract conditioned on the other unit's performance requires some form of profit sharing between the owners, and thus, realistically, integration: under independent ownership, owner 1 has no reason to pay her manager a reward for 2's good performance unless the owner herself gets a share as well.

**Proposition 6** *Assume the integrated firm's owner wants the managers to exert high effort, and wants resources to be allocated efficiently under horizontal exchange. If  $\varphi \geq y_1^2/(y_2 - y_1)^2$ , then the optimal contract for each manager is given by Proposition 3. If  $\varphi < y_1^2/(y_2 - y_1)^2$ , then the incentive constraints are unambiguously more restrictive than those in the CEO hierarchy.*<sup>28</sup>

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Proposition 6 states that for large enough  $\varphi$ , the incentive-related costs at the division level are the same as in the CEO hierarchy. If employing a CEO entails extra costs (which is plausible even though we have been ignoring them in our analysis), horizontal exchange then dominates. For smaller  $\varphi$ , the CEO hierarchy is strictly optimal in terms of the incentive constraints involved.

The intuition for Proposition 6 is very simple, since horizontal exchange differs from the CEO hierarchy only in the constraints that lead to an efficient resource allocation. That is, assuming that resources are eventually used efficiently (according to  $k^*$ ), both managers' effort and truthtelling incentive constraints are the same as before. Since with separable contracts the resource allocation constraint (RA) is never binding in the CEO hierarchy, it follows that the wage contract of Proposition 3 is also the optimal contract to induce high effort and truthtelling under horizontal exchange.

But achieving an efficient resource allocation under horizontal exchange also requires that a manager with a bad project is willing to lend his resources to the other division if that division has a good project. The relevant constraint is similar but not identical to the truthtelling constraint (IC-B) that we already imposed anyway, and may be more or less restrictive than (IC-B) depending on the magnitude of  $\varphi$ . This leads to the case distinction stated in the proposition. Specifically, the larger  $\varphi$ , the *easier* it is to get a manager to lend his resources to the other division (since the manager stands to gain from a larger  $\varphi$ ), and for  $\varphi \geq y_1^2/(y_2 - y_1)^2$  the corresponding incentive constraint is no longer binding given that (IC-B) must already hold.

Together, Propositions 5 and 6 shed light on why — as emphasized by Chandler — firms are often structured as pyramidal hierarchies, with a top management that is in charge of coordinating a firm's activities but is not directly involved in production. We already know part of the story from Athey and Roberts (2001): incentive contracts that provide good incentives for

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<sup>28</sup>Note that Assumption 2 implies a lower bound of  $y_1/(y_2 - y_1)$  on  $\varphi$ , which given Assumption 1 is less than  $y_1^2/(y_2 - y_1)^2$ .

<sup>29</sup>As far as we can tell, Proposition 6 extends to the more general non-separable contracts, but we have not been able to prove this for all possible cases. We can show that the incentive constraints for horizontal exchange are unambiguously more restrictive when the optimal contract according to Proposition 8 specifies  $\gamma = 0$  or  $\beta = 0$ , while when  $\delta = 0$  that is the case if  $\varphi$  is low enough. These results are available on request. We have not been able to generalize Proposition 6 to the case where  $\beta$ ,  $\gamma$  and  $\delta$  are all positive.

effort tend to provide bad incentives for decisions relevant to other units. The latter decisions are then perhaps best left to an unbiased decision maker. But how does the decision maker obtain the information needed to make good decisions? Athey and Roberts assume this can be done through monitoring, at a fixed cost. In our model, the information must be communicated by division managers, which introduces a new agency problem. Nevertheless, as we show, the associated costs can be less than those of decentralized decision making.

Proposition 6 is related to a result by Alonso et al. (2006) that decentralized decision-making may be preferred to centralization (involving a CEO) if coordination between two divisions is very important. Whether that is the case in their model depends on the managers' bias parameter. Here, in contrast, the condition in Proposition 6 depends only on parameters of the production technology.

Specifically, the condition  $\varphi \geq y_1^2/(y_2 - y_1)^2$  leads to the testable predictions that decentralized resource allocation through horizontal exchange is more likely the more closely related the divisions are (the larger  $y_2$ ), and the more variable the division payoffs are (measured by  $\varphi$ ). While we are not aware of direct evidence linking relatedness and horizontal exchange, our prediction is consistent with broad changes in the way modern firms organize themselves, as described by Roberts (2003, p.2). One is a trend towards less diversification and an increased focus on a firm's core strengths; another is a trend among firms to facilitate horizontal communication and coordination between different units, that is, a decrease in reliance on vertical communication. A more specific example is the case of *BP*, which in the course of extensive restructuring efforts in the 1990s introduced “ ‘peer groups’ that linked assets [e.g. oil fields] facing similar technical and commercial challenges to provide mutual support” (Roberts 2003, p.26 and p.187).

## 7 Concluding Remarks

The purpose of our paper is to shed light on the benefits and costs of integration by focusing on one of the key tasks of managers, the allocation of resources. We have shown that this task is associated with incentive conflicts that do not exist in non-integrated firms. More generally, our focus reflects a belief that has its origins in the work of Coase, Barnard, Simon, Williamson and others: understanding hierarchies, and the role of managers in them as people who coordinate others' actions or resolve conflicts through the use of authority, is key to understanding why firms exist and what determines their boundaries.

Methodologically, we borrow from both the incentive-system and the property-rights theories of the firm. But while each theory emphasizes either incentive contracts or control rights, in

ours the two are inseparable, and are part of the same organizational design problem. We add an element that plays a prominent role of much of recent organizational economics but has been missing in the theory of the firm: the dispersion of information in a firm, the need to communicate critical information to decision makers, and the resulting incentive problems involving the agents who are the sources of information. In our view, this Hayekian perspective is essential to understanding not only how firms work, but why they exist.

It may be considered a limitation of our analysis that we do not consider agency problems between shareholders and top management. Bolton and Scharfstein (1998) discuss how to integrate these problems into the theory of the firm. It seems intuitive, however, that there are limits to organization even in the absence of shareholder-manager conflicts. While the separation of ownership and control creates its own agency problems, “managerial diseconomies of scale” — a subject of much discussion in economics since at least the 1930s — most likely also exist in firms that are run by their owners. As we have seen, focusing the spotlight on agency problems at lower levels in a firm, while assuming that top management is benevolent, leads to many new insights into this old problem.

## Appendix A: Proofs

**Proof of Lemma 1:**  $E_{\theta_i}[z_i(1, \theta_i)|e_i = 1]$  in (3) is given by  $p\varphi y_1 + (1-p)y_1$ , while  $E_{\theta_i}[z_i(1, \theta_i)|e_i = 0]$  equals  $q\varphi y_1 + (1-q)y_1$ . The manager’s incentive constraint can thus be rephrased as  $\beta(p - q)(\varphi - 1)y_1 \geq c$ . Under an optimal contract, this condition must be binding, which leads to the bonus  $\beta$  stated in the Lemma. ■

**Proof of Lemma 2:** If owner  $i$  wants to implement high effort, she needs to pay the bonus  $\beta^{ni}$  given by Lemma 1, and the resulting profit is  $(1 - \beta^{ni})[p\varphi + 1 - p]y_1$ . If the owner wants to implement low effort, no wage needs to be paid (under the assumption that the manager’s reservation wage is zero), and the resulting profit is  $[q\varphi + 1 - q]y_1$ . It is straightforward to show that the difference between these two expressions is positive if and only if (6) holds. ■

**Proof of Proposition 1:** Under perfect information, if manager 1 has a good project, then with probability  $p$  manager 2 has a good project as well, and each is allocated one unit of resources. Manager 1 then earns  $\beta\mu$  if division 1 has a high payoff and  $\gamma\mu$  if division 2 does; the probability of each event is  $\varphi y_1/\mu$ . With probability  $1 - p$ , manager 2 has a bad project. All resources are then allocated to division 1, and manager 1 earns  $\beta\mu$  with probability  $\varphi y_2/\mu$ .

Manager 1's expected wage from having a good project is thus

$$W(G) = \varphi[p(\beta + \gamma)y_1 + (1 - p)\beta y_2]. \quad (11)$$

Similarly, if manager 1 has a bad project, then with probability  $p$  manager 2 has a good one, and all resources go to division 2; whereas with probability  $1 - p$  manager 2 has a bad project as well. Manager 1's expected wage from having a bad project then is

$$W(B) = p\gamma\varphi y_2 + (1 - p)(\beta + \gamma)y_1. \quad (12)$$

These expressions are the same for manager 2, and so each manager will exert high effort if  $pW(G) + (1 - p)W(B) - c \geq qW(G) + (1 - q)W(B)$ , or equivalently,

$$(p - q)[(1 - p)(\varphi y_2 - y_1) + p\varphi y_1]\beta - [p\varphi(y_2 - y_1) + (1 - p)y_1]\gamma \geq c. \quad (13)$$

The term in brackets before  $\gamma$  in (13) is positive; hence,  $\gamma$  has a negative effect on effort incentives.

Next, let us determine the firm's net profit. With separable contracts and for any division  $i$ , the firm pays  $\beta$  (expressed as share of  $\mu$ ) to manager  $i$  and  $\gamma$  to manager  $j \neq i$  if division  $i$  has a high payoff. The firm's net profit therefore is given by  $1 - \beta - \gamma$  times the expected total payoff in the two divisions, where the latter is given by:

$$Z^{int} = 2p^2\varphi y_1 + 2p(1 - p)\varphi y_2 + 2(1 - p)^2 y_1. \quad (14)$$

This expression is obtained as follows: with probability  $p^2$ , both divisions have a good project and receive one unit of resources each. The resulting expected payoff in each division then is  $\varphi y_1$ , which leads to the first term in (14). With probability  $2p(1 - p)$ , one division has a good project and the other a bad one. All resources are allocated to the good project, whose expected payoff is  $\varphi y_2$ . This is the second term in (14). The third term is obtained in the same way as the first. The firm's expected net profit therefore is

$$2(1 - \beta - \gamma)[p^2\varphi y_1 + p(1 - p)\varphi y_2 + (1 - p)^2 y_1], \quad (15)$$

which is decreasing in  $\beta$  and  $\gamma$ .

Since  $\gamma$  has a negative effect on profit *and* on effort incentives, it is optimal to set  $\gamma = 0$ . The optimal  $\beta$  is then obtained by solving (13) as equality for  $\beta$ , setting  $\gamma = 0$ . The result is  $\beta^{pi}$  as stated in the Proposition.

To show that  $\beta^{pi} < \beta^{ni}$ , we need to compare  $(1 - p)[\varphi(y_2 - y_1) - y_1] + \varphi y_1$  in the denominator of  $\beta^{pi}$  with  $(\varphi - 1)y_1$  in the denominator of  $\beta^{ni}$  in (5). The difference between these two expressions equals  $py_1 + (1 - p)\varphi(y_2 - y_1)$ , which is positive, proving our claim. ■

**Proof of Proposition 2:** Instead of deriving the optimal mechanism directly, we first show that no contract can lead to a wage bill that is lower than under non-integration. Then, we show that the wage bill associated with the contract stated equals that under non-integration. Since the principal's profit is expected total payoff minus the expected wage bill, it then follows that the stated contract must be an optimal (though not unique) contract. This indirect strategy of proof makes precise which assumptions of the model drive the result.

If manager 1 exerts high effort and communicates truthfully, his expected wage is

$$p\bar{w}_1(G, G) + (1 - p)\bar{w}_1(B, B) = p[\bar{w}_1(G, G) - \bar{w}_1(B, B)] + \bar{w}_1(B, B), \quad (16)$$

while his effort incentive constraint can be written as

$$\bar{w}_1(G, G) - \bar{w}_1(B, B) \geq \frac{c}{p - q}. \quad (17)$$

Since (17) is binding under non-integration, the difference  $\bar{w}_1(G, G) - \bar{w}_1(B, B)$  in (16) cannot decrease under non-integration, and thus the firm's expected wage payment to manager 1 must increase whenever  $\bar{w}_1(B, B)$  is higher than a bad manager's expected wage under non-integration, which is given by  $\beta^{ni}y_1 = c/[(p - q)(\varphi - 1)]$ , cf. (5).

The key property that our general argument relies on is that

$$\varphi\bar{w}_1(B, G) \geq \bar{w}_1(G, G) \quad \text{or} \quad \frac{\bar{w}_1(B, G)}{\bar{w}_1(G, G)} \geq \frac{1}{\varphi}. \quad (18)$$

To see why, notice that since the resource allocation depends only on the manager's reported type, both a truthful manager with a good project and a lying manager with a bad project receive the same resources. It follows that the two managers' expected payoffs generated by those resources differ by the factor  $\varphi$ , and hence their associated wages do too (regarding the last step, recall that realized output is 0 or  $\mu$ , so there is no way to infer type from output). However, depending on the contract, manager 1 might also be compensated based on unit 2's output. In that case, since a  $(G, G)$ -manager and a  $(B, G)$ -manager benefit from such a payment in the same way, (18) would be strict; however, it can never go the other way around.

The truth-telling constraint for a manager 1 with a bad project is given by

$$\bar{w}_1(B, B) \geq \bar{w}_1(B, G). \quad (19)$$

Then, (19), (18) and (17), respectively, imply that

$$(\varphi - 1)\bar{w}_1(B, B) \geq (\varphi - 1)\bar{w}_1(B, G) \geq \varphi\bar{w}_1(B, G) - \bar{w}_1(B, B) \geq \bar{w}_1(G, G) - \bar{w}_1(B, B) \geq \frac{c}{p - q}, \quad (20)$$

and hence  $\bar{w}_1(B, B) \geq c/[(p - q)(\varphi - 1)]$ . We have thus shown that  $\bar{w}_1(B, B)$  must be weakly greater than a bad manager's expected wage under non-integration. Notice also that the condition  $\varphi\bar{w}_1(B, G) \geq \bar{w}_1(G, G)$  is tight: if  $\varphi\bar{w}_1(B, G) = \bar{w}_1(G, G)$ , the inequality signs in (20) can normally be replaced by equality signs, provided the contract space is large enough (i.e. allows for enough wage variables to specify) to make the effort and truthtelling constraints binding.

Let us now verify that the contract stated in the proposition is a solution to the problem (4). Consider first manager 1's incentive to choose high effort, assuming that manager 2 does too. If manager 1 has a good project, then with probability  $p$  manager 2 does too, and both get one unit of resources. With probability  $1 - p$ , manager 2's project is bad, and manager 1 gets all resources. If manager 1 has a bad project, he gets  $w_B$  for sure. Manager 1 then chooses high effort if (cf. 13)

$$(p - q) \left[ p\varphi \frac{y_1}{\mu} w^1 + (1 - p)\varphi \frac{y_2}{\mu} w^1 - w_B \right] \geq c. \quad (21)$$

Next, consider manager 1's incentive to report his project type truthfully if his project is bad. If he reports a bad project, he gets  $w_B$  for sure. If instead he reports to have a good project, then he receives one unit of resources if manager 2 has a good project (which occurs with probability  $p$ ), and all resources if manager 2 has a bad project (with probability  $1 - p$ ). Manager 1 then reports truthfully if

$$w_B \geq p \frac{y_1}{\mu} w^1 + (1 - p) \frac{y_2}{\mu} w^1. \quad (22)$$

Similarly, if manager 1 has a good project, he will report his type truthfully if

$$p \frac{\varphi y_1}{\mu} w^1 + (1 - p) \frac{\varphi y_2}{\mu} w^1 \geq w_B. \quad (23)$$

The expressions for  $w^1$  and  $w_B$  stated in the proposition are the unique solution of (21) and (22) as equalities. It is easy to verify that this contract also satisfies (23). To complete the proof that the contract is feasible, we need to check the CEO's resource allocation constraint (RA) in (4). If both projects are equally good or bad, the CEO allocates the resources equally as is efficient (for two good projects, this follows from  $2y_1 > y_2$ , cf. Assumption 1). Suppose now that project 1 (say) is good and project 2 bad. If the CEO allocates all resources to division 1, then the firm's resulting expected profit is  $\varphi \frac{y_2}{\mu} (\mu - w^1) - w_B$ , where  $w^1$  is paid to manager 1 if  $\tilde{z}_1 = \mu$  and  $w_B$  is paid to manager 2 with certainty. If instead the CEO allocates the resources equally, then the firm's resulting expected profit is  $\varphi \frac{y_1}{\mu} (\mu - w^1) + \frac{y_1}{\mu} \mu$ . It is then straightforward to show that for the CEO to allocate all resources to division 1,  $w^1$  must not exceed the stated upper bound.

Under non-integration, the expected wage bill per firm is  $\beta^{ni}$  times the expected payoff

$[p\varphi + (1 - p)]y_1$ , which simplifies to

$$\frac{c[p\varphi + (1 - p)]}{(p - q)(\varphi - 1)}. \quad (24)$$

The expected wage bill per manager in the integrated firm is given by

$$\frac{1}{2} \left[ p^2 2\varphi \frac{y_1}{\mu} w^1 + (1 - p)^2 2w_B + 2p(1 - p) \left( \frac{\varphi y_2}{\mu} w^1 + w_B \right) \right]$$

which upon substituting the expressions in the proposition simplifies to (24), i.e. the same as under non-integration. As argued above, it follows that the contract  $(w^1, w_B)$  must be optimal. ■

**Proof of Lemma 3:** For a manager 1 with a *good* project, the expected payoff from reporting truthfully, and under the assumption that manager 2 reports truthfully too, is given by  $\bar{w}_1(G, G) = W(G)$  as given by (11). Suppose manager 1 reports “B” instead. Then with probability  $p$ , manager 2 has a good project, in which case all resources go to division 2 (given that manager 2 reports truthfully and that the CEO assumes truthtelling on part of both managers). With probability  $1 - p$ , manager 2 has a bad project, and each division is allocated one unit of resources. The resulting expected wage for manager 1 thus is

$$\bar{w}_1(G, B) = p\gamma\varphi y_2 + (1 - p)(\beta\varphi y_1 + \gamma y_1). \quad (25)$$

The truthtelling constraint (IC-G) given by  $\bar{w}_1(G, G) \geq \bar{w}_1(G, B)$  can therefore be expressed as

$$\varphi[y_2 - y_1 + p(2y_1 - y_2)]\beta \geq [(1 - p)y_1 + p\varphi(y_2 - y_1)]\gamma, \quad (26)$$

which is equivalent to (8). The difference between the numerator and denominator on the right-hand side of (8) simplifies to  $(1 - p)[\varphi(y_2 - y_1) - y_1] + p\varphi(2y_1 - y_2) > 0$ , which means the fraction is greater than 1.

For a manager 1 with a *bad* project, the expected payoff from reporting truthfully (under the same ancillary assumptions as above), is  $\bar{w}_1(B, B) = W(B)$  as given by (12). Suppose manager 1 reports “G” instead. Then with probability  $p$ , manager 2 has a good project too, in which case each division gets one unit of resources. With probability  $1 - p$ , manager 2 has a bad project, and all resources go to division 1. The resulting expected wage for manager 1 is

$$\bar{w}_1(B, G) = p(\beta + \varphi\gamma)y_1 + (1 - p)\beta y_2. \quad (27)$$

The truthtelling constraint (IC-B) given by  $\bar{w}_1(B, B) \geq \bar{w}_1(B, G)$  can therefore be expressed as

$$\{p[\varphi(y_2 - y_1) - y_1] + y_1\}\gamma \geq [p(2y_1 - y_2) + y_2 - y_1]\beta, \quad (28)$$

which is equivalent to (7). The difference between the numerator and denominator on the right-hand side of (7) simplifies to  $-(1-p)(2y_1 - y_2) - p[\varphi(y_2 - y_1) - y_1] < 0$ , which means the fraction is smaller than 1 (but positive, since both numerator and denominator are). ■

**Proof of Proposition 3:** The optimal separable contract maximizes (15) with respect to  $\beta$  and  $\gamma$ , subject to (i) the effort incentive constraint (13), (ii) the truthtelling constraints (8) and (7), (iii) the constraint (RA) that it must be (ex post) optimal for the CEO to allocate resources efficiently, assuming high effort and truthtelling on part of both managers, and (iv) nonnegativity constraints  $\beta, \gamma, \delta \geq 0$ . This optimization problem is a linear program, and therefore the optimal solution must be a corner point of the parameter set defined by the constraints.

The resource allocation constraint (RA) is never binding with separable contracts. This follows immediately from the fact that the firm's profit is  $1 - \beta - \gamma$  times expected total payoff, which by definition is maximized if resources are allocated efficiently, cf. the discussion in Section 3.1 on the role of our parameter constraints. Thus, for  $\beta + \gamma < 1$ , ex-post profit maximization on part of the CEO leads to an efficient resource allocation, whereas  $\beta + \gamma > 1$  would never be chosen since the resulting profit would be negative.

The relevant constraints are (IC-e) and (IC-B), whereas (IC-G) is redundant. To see why, observe first that the effort constraint (13) must be binding, for otherwise truthtelling would be optimally achieved by setting  $\beta = \gamma = 0$ , when (13) is clearly violated. Second, we already know from Proposition 1 the solution to the relaxed problem in which (IC-G) and (IC-B) are not imposed, and from Lemma 3 we know that it satisfies (IC-G) but not (IC-B). Hence (IC-B) must be binding. Finally, given Lemma 3, any  $(\beta, \gamma)$  that satisfies (IC-B) also satisfies (IC-G). The optimal contract is therefore given by solving (7) and (13) as equalities for  $\beta$  and  $\gamma$ ; the solution is stated in the Proposition.

Since both  $(\beta^{pi}, 0)$  and  $(\beta^{int}, \gamma^{int})$  solve (13) with equality and since (13) is increasing in  $\beta$  and decreasing in  $\gamma$ , it follows that  $\beta^{int} > \beta^{pi}$ , see Figure 2. That  $\beta^{int} > \beta^{ni}$  follows from comparing the expressions for both variables, where  $py_1 + (1-p)y_2$  in the denominator of  $\beta^{int}$  is greater than  $y_1$  in the denominator of  $\beta^{ni}$ , and the expressions are otherwise the same.

The integrated firm's total wage bill is given by  $\beta + \gamma$  times total expected output  $Z^{int}$ , while the total wage bill for two independent firms is given by  $\beta$  times  $2[p\varphi + (1-p)]y_1$ . Upon substituting  $(\beta^{int}, \gamma^{int})$  into the wage bill of the integrated firm and  $\beta^{ni}$  into that of the non-integrated firms, the difference between the two simplifies to

$$2 \frac{cp\varphi y_1}{(p-q)(\varphi-1)} \frac{y_2 - y_1 + p(2y_1 - y_2)}{[py_1 + (1-p)y_2][p(\varphi(y_2 - y_1) - y_1) + y_1]},$$

which under Assumptions 1 and 2 is strictly positive. ■

**Proof of Proposition 4:** (a) Comparing non-integration with integration with high effort, we have that for  $p = 1$ , the expected total payoff of both the nonintegrated firms (given by  $2(p\varphi + 1 - p)y_1$ ) and the integrated firm with high effort (given by  $Z^{int}$  in (14)) equal  $2\varphi y_1$ . Moreover, both  $\beta^{ni}$  and  $\beta^{int}$  equal  $c/[(1 - q)(\varphi - 1)y_1]$ , whereas  $\gamma^{int}$  is strictly positive. Thus, for  $p = 1$  the total profit of the non-integrated firms strictly exceeds the integrated firm's profit. By continuity, the same holds for some interval of  $p$  close enough to 1, which establishes our claim.

Comparing non-integration with integration with low effort, observe that the non-integration profit is increasing in  $p$  while the integration effort does not depend on  $p$ . It follows that for large enough  $p$ , non-integration dominates as long as the non-integration profit for  $p = 1$  exceeds the integration profit. The difference between these profits is given by

$$2\varphi \left( 1 - \frac{c}{(1 - q)(\varphi - 1)y_1} \right) - 2[(1 - q)^2 y_1 + q^2 \varphi y_1 + q(1 - q)\varphi y_2],$$

which (using Assumption 1) is increasing in  $\varphi$  and grows without bound as  $\varphi$  does. Moreover, we know that the difference must be negative whenever (6) is violated, which completes the proof.

(b) First, compare integration with high vs. low effort. The difference between the expected payoff  $Z^{int}$  minus the analogous expression in (14) with  $q$  instead of  $p$  is positive and increases without bound with  $\varphi$ , which can be seen by inspection the derivative  $2(p - q)[y_2 - (p + q)(y_2 - y_1)]$ , which by Assumption 1 is positive. Moreover, the managers' total wages,  $2(\beta^{int} + \gamma^{int})$  times  $Z^{int}$ , converge to  $4cp/(p - q)$  as  $\varphi$  approaches infinity. It follows that the net profits under integration with high effort must be larger than with low effort for  $\varphi$  sufficiently large.

Comparing integration with high effort with non-integration, observe that  $\beta^{ni}$ ,  $\beta^{int}$  and  $\gamma^{int}$  also converge to zero as  $\varphi$  approaches infinity; the owners' share of the payoff thus converges to 1 in both cases. The integration payoff eventually exceeds the total no-integration payoff, which can be seen by comparing  $\lim_{\varphi \rightarrow \infty} \frac{1}{\varphi} Z^{int} = 2p[p y_1 + (1 - p)y_2]$  and  $\lim_{\varphi \rightarrow \infty} \frac{1}{\varphi} 2[p\varphi + (1 - p)]y_1 = p y_1$ . Thus, since integration with high effort dominates non-integration for  $\varphi \rightarrow \infty$ , the same holds by continuity for an interval of  $\varphi$  above some threshold value.

(c) The non-integration profits do not depend on  $y_2$ . All that remains to show is that an integrated firm's profit is increasing in  $y_2$ . First, under integration with low effort, the firm pays the managers a constant and keeps the rest of the expected payoff  $2q^2 \varphi y_1 + 2q(1 - q)\varphi y_2 + 2(1 - q)^2 y_1$ , which is increasing in  $y_2$ . The same is true for the expected payoff  $Z^{int}$  under high effort. Here, the firm keeps only the share  $(1 - \beta^{int} - \gamma^{int})$  of the payoff, but as we show next, this share is increasing in  $y_2$  too, implying that the product  $(1 - \beta^{int} - \gamma^{int})Z^{int}$  is increasing in  $y_2$ .

We show that  $\beta^{int} + \gamma^{int}$  is decreasing in  $y_2$ . This sum can be expressed as

$$\frac{c[y_2 + p(\varphi - 1)(y_2 - y_1)]}{(p - q)(\varphi - 1)[py_1 + (1 - p)y_2][y_1 + p(\varphi(y_2 - y_1) - y_1)]}.$$

Its derivative has the same sign as

$$- \left[ p(1 - p)(\varphi - 1)(y_2 - y_1)^2 - (1 - p)(y_2^2 - y_1^2) - py_1^2 \right],$$

which is negative, as claimed. ■

**Proof of Proposition 5:** Manager 2 must be willing to report his project truthfully, under the assumption that manager 1 exerts high effort, and that manager 1 allocates resources according to  $k^*$ . But these constraints for manager 1 are the same as for the CEO hierarchy, and are given by (8) and (7). Hence manager 2's truthtelling constraints is the same as in the CEO hierarchy. Similarly, manager 2 must be willing to exert effort, again under the same assumptions as above. Since the managers' contracts can be different, we can solve for manager 2's optimal contract independently from manager 1's. Thus, the arguments of Proposition 3 apply without change for manager 2, and his optimal contract  $(\beta_2, \gamma_2)$  is given by Proposition 3.

For manager 1, the effort incentive constraint is likewise the same as before. In place of a truthtelling constraint, there are now incentive constraints for manager 1 to be willing to allocate resources efficiently. There are two relevant new constraints; suppose in both cases that manager 1 has a bad project. First, if manager 2's project is bad as well, allocating the resources equally (which is the efficient choice) leads to an expected wage of  $(\beta_1 + \gamma_1)y_1$  for manager 1, while allocating all resources to division 1 leads to an expected wage of  $\beta_1 y_2$ . Manager 1 therefore allocates resources efficiently if

$$(\beta_1 + \gamma_1)y_1 - \beta_1 y_2 \geq 0 \tag{29}$$

or equivalently  $\gamma_1/\beta_1 \geq (y_2 - y_1)/y_1$ . Second, if manager 2's project is good, then allocating all resources to division 2 (the efficient choice) leads to a payoff of  $\gamma\varphi y_2$  for manager 1, whereas allocating the resources equally instead leads to a wage of  $(\beta + \varphi\gamma)y_1$ . Manager 1 therefore allocates resources efficiently if

$$\gamma_1\varphi y_2 - (\beta_1 + \gamma_1)y_1 \geq 0 \tag{30}$$

or equivalently  $\gamma_1/\beta_1 \geq y_1/[\varphi(y_2 - y_1)]$ . There are more constraints, in particular since there are other ways to allocate resources inefficiently, but all of them are equivalent to or less restrictive than (29) or (30), and therefore need not be considered.

To prove the proposition, we show that (29) and (30) are jointly more restrictive than the truthtelling constraint (28) in the symmetric hierarchy that they replaced. This follows simply

from the fact that if we write (28) in this form,

$$\{p[\varphi(y_2 - y_1) - y_1] + y_1\}\gamma - [p(2y_1 - y_2) + y_2 - y_1]\beta \geq 0,$$

the left-hand side is equal to  $1 - p$  times the left-hand side of (29) plus  $p$  times the left-hand side of (30), as is easy to verify. Thus, in the hybrid organization, the more restrictive condition of (29) and (30) must hold, whereas in the symmetric hierarchy only a weighted average of the two constraints must hold, which is an unambiguously weaker constraint. ■

**Proof of Proposition 6:** As explained in the text, the effort and truth-telling incentive constraints (IC-e), (IC-B) and (IC-G) are the same under horizontal exchange as in the CEO hierarchy. In addition, an efficient allocation of resource requires that a manager with a bad project lend his resources to the other division if the latter has a good project. If he does, his payoff is  $\gamma\varphi y_2$ , whereas if he keeps his resource his payoff is  $(\beta + \gamma\varphi)y_1$ . It follows that he will lend his resources if  $\gamma/\beta \geq y_1/[\varphi(y_2 - y_1)]$ . It is straightforward to show that this lower bound on  $\gamma/\beta$  is lower than the lower bound stated in Lemma 3 if and only if  $\varphi < y_1^2/(y_2 - y_1)^2$ . When that is this case, the new constraint  $\gamma/\beta \geq y_1/[\varphi(y_2 - y_1)]$  is not binding given that (IC-B) is already imposed; and the contract of Proposition 3 remains optimal. Otherwise, the new constraint is more restrictive than (IC-B), leading to an optimal wage contract that implies a higher wage bill for the firm than under the CEO hierarchy. ■

## Appendix B: General, Non-separable Contracts

General non-separable contracts specify a wage for each possible realization of  $(\tilde{z}_1, \tilde{z}_2)$ . For a non-integrated firm, the contract given by Lemma 1 remains optimal when non-linear contracts are allowed, since there is no reason to condition wage payments on the other firm's payoff, and since in each firm realized payoff is only  $\mu$  or zero, requiring only one non-zero wage variable.

We therefore need to study non-separable contracts only for the integrated firm. As before, with limited liability but a non-binding participation constraint, it is optimal to pay each manager zero if both divisions have a zero payoff. For this reason, it is without loss of generality to describe each manager's wage as a share of  $\mu$  as in the separable case. The managers' (symmetric) contracts can then be described by the triple  $(\beta, \gamma, \delta)$ , where manager  $i$  is paid  $\beta$  (as a share of  $\mu$ ) if only  $\tilde{z}_i = \mu$ ,  $\gamma$  if only  $\tilde{z}_j = \mu$ , and  $\delta$  if both units have a high payoff. The case of separable contracts discussed in the main text is a special case of this more general setting, and corresponds to the restriction  $\delta = \beta + \gamma$ . Our first result generalizes Proposition 1:

**Proposition 7** *In an integrated firm in which the CEO has perfect information about  $\theta$  and allocates resources efficiently, the optimal contract for each division manager is given by  $\gamma = 0$ , and*

$$\begin{aligned} \text{if } p < \frac{y_2}{(\varphi - 1)y_1 + y_2}: \quad & \beta^{pi} = \frac{c\mu}{(p - q)\{(1 - p)(\varphi y_2 - y_1) + p\varphi y_1\}\mu - (p\varphi^2 - 1 + p)y_1^2} \quad \text{and} \\ & \delta^{pi} = 0, \\ \text{if } p > \frac{y_2}{(\varphi - 1)y_1 + y_2}: \quad & \beta^{pi} = 0 \quad \text{and} \quad \delta^{pi} = \frac{c\mu}{(p - q)(p\varphi^2 - 1 + p)y_1^2}. \end{aligned} \quad (31)$$

*In either case, the expected total wage bill is lower under integration than under non-integration.*

*Proof:* Under perfect information, if manager 1 has a good project, then with probability  $p$  manager 2 has a good project as well and each is allocated one unit of resources. Manager 1 can then earn either  $\delta\mu$ ,  $\beta\mu$  or  $\gamma\mu$  (with appropriate probabilities), depending on which of the two divisions has a high payoff. With probability  $1 - p$ , manager 2 has a bad project, all resources are allocated to division 1, and manager 1 earns  $\beta\mu$  with probability  $\varphi y_2/\mu$ . Overall, manager 1's expected wage from having a good project is

$$W(G) = \left\{ p \left[ \frac{\varphi^2 y_1^2}{\mu^2} \delta + \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) (\beta + \gamma) \right] + (1 - p) \frac{\varphi y_2}{\mu} \beta \right\} \mu. \quad (32)$$

If manager 1 has a bad project, then with probability  $p$  manager 2 has a good one, and all resources go to division 2; whereas with probability  $1 - p$  manager 2 has a bad project as well. Manager 1's expected wage from having a bad project then is

$$\left\{ p \frac{\varphi y_2}{\mu} \gamma + (1 - p) \left[ \frac{y_1^2}{\mu^2} \delta + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta + \gamma) \right] \right\} \mu. \quad (33)$$

By symmetry, these expressions are the same for manager 2, and so each manager will exert high effort if  $pW(G) + (1 - p)W(B) - c \geq qW(G) + (1 - q)W(B)$ , or equivalently,

$$\begin{aligned} (p - q) \left\{ \left( p\varphi^2 - 1 + p \right) \frac{y_1^2}{\mu} \delta + \left[ (1 - p) \left( \varphi y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + p\varphi y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right] \beta \right. \\ \left. - \left[ p\varphi \left( y_2 - y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right) + (1 - p)y_1 \left( 1 - \frac{y_1}{\mu} \right) \right] \gamma \right\} \geq c. \end{aligned} \quad (34)$$

As in the separable case, the left-hand side of (34) is decreasing in  $\gamma$ , meaning that  $\gamma$  has a negative effect on effort incentives. The firm's expected net profit is  $\mu$  times

$$\left[ p^2 \varphi^2 + (1 - p)^2 \right] \frac{y_1^2}{\mu^2} (2 - 2\delta) + 2(1 - \beta - \gamma) \left[ p^2 \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) + (1 - p)^2 \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) + p(1 - p)\varphi \frac{y_2}{\mu} \right]. \quad (35)$$

This expression is obtained as follows. If both divisions have a high payoff, the firm's profit is  $(2 - 2\delta)\mu$ . For this to occur requires that both projects are either good (probability  $p^2$ ) or bad

(probability  $(1 - p)^2$ ), which leads to the first term in (35). Otherwise, if only one division has high payoff, the firm's profit is  $(1 - \beta - \gamma)\mu$ ; cf. the coefficient of the second term in (35). This occurs if both divisions have equally good or bad projects, and both get resources, but only one has a high payoff (the first two terms in []-brackets in (35)), or if only one division has a good project and gets all resources (the last term in []-brackets in (35)).

Since both (35) and the left-hand side of (34) are decreasing in  $\gamma$ , it is optimal to set  $\gamma = 0$ . This leaves two variables to choose but only one constraint. Since both  $\beta$  and  $\delta$  decrease the firm's profit and the slopes of (35) and the left-hand side of (34) are different, it is optimal to set either  $\beta$  or  $\delta$  to zero and solve (34) for the other variable. The solutions for each case are stated in the proposition.

Which of these two cases is optimal depends on the slopes of (35) and the left-hand side of (34) in  $\beta, \delta$ -space. Choosing  $\delta > 0$  and  $\beta = 0$  is optimal if the iso-profit curves are steeper than the (34)-line. The formal condition can be expressed as  $-\frac{\partial \Pi}{\partial \beta} / \frac{\partial \Pi}{\partial \delta} < -\frac{\partial \text{IC-e}}{\partial \beta} / \frac{\partial \text{IC-e}}{\partial \delta}$ , or equivalently  $\frac{\partial \Pi}{\partial \beta} / \frac{\partial \Pi}{\partial \delta} > \frac{\partial \text{IC-e}}{\partial \beta} / \frac{\partial \text{IC-e}}{\partial \delta}$ , with  $\Pi$  given by (35) and "IC-e" given by the left-hand side of (34). The difference between the two slopes,  $\frac{\partial \Pi}{\partial \beta} / \frac{\partial \Pi}{\partial \delta} - \frac{\partial \text{IC-e}}{\partial \beta} / \frac{\partial \text{IC-e}}{\partial \delta}$ , can be shown to be equal to

$$\mu(1 - p)\varphi \frac{p(\varphi - 1)y_1 - (1 - p)y_2}{y_1^2[(1 - p)^2 + p^2\varphi^2](p\varphi^2 - 1 + p)}. \quad (36)$$

The last expression in the denominator of (36) must be positive since it appears in the denominator of  $\delta$  in (31), and so must be positive for this solution to be feasible at all. All other terms in (36) are unambiguously positive, except for  $p(\varphi - 1)y_1 - (1 - p)y_2$ , which is positive if and only if  $p > \frac{y_2}{(\varphi - 1)y_1 + y_2}$ . This leads to the condition for the case distinction stated in the proposition.

Finally, since  $\beta^{pi} < \beta^{ni}$  in Proposition 1, the optimal separable contract under perfect information already leads to a lower expected wage bill for the firm. Since the set of separable contracts is a strict subset of the set of non-separable contracts, it follows that the wage bill must also be lower with an optimal non-linear contract. ■

The optimal solutions stated in Proposition 7 look quite different from the contract in Proposition 1 for the separable case. Given that  $\gamma = 0$  is optimal, separability imposes the restriction  $\delta = \beta$  on the contract. Without this restriction, it is optimal to set one of  $\beta$  or  $\delta$  to zero (this is also why the expressions for  $\beta$  in Propositions 1 and 7 look very different).

The main conclusion from Proposition 1, however, remains intact when non-separable contracts are feasible: with perfect information, the *competition effect* of centralized resource allocation *improves* effort incentives relative to non-integration, as reflected in a lower wage bill.

Let us now turn to the case where project types are communicated strategically by the managers. As in Section 4.4, additional constraints come into play. In the following, we first

derive these constraints formally, and then generalize Proposition 3.

First, it must be optimal for each manager to report his type truthfully. For a manager 1 with a *good* project, the expected payoff from reporting truthfully, and under the assumption that manager 2 reports truthfully too, is given by  $\bar{w}_1(G, G) = W(G)$  as given by (32). Suppose manager 1 reports “B” instead. Then with probability  $p$ , manager 2 has a good project, in which case all resources go to division 2 and manager 1 earns  $\gamma$  if division 2 has high payoff. With probability  $1 - p$ , manager 2 has a bad project, each division is allocated one unit of resources, and the manager can earn  $\delta$ ,  $\beta$  or  $\gamma$  times  $\mu$ , depending on both divisions’ payoffs. The resulting expected wage for manager 1 is

$$\bar{w}_1(G, B) = \left\{ p \frac{\varphi y_2}{\mu} \gamma + (1 - p) \left[ \varphi \frac{y_1^2}{\mu^2} \delta + \frac{\varphi y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) \beta + \left( 1 - \frac{\varphi y_1}{\mu} \right) \frac{y_1}{\mu} \gamma \right] \right\} \mu. \quad (37)$$

The truthtelling constraint (IC-G) given by  $\bar{w}_1(G, G) \geq \bar{w}_1(G, B)$  can therefore be expressed as

$$\begin{aligned} \varphi \frac{y_1^2}{\mu} (p\varphi - 1 + p)\delta + \varphi \left[ (1 - p)y_2 - (1 - p)y_1 \left( 1 - \frac{y_1}{\mu} \right) + py_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right] \beta \\ + \left[ (p\varphi - 1 + p)y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) - p\varphi y_2 \right] \gamma \geq 0. \end{aligned} \quad (38)$$

For a manager 1 with a *bad* project, the expected payoff from reporting truthfully is  $\bar{w}_1(B, B) = W(B)$  as given by (33). Suppose manager 1 reports “G” instead. Then with probability  $p$ , manager 2 has a good project too, in which case each division gets one unit of resources, and manager 1 can earn  $\delta$ ,  $\beta$  or  $\gamma$  times  $\mu$  depending on the divisions’ payoffs. With probability  $1 - p$ , manager 2 has a bad project, and all resources go to division 1. The resulting expected wage for manager 1 is

$$\bar{w}_1(B, G) = \left\{ p \left[ \frac{\varphi y_1^2}{\mu^2} \delta + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta \right] + (1 - p) \frac{y_2}{\mu} \beta \right\} \mu. \quad (39)$$

The truthtelling constraint (IC-B) given by  $\bar{w}_1(B, B) \geq \bar{w}_1(B, G)$  can then be expressed as

$$\begin{aligned} -(p\varphi - 1 + p) \frac{y_1^2}{\mu} \delta - \left[ (1 - p) \left( y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + py_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right] \beta \\ + \left[ p\varphi \left( y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + (1 - p)y_1 \left( 1 - \frac{y_1}{\mu} \right) \right] \gamma \geq 0. \end{aligned} \quad (40)$$

Second, it must be optimal for the CEO to allocate resources efficiently if he assumes that project types are reported truthfully. This constraint was never binding with separable contracts, but may become binding with non-separable contracts. Suppose first that both projects are good. If the CEO allocates one unit of resources to each division (the efficient allocation), the expected profit for the firm is

$$2(1 - \delta) \frac{\varphi^2 y_1^2}{\mu^2} + 2(1 - \beta - \gamma) \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \quad (41)$$

times  $\mu$  (in the following next equations, all profit expressions are stated as shares of  $\mu$ ). If instead the CEO were to allocate all resources to one division, then the expected profit would be

$$(1 - \beta - \gamma) \frac{\varphi y_2}{\mu}. \quad (42)$$

For the CEO to choose the efficient allocation requires that (41) be at least as large as (42), or

$$2(\beta + \gamma - \delta) \varphi \frac{y_1^2}{\mu} + (1 - \beta - \gamma)(2y_1 - y_2) \geq 0. \quad (43)$$

By similar reasoning, it can be shown that the condition for the CEO to allocate resources efficiently if both projects are bad is given by

$$2(\beta + \gamma - \delta) \frac{y_1^2}{\mu} + (1 - \beta - \gamma)(2y_1 - y_2) \geq 0. \quad (44)$$

Finally, suppose that division 1's project is good and division 2's bad. If the CEO allocates all resources division to 1 (the efficient allocation), the firm's expected profit is

$$(1 - \beta - \gamma) \frac{\varphi y_2}{\mu} \quad (45)$$

If instead the CEO were to allocate the resources equally, then the expected profit would be

$$2(1 - \delta) \frac{\varphi y_1^2}{\mu^2} + (1 - \beta - \gamma) \left[ \frac{\varphi y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) + \left( 1 - \frac{\varphi y_1}{\mu} \right) \frac{y_1}{\mu} \right]. \quad (46)$$

For the CEO to choose the efficient allocation requires that (45) be at least as large as (46), or equivalently

$$(1 - \beta - \gamma)[\varphi(y_2 - y_1) - y_1] - 2(\beta + \gamma - \delta) \frac{\varphi y_1^2}{\mu} \geq 0. \quad (47)$$

Of these three constraints, (44) is redundant. To see why, notice that since both (43) and (47) must hold, the sum of their left-hand sides, which yields  $\mu(\varphi - 1)(y_2 - y_1)(1 - \beta - \gamma)$ , must be positive. This in turn requires that  $\beta + \gamma < 1$ . Next, given the last result, both (43) and (44) can be binding only if  $\delta > \beta + \gamma$ ; but in that case (42) is clearly the more restrictive condition. We can therefore ignore (44).

The optimization problem (4) can hence be stated more precisely as the problem of maximizing (35) with respect to  $\beta$ ,  $\gamma$  and  $\delta$ , subject to the effort incentive constraint (34), the truth-telling constraints (38) and (40), the resource allocation constraints (43) and (47), and the nonnegativity constraints  $\beta, \gamma, \delta \geq 0$ .

With three variables to specify and eight linear constraints, there are as many as  $8!/(3! 5!) = 56$  different corner points as possible candidates for an optimal solution. The following result, which generalizes Proposition 3, narrows the number of possible solutions down to only a few:

**Proposition 8** *In an integrated firm, the optimal non-separable contract for each division manager that leads to high effort, truthful reports about investment projects, and an efficient resource allocation, depends on  $p$  as follows:*

(a) *If  $p < 1/(1 + \varphi)$ , then an optimal contract is given by*

$$\begin{aligned}\beta &= c \frac{p\varphi - 1 + p}{(1-p)(p-q)(\varphi-1)[p(\varphi-1)y_1 - (1-p)y_2]} \\ \gamma &= 0, \\ \delta &= c \frac{(\mu - \varphi y_1)[p(2y_1 - y_2) + y_2 - y_1] + (1-p)y_1[\varphi(y_2 - y_1) + y_1]}{(1-p)(p-q)(\varphi-1)y_1^2[(1-p)y_2 - p(\varphi-1)y_1]}.\end{aligned}\quad (48)$$

*The resulting expected wage per agent is the same as under non-integration.*

(b) *If  $p > 1/(1 + \varphi)$ , then the optimal contract is given by the unique solution for  $\beta$ ,  $\gamma$  and  $\delta$  of one of six different systems of conditions (with inequalities interpreted as equations): in four of the possible solutions, (34) and (40) hold, and in addition either  $\beta = 0$ ,  $\delta = 0$ , (43), or (47). In the remaining two possible solutions, both (43) and  $\beta = 0$  hold, and in addition either (34) or (40). In each of these solutions,  $\gamma$  is strictly positive, and the resulting expected wage per agent is strictly higher than under non-integration.*

*Proof:* Part (a): The solution stated is the unique solution for which  $\gamma = 0$  and both (34) and (40) are binding. Feasibility of this solution requires  $\beta, \delta \geq 0$ . Since the numerator of  $\delta$  in (48) is positive, we need  $(1-p)y_2 > p(\varphi-1)y_1$  for the denominator of  $\delta$  to be positive. But if the latter condition holds, the denominator of  $\beta$  in (48) is negative, which means that for  $\beta$  to be positive, we need  $1-p > p\varphi$  for the numerator to be negative as well. Conversely, if  $1-p > p\varphi$ , then it follows that  $(1-p)y_2 > p\varphi y_1 > p(\varphi-1)y_1$ , i.e. the same condition we started with. We can conclude that  $1-p > p\varphi$  or  $p < 1/(1 + \varphi)$  is both necessary and sufficient for the stated solution to be feasible (although we haven't established optimality yet).

For general  $\beta$ ,  $\gamma$  and  $\delta$ , the total expected wage bill for the integrated firm is  $\mu$  times

$$2\delta \left[ p^2\varphi^2 + (1-p)^2 \right] \frac{y_1^2}{\mu^2} + 2(\beta + \gamma) \left[ p^2 \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) + p(1-p) \frac{\varphi y_2}{\mu} + (1-p)^2 \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) \right], \quad (49)$$

cf. the expression for the firm's net profit in (35). Substituting the contract (48) into (49) and simplifying leads to  $2c(p\varphi - 1 + p)/[(p-q)(\varphi-1)]$ , which is the same as the total wage bill for both firms under non-integration, cf. (24). Optimality of the solution (48) then follows from Proposition 2 (the contract (48) need not be the unique optimal contract, though).

Part (b): If  $p > 1/(1 + \varphi)$  or  $p\varphi - 1 + p > 0$ , then the truth-telling constraint (40) is decreasing in  $\delta$ . Since it is also decreasing in  $\beta$ , the only way to satisfy (40) is to set  $\gamma > 0$ . Next, recall from the proof of Proposition 2 that the wage bill under integration is strictly higher than under

non-integration if  $\varphi\bar{w}(B, G) > \bar{w}(G, G)$ . Evaluating the difference  $\varphi\bar{w}(B, G) - \bar{w}(G, G)$ , using the expressions in (32) and (39), simplifies to  $p\varphi(\varphi - 1)y_1\gamma$ , which is positive whenever  $\gamma$  is. Both results together imply that any solution for the case  $p > 1/(1 + \varphi)$  leads to a strictly higher wage bill than under non-integration, as stated.

To sort through the numerous possible candidates for an optimal contract, let us proceed in order of the number of variables that are set to zero. First,  $\beta = \gamma = \delta = 0$  is clearly not a feasible solution since it violates the effort constraint (34). Second, given the requirement  $\gamma > 0$ , the only candidate for a solution with two variables equal to zero has  $\beta = \delta = 0$  and  $\gamma > 0$ ; but in this case (34) is again violated.

Third, consider solutions in which exactly one variable is zero; this can only be either  $\beta$  or  $\delta$ . The other two variables are then determined by any two of (34), (40), (42) and (47) as binding conditions. Under an optimal contract, the resource allocation constraints (42) and (47) will never simultaneously be binding, for this would require  $\beta + \gamma = \delta = 1$  and would lead to a profit of zero. (Intuitively, the CEO is indifferent between all resource allocations only if the entire division payoffs are paid out to the managers.)

Suppose first that  $\delta = 0$ . One solution is given where both (34) and (40) are binding. It satisfies the nonnegativity and resource allocation constraints for some but not all parameters, and therefore belongs to the set of possible solutions. Next, the resource allocation constraint (43) is always satisfied if  $\delta = 0$ ; therefore only (47) can possibly be binding. This leaves two remaining potential solutions in this group, where (47) and either (34) or (40) are binding. Neither can be optimal, however: if (34) and (47) are to be binding, it would be optimal to set  $\gamma = 0$  since both constraints are decreasing in  $\gamma$ , which we already know would violate (40). And if (40) and (47) are to be binding, it would be optimal to set  $\beta = 0$  since both constraints are decreasing in  $\beta$ , but doing so would violate our assumption that only one variable is set to zero. Hence, the only contract with  $\delta = 0$  that can be optimal is the one where (34) and (40) are binding.

Now, suppose that  $\beta = 0$ . Again, one possible solution is given where both (34) and (40) are binding. Next, we show that (47) can never be binding: with  $\beta = 0$ , (47) could bind only if  $\delta < \gamma$ . But if  $\beta = 0$  and  $\delta = \gamma$ , then (34) is definitely violated, as is easy to verify, and since (34) is increasing in  $\delta$  in this range of  $p$ , the same must be true for even smaller  $\delta$ . It follows that out of the two resource allocation constraints, only (43) can possibly be binding. This leaves two remaining potential solutions in this group, where (43) and either (34) or (40) are binding. Both are feasible for some parameters, and since we have not been able to rule out their optimality,

they too are possible solutions.<sup>30</sup>

Fourth, consider strictly interior solutions, which are obtained by making any three out of (34), (40), (42) and (47) bind. Since we already showed that (42) and (47) will never simultaneously be binding, this leaves two possible solutions, in which both (34), (40), and either of (42) or (47) are binding. Both solutions are feasible, and can be shown to be optimal for certain parameters.

Overall, we have narrowed the range of possible solutions for case (b) down to the six stated in the proposition. ■

Proposition 8 states that if  $p > 1/(1 + \varphi)$  (part b), then the conclusions of Proposition 3 carry over to the non-separable case: any *feasible* solution entails  $\gamma > 0$ , and from the proof of Proposition 2 it follows that the wage bill must be higher than under non-integration. If  $p \leq 1/(1 + \varphi)$ , on the other hand, the managers' information can be elicited without any additional cost relative to the case of non-integration, similar to the message-contingent contract of Proposition 2.

The condition on  $p$  that distinguishes the two cases results from the truth-telling constraint (40) for a manager with a bad project, which in turn is derived from (33) and (39). Specifically, the condition follows from the derivative of the left-hand side of (40) with respect to  $\delta$ : case (a) of the Proposition applies when  $\delta > 0$  increases the manager's incentive to report truthfully, and otherwise case (b) applies. To understand the sign of the derivative, observe first that a manager can earn  $\delta$  only if both divisions have high payoffs, which requires that the CEO allocates the resources equally between the divisions (if one division has no resources, it cannot attain a high payoff). If manager 1 has a bad project and reports truthfully, he earns  $\delta$  if manager 2 also has a bad project (the probability of which is  $1 - p$ ), and both have high payoff, which occurs with probability  $y_1^2/\mu^2$ . In contrast, if manager 1 claims to have a good project, he earns  $\delta$  if manager 2 has a good project (the probability of which is  $p$ ), and both have high payoff, which occurs with probability  $\varphi y_1^2/\mu^2$ . Thus, the effect of  $\delta$  to report truthfully is given by  $(1 - p - p\varphi)y_1^2/\mu$ , which is positive if and only if  $p \leq 1/(1 + \varphi)$ .

Our next result generalizes Proposition 5. Like in Section 6, we allow for asymmetric contracts for the managers. The managers' wages for the different possible payoff outcomes can therefore be described by  $\delta_1, \beta_1, \gamma_1$  and  $\delta_2, \gamma_2, \beta_2$ , respectively.

**Proposition 9** *Assume that the owner of an integrated firm wants the managers to exert high effort, and wants resources to be allocated efficiently. Then the incentive constraints in the CEO*

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<sup>30</sup> In numerical simulations, though, these two solutions, where feasible, have turned out to be inferior to the one in which  $\beta = 0$  and (34) and (40) are binding.

hierarchy are unambiguously less restrictive than those in the hybrid organization.

*Proof:* As in the separable case, in the hybrid organization all incentive constraints for manager 2, as well as the effort incentive constraint for manager 1, are the same as in the CEO hierarchy, cf. the proof of Proposition 5. It remains to show how the resource allocation constraints for a manager 1 with a bad project compare to his truthtelling constraint in the hierarchy with CEO. Suppose manager 1 has a bad project. If manager 2's project is bad too, and manager 1 allocates the resources equally as would be efficient, his expected wage is

$$\left[ \frac{y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta_1 + \gamma_1) \right] \mu.$$

If instead he allocates all resources to himself, his expected wage is  $\beta_1 y_2$ . Manager 1 will therefore allocate resources efficiently if

$$\left[ \frac{y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta_1 + \gamma_1) \right] \mu - \beta_1 y_2 \geq 0. \quad (50)$$

If manager 2's project is good and manager 1 allocates all resources to division 2 as would be efficient, his expected wage is  $\gamma_1 \varphi y_2$ . If instead he allocates the resources equally, his expected wage is

$$\left[ \frac{\varphi y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta_1 + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma_1 \right] \mu.$$

Manager 1 will therefore allocate resources efficiently if

$$\gamma_1 \varphi y_2 - \left[ \frac{\varphi y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta_1 + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma_1 \right] \mu \geq 0. \quad (51)$$

It can then be shown that left-hand side of (40) is equal to  $(1-p)$  times the left-hand side of (50) plus  $p$  times the left-hand side of (51), which completes the proof (see the proof of Proposition 5 for further details). ■

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