On the Co-Evolution of Policy and Political Power

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Abstract

This paper examines the dynamic evolution of policy and power. We posit a general (non-parametric) class of dynamic stochastic games in which a leader's policy has distributional consequences that may alter the future balance of power. Policies such as taxes and public goods chosen by an authority in date t alter the economic fundamentals such as wealth distribution in date $t + 1$. Changes in the distribution of wealth change the balance of political power in $t + 1$ which, in turn, determines the policy choices at that date.

Because the current decision maker cannot de-couple the direct effect of his policy from its indirect effect on future power, a transfer of power can result. We refer to this as a case of *policy-endogenous political power*. Under policy-endogenous power, the current political authority crafts a "Faustian bargain" from the following trade-off: if he chooses his preferred policy, then he sacrifices future political power; yet if he wants to preserve his future power, he must sacrifice his policy objectives in the present. We examine this trade-off in the set of smooth Markov Perfect equilibria. In each such equilibrium, the motives of a political leader can be decomposed into two basic rationales. The "political preservation effect" induces the authority to choose "more conservatively" than if his policy choice did not affect his political fortunes. However, the "reformation effect" induces less conservative choices in order to exploit the gains from policies of more aggressive successors. We identify sufficient conditions under which policy-endogenous power differs from a regime in which power is permanent. The co-evolution of policy and power is illustrated in a parametric example.

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Key Words and Phrases: Policy-endogenous political power, permanent power, authority, Political preservation effect, political reformation effect, biased political system

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1 Introduction

Power can be fragile. At any given time, a "politically wrong" policy can alter the balance of power, leading to a government's demise and replacement. Even a purely policy-motivated leader must often choose between his current policy objectives and the political power that would safeguard his policy objectives in the future.

This paper explores the dynamic link between policy and power. In particular, we focus on the distributional and/or demographic channels through which this link occurs. For example, Schonhardt-Bailey (2006) documents the ending of mercantilist policies in Britain in the early 19th century, culminating in the repeal of the Corn Law in 1846. These policies had the effect of enlarging the merchant and middle classes which eventually produced sweeping changes in the composition of British Parliament.[1] In another example, Tichenor (2002) describes alternating periods of restrictive and expansive immigration policies in the U.S. from the mid 19th century onward. Immigration ultimately brought about large political shifts toward urban regions, as reflected in congressional and presidential elections.

As the examples suggest, changes in policy alter the distribution of population characteristics which then bring about changes in future power. In turn, these changes affect the types of policies chosen in the future. This paper studies the feedback loop from policy to power and back to policy, in a class of infinite horizon, dynamic stochastic games. Our aim is to identify fundamental attributes of a political actor's choice between policy and power. For this reason, we eschew detailed models of political interaction in favor of a more stripped down approach. We consider a society inhabited by a continuum of infinitely lived citizens. At each date t, one of the citizens, a *pivotal decision maker* (or "*PDM*") has the authority to choose a policy that affects all the citizens in society. His policy choice in date t determines, through changes in next period's characteristics of voters, the identity of next period's PDM.

Significantly, the current decision maker is unable to de-couple the tangibly direct effect of his policy from its indirect effect in determining the identity of a future decision maker. The change in *de facto* power therefore occurs despite no change in the formal (or *de jure*) rules of the political system.

Because political power is endogenously driven by policy change, we refer to this as a case of *policy-endogenous political power*. Under policy-endogenous political power, the current authority faces a classic "Faustian" trade-off: if he tries to meet his policy objectives, then he changes economic fundamentals in a way that later strips him of power, placing it in the hands of a less desirable ruler. If, on the other hand, the current authority tries to preserve power, he sacrifices his policy objectives. Significantly, the PDMs in our model are purely policy-motivated. Hence, their concern about losing power arises only because future decision

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[1] See also Finer (1997).

makers have different policy objectives than their own.

While the restriction to political systems that admit a pivotal decision maker entails some loss of realism and breadth, it allows a clearer focus on the dynamic trade-offs that determine policy and political power. We study these trade-offs in the set of smooth Markov Perfect equilibria (MPE) of the game. These are Subgame Perfect equilibria in which strategies depend smoothly (i.e., differentiably) on the payoff-relevant state. Smooth equilibria are natural objects to study, and have recently been shown by Judd (2004) to be effective equilibrium selection devices when, as is commonly the case in time consistent models of policy, multiple equilibria exist.

Our main results compare the equilibrium paths of policy and power to the benchmark case in which political power is permanent - authority never changes hands. We characterize the incentives of political leaders when certain monotone comparative statics conditions on payoffs and technologies obtain. From the Euler equation, a pivotal decision maker's motives may be decomposed into two basic rationales. First, the "political preservation effect" induces the leader to choose "more conservatively" or "less aggressively" than if his hold on power did not depend on his policy choice. In terms of our monotone comparative statics assumptions, this means that he chooses a lower level of the policy, e.g., a lower level of public investment, than if his power were permanent.

However, this preservation effect is mitigated by a second rationale, the "reformation effect," which induces less conservative policy. The reformation effect comes from fact that a more aggressive/less conservative policy alters the balance of power in a way that places future authority in the hands of more aggressive types of leaders. These leaders choose larger investments in the future, which increases the marginal productivity of a policy increase in the present.

Which of the two, the preservation or reformation effect, dominates depends on parameter values and functional forms. However, even if, say, the preservation effect dominates, the *overall* effect on policy may be more aggressive than under permanent power. Although each leader is individually less aggressive, political power may evolve toward more aggressive leaders, and so the policy trajectory may be more aggressive on the whole.

While there is an abundant literature in political economy that studies the link from political power to policy,[2] the "reverse causal link," i.e., from policy to power, has not been studied to the same degree. In the growth models with infinitely-lived agents, Bertola (1993), Alesina and Rodrik (1994), Krusell et al (1996, 1997) and Krusell and Rios-Rull (1999) studied the determination of taxation and capital accumulation under majority voting.[3] Because the tax policy does not change the relative wealth distribution in these models, a fixed median

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[2] Persson and Tabellini (2003) is a standard reference, although the literature has mushroomed even since then.

[3] For a detailed review and comparison of literature in this tradition, see Krusell et al (1997).

voter emerges as a permanent authority in the equilibrium.

These models are indicative of a general attribute of many politico-economic models. Namely, that although the potential for policy-endogenous change in power exists in these models, it does not occur in equilibrium. In other words, permanent authority arises endogenously. The reason is that standard majority voting is most often assumed, and this rule typically admits no policy-endogenous changes in equilibrium.[4] This is so because the distributional effects of many policies are order-preserving, and the median voter depends only on an order statistic of the population distribution. For instance, an increase in a country's income tax preserves the order statistics of the population through time (as long as marginal rates do not exceed 100%). An individual who is richer than another today, is still expected to be richer after the tax increase goes into effect. Hence, under majority voting the median voter's identity does not change, and so there is no change in political power - at least not through this channel.

Yet, there are other political mechanisms that could be considered, notably those factor in wealth and other cardinal characteristics, that *do* exhibit policy-endogenous power. We examine a parametric special case of the model in Section 3. There we derive an equilibrium from an explicit voting rule. The example displays some of the salient features of political systems that gives rise to policy-endogenous power. One necessary condition is that the political system must be, in some sense, biased. Distributional characteristics beyond just aggregate order statistics must matter. But this requirement probably is satisfied in all real world political systems. In the U.S., for example, Senate representation is biased in favor of less populous states, hence toward characteristics of rural rather than urban voters. Often in parliamentary systems, small minority parties, and the voters they represent, have disproportionate influence in the formation of majority governments. Finally, campaign contributions and the large expense of running modern political campaigns bias the outcomes in favor of wealthier citizens.

The upshot is that standard politico-economic models characterize the effects of permanent political power on the determination of policies, but are not designed to answer the question of endogenous evolution of political power and its subsequent effects on the economic policies.

Our paper is tailor-made to answer the latter questions left by these permanent-authority studies. First of all, our model emphasizes an endogenous evolution of political power with a biased political institution. This opens the door for explaining the de facto authority change even under a stable (*de jure*) political institution. Our framework also helps to isolate the effect of a specific political institution on the equilibrium path when power is policy-endogenous. As shown in this paper, the equilibrium path with a policy-endogenous political power can be quite different from that under permanent power.

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[4] Though some exceptions are later mentioned in our review of the literature.

A few recent studies do allow for policy-endogenous changes in power, each using a different mechanism. Hassler, et. al. (2003) investigates the evolution of the welfare state in a parametric overlapping-generations model. A majority vote determines the level of transfers to unsuccessful agents. Because the population size of different types is endogenously determined by individual investment decisions, their model can generate a shift of political power even with majority voting. Ortega (2005) examines policy-endogenous power when the policy is an immigration quota. Each period, a majority vote by current citizens determines which types of immigrants to let in. Ceteris paribus, current residents want to admit immigrants with complementary skills. On the other hand such immigrants are future voters who will vote to admit future immigrants whose skills are substitutes to those of the current residents. This trade-off can be viewed as a particular instance of the preservation and reformation effects we identify. Finally, Bowen and Poon (2007) examines another explicit mechanism through policy-endogenous power occurs. In their model, elites exert influence through campaign contributions.

More generally, our model differs from and is complementary to these both in terms of the modeling framework and the level of abstraction. Whereas these models flesh out the dynamics of a specific political process through which policy affects power, our aim is to highlight representative attributes of the policy-power trade-off across a wide spectrum of political institutions and policy objectives.

A related literature allows for policy-endogenous power, but also allows the participants to undo its effects by de-coupling the policy and institutional decisions. For instance, Acemoglu and Robinson (2000, 2001, 2005), Cervelatti, et. al. (2006), and Jack and Lagunoff (2006a,b) and Lagunoff (2006a) all examine models of explicit institutional (de jure) choices by current rulers, as a way of reversing or mitigating the deleterious effects of current policy on one's future political fortunes.

Similarly, Person and Tabellini (2007) examine the nexus of political and economic capital. They build a model in which economic and political investments are mutually reinforcing in democracies. The success of an attempted coup depends on the political investments of the citizenry. Because democracies generate higher returns to economic investments, its citizens are willing to invest more political capital to defend it against coups.

Another paper by Acemoglu and Robinson, (2006), explores this "de-coupling" idea explicitly. They examine persistence of *de facto* power in the face of institutional change. Building on their earlier framework laid out in Acemoglu, Johnson, and Robinson (2005), they model the difference between *de facto* and *de jure* power explicitly by examining the economic mechanisms through which the elites can prevent or undo the steps taken by a country to democratize. They identify "captured democracies" as those in which the elite's investments succeed in preserving its de facto power despite the democratizing de jure changes in institutions.

The key difference between their model and ours is that in our model, policies generate political change, while in theirs, policies are used by elites to undo political change. Thus, our mechanism is, in a sense, the reverse of theirs.[5] Clearly, there are examples of both. Acemoglu and Robinson look to 20th century Latin America for numerous instances of captured democracies. On the other hand, the collapse of the Soviet Union, and the role of Glasnost in facilitating the change in power, is an instance where a decision maker (Gorbachev) made hard choices leading to a, perhaps unavoidable, change in power.

In comparing our work to the papers mentioned here, ours is a stripped down version of all these papers in the sense that we eliminate political details in an attempt to identify the most elemental trade-offs between power and policy facing all political actors. The basic model is introduced in Section 2. Section 3 elaborates on a parametric example to fix ideas. There, we describe conditions under which policy-endogenous political power matters. Section 4 returns to the abstract model and contains the main decomposition result. Section 5 examines the issues of existence of Markov equilibria. Section 6 gives concluding remarks. Section 7 is an Appendix with the proofs of all the results.

2 The Model

2.1 Basic Environment

Society is comprised of a continuum $I = [0, 1]$ of infinitely lived *citizen-types.* Each citizen-type derives a distinct payoff in each period $t = 0, 1, \ldots$, from a state variable and a policy decision. We refer to the index i as a "type" rather than an individual in order to allow for demographic changes in the population distribution as the state evolves. Denote the state by $\omega_t \in \Omega$, and the policy by $a_t \in A$. The policy a_t is a "public decision" in the sense that it can enter each citizen's payoff. However, we allow for the possibility that a_t can have individual-specific components that affect each citizen separately. For instance, the state ω_t could correspond to the date t distribution of income, and the policy a_t an income tax schedule. Assume both A and Ω are subsets of a finite dimensional space $\mathbb{R}^{\ell}$, with A a hypercube $[\underline{a}, \bar{a}]^{\ell}$, and Ω a convex cone $[\underline{\omega}, \infty)^{\ell}$.[6]

Given any sequence of states $\{\omega_t\}$ and policies $\{a_t\}$, the dynamic payoff to citizen-type $i \in I$ is

$$\sum_{t=0}^{\infty} \delta^t u(i, \omega_t, a_t) \tag{1}$$

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[5] To wit, one might dub the difference as one of "captured democracy" vs "released democracy." Though we should mention that the mechanism we explore does not automatically work in the direction of democratization.

[6] This implies that the distribution of population characteristics a summarized by a finite dimensional parameter set.

where δ is a common discount factor, and the payoff function u is smooth (in all variables), bounded, and strictly concave in a.

A Markov process determines the distribution over future states as a function of current states and actions. Formally, let $\omega_{t+1} = \nu_{t+1} + z_t$ with $z_t = Q(\omega_t, a_t) \in \Omega$ interpreted as end-of-period capital, and ν_{t+1} is an $\ell \times 1$ vector of iid shocks that are realized at the beginning of period $t+1$. The transition function Q is assumed nonnegative, smooth, and weakly concave. The shocks are distributed according to a unimodal density π with compact support.

Political power is modeled in reduced form by assuming that the political system is always rationalized by the preferences of a pivotal individual. Specifically, in each period, a particular citizen is assumed to hold decisive political authority. This citizen (e.g., "pivotal voter") is effectively endowed with the exclusive right in period t to choose the policy action a_t. Henceforth, we refer to this individual as the *pivotal decision maker (or PDM)*. Political power is therefore represented by a mapping from states (e.g., capital stocks, income distributions) to citizen-types. Formally, the mapping is assumed to be a smooth function $\mu : \Omega \to I$. Given a state ω_t, the *PDM* is a citizen-type $i_t = \mu(\omega_t)$.

Clearly, assuming that political process admits a pivotal decision maker is restrictive. It is well known, for instance, voting aggregation is not so well behaved when policies are multi-dimensional. There are commonly assumed conditions on policy preferences, notably single crossing properties Gans and Smart (1996) and Rothstein (1990) that do admit pivotal voters. In the present model, we simply assume it without postulating these conditions formally.

Because μ determines "who's in charge" in each state, we refer to it as the *authority function.* The underpinnings of an authority function μ is open to interpretation. We have in mind a polity summarized by a fixed set of (*de jure*) rules, with *de facto* political power evolving naturally in economic state. There are two natural interpretations of how this might happen.

- **Distributional channels**. The state ω_t identifies temporal characteristics such as wealth, income, or social status of the citizen-types. Tax cuts, for instance, could change the identity of the pivotal decision maker if citizens are differentiated by income dimension. Significantly, this type of endogeneity does not arise under standard majority voting rules (because of invariance of the median as policy changes). However, it does appear in "biased" political systems. A direct form of bias occurs when wealth buys political favors. An indirect bias exists when a federal system aggregates outcomes of local elections, and there are regional differences in wealth or other population characteristics.
- **Demographic channels**. The state ω_t directly identifies the population distribution or the voter turnout of citizen types. Changes in, say, immigration laws or fertility policy can have (longer run) electoral effects even under median voter rules, since the demographic changes themselves may be biased toward certain groups or social classes.

2.2 Permanent Political Power

Consider the benchmark case in which some individual i_0 maintains political power regardless of his policy action. This would be true in the model if the authority function μ were constant, i.e., $\mu(\omega_t) = i_t = i_0$ for all ω_t. We refer to this as a *permanent political power* or *permanent authority* regime, and we refer to i_0 as the *permanent PDM.* For purposes of comparison, we will always use i_0 to denote the PDM under permanent power.

Permanent authority may exist if society has the ability to commit itself constitutionally to a ruler such as a king or monarch. Commitments of this type are not common in modern democracies. However, they were the norm in many European monarchies prior to the 19th century. Alternatively, permanent authority can *result* if the economic state does not affect the identity of the PDM, or if fundamentals do not produce differences of opinion regarding policy. Regardless of the source, the case of permanent power is the natural benchmark against which to compare the "normal" case of policy-endogenous power.

Under the assumptions of the basic model, the permanent PDM derives a policy function ψ^* in which $\psi^*(\omega_t) = a_t$ is the policy that would be taken by citizen-type i_0 in state ω_t under the presumption that his own authority is perpetual. The policy function ψ^* solves the Bellman equation

$$U(i_0, \omega_t; \psi^*) \;=\; \max_{a_t \in A} \; \left[u(i_0, \omega_t, a_t) \;+\; \delta \int U(i_0, \nu_{t+1} + z_t;\; \psi^*) \pi(\nu_{t+1}) d\nu_{t+1} \right] \tag{2}$$

subject to $z_t = Q(\omega_t, a_t)$. This problem is standard and serves as the baseline case for purposes of comparison.

2.3 Policy-Endogenous Political Power

When political power is policy-endogenous, current policy changes produce changes in the state which, in turn, produce changes in the identity of the pivotal decision maker through μ. Let $i_t = \mu(\omega_t)$ denote the pivotal decision maker (PDM) in state ω_t. The change in authority, as described by μ, defines a dynamic, stochastic game with a potentially infinite set of players. Moreover, because the state ω_t is itself stochastic, the realized sequence of players $\{i_t\}$ is stochastic as well.

In principle, the PDM could make his choice contingent on the entire history of the game. However, we restrict attention to Markov Perfect equilibria (i.e., Subgame Perfect equilibria in Markov strategies). There are two reasons for this. First, Markov strategies facilitate a direct comparison with the permanent authority regime which is inherently Markov. Second, we think the restriction is a natural one in this context. With an infinite number of decision makers, each of whom is endogenously chosen from the continuum, coordination costs of history-contingent equilibria are particularly high. Strategies that therefore depend only on the current, payoff relevant state bypass some of the difficulty.

A Markov strategy profile for this political system is expressed as a function Ψ in which $\Psi(i, \omega_t) = a_t$ is the policy choice of citizen-type i in any state ω_t in which i is the pivotal decision maker, i.e., in any state ω_t satisfying $\mu(\omega_t) = i$. This choice is made by i knowing the future sequence of PDMs are determined endogenously by μ, and knowing that the choices of future PDMs are determined by Ψ. Letting Ω_i denote the subset of states (possibly empty) in which i is the pivotal decision maker,[7] a *Markov Perfect equilibrium (MPE)* for this game is a Markov profile Ψ such that for all i and all states $\omega_t \in \Omega_i$, Ψ is a best response against any history contingent strategy that differs from Ψ only on the states in Ω_i.

A MPE can also be expressed as a Markov strategy function Ψ^* defined on states alone:

$$\Psi^*(\omega_t) \;=\; \Psi(\mu(\omega_t), \omega_t) \tag{3}$$

Because behavior can be summarized by the Markov function in the sense of (3), it will often prove more convenient to work with the Ψ^* instead of Ψ. We will use the notation Ψ only when it is important to emphasize the explicit dependence on the identity i of the PDM.

At any date t, the continuation payoff to a citizen-type i under Markov strategy Ψ^* is given by

$$U(i, \omega_t;\; \Psi^*) \;=\; E\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} u(i, \omega_\tau, \Psi^*(\omega_\tau)) \,\middle|\, \omega_t\right] \tag{4}$$

The expectation $E[\cdot]$ is taken with respect to the transition technology and the distribution over shocks. A standard argument shows that MPE policies are those that are best responses against one-shot deviations. A straightforward consequence of this fact is that Ψ is a MPE if and only if it induces a Markov strategy Ψ^* defined by (3) such that for all ω_t with $i_t = \mu(\omega_t)$, for all $a \in A$,

$$U(i_t, \omega_t;\; \Psi^*) \;\geq\; u(i_t, \omega_t, a_t) \;+\; \delta \int U(i_t,\; \nu_{t+1} + z_t;\; \Psi^*)\; \pi(\nu_{t+1}) d\nu_{t+1} \tag{5}$$

subject to $z_t = Q(\omega_t, a_t)$. It is not hard to show that if Ψ (or the corresponding Ψ^*) is Markov Perfect, then it solves

$$V(\omega_t;\; \Psi^*) \;=\; \max_{a_t \in A} \left[u(i_t, \omega_t, a_t) \;+\; \delta \int U(i_t,\; \nu_{t+1} + z_t;\; \Psi^*)\; \pi(d\nu_{t+1}) \right] \quad \forall \omega_t \; \forall t \tag{6}$$

subject to $z_t = Q(\omega_t, a_t)$ and $i_t = \mu(\omega_t)$.

The function V lacks the recursive structure of a standard Bellman equation. It is not generally true that $V(\omega_\tau; \Psi^*) = U(i_t,\; \omega_\tau; \Psi^*)$. The two are only equal if $i_t = \mu(\omega_\tau)$. Equation

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[7] Formally, $\Omega_i = \{\omega \in \Omega :\; i = \mu(\omega)\}$.

(6) creates a conflict between the current PDM and the individuals that acquire decision authority in the future. The problem is reminiscent of the much-studied hyperbolic $\beta-\delta$ policy models of Krusell and Smith (2002), Krusell, Kuruscu and Smith (2002), Judd (2004), and many others. However, the policy-endogenous model is more complicated in several important respects. First, while the hyperbolic problem is fundamentally about the conflict between the decision maker at a date t and all his future incarnations, each of whom share the same preference at date t, the policy-endogenous problem is about a conflict between *every* decision maker at every date $\tau \geq t$, separately. Second, while the conflict in the hyperbolic model is exogenously determined by the $\beta-\delta$ parameters, the degree of conflict in the current model is endogenous. At each decision date, the policy determines the degree of conflict between the current and next decision date.

Despite the differences, the present model has one critical feature in common with the hyperbolic model. Namely, the dynamic inconsistency between current and future pivotal decision makers leads to policies that differ from the "full commitment" case of permanent political power.

2.4 What Does "Comparing the Two Regimes" Mean?

The framework is general enough to admit a comparison of any two political institutions represented by authority functions. However, in order to highlight the qualitative effect of a policy-endogenous change within a formal political institution, we concentrate on the simpler comparison between the policy-endogenous and permanent political power regimes.

Even in this case, the nature of the comparison is not obvious. Consider fixing an arbitrary initial state, ω_0. Given this state, one can pin down a permanent authority function, μ^P such that $i_0 = \mu^P(\omega_0)$ is the permanent PDM under μ^P. One the other hand, starting from the same state ω_0, a policy-endogenous authority function μ^{PE} determines an initial decision maker $i^* = \mu^{PE}(\omega_0)$. In general the two decision makers will not be the same, i.e., $i_0 \neq i^*$. But this means the comparison is not so meaningful. If each regime begins with a very different type of decision maker, then the difference in two regimes may be entirely attributable to different initial conditions.

Consequently, to make the comparison meaningful, we need to control for economic fundamentals in the experiment. Specifically, we start with two identical economies with the same preference, technology, policy sets, and so on. Then, we start off the comparison under the same initial decision maker. In other words, type i_0 is both the permanent decision maker under μ^P and the initial decision maker under μ^{PE}.[8] This entails finding an initial state ω_0 such that $\mu^{PE}(\omega_0) = i_0 = \mu^P(\omega_0)$. If μ^{PE} is a monotone function (as will be assumed

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[8] Since μ^{PE} is not necessarily onto, the citizen-type i_0 is chosen from the support of μ^{PE}.

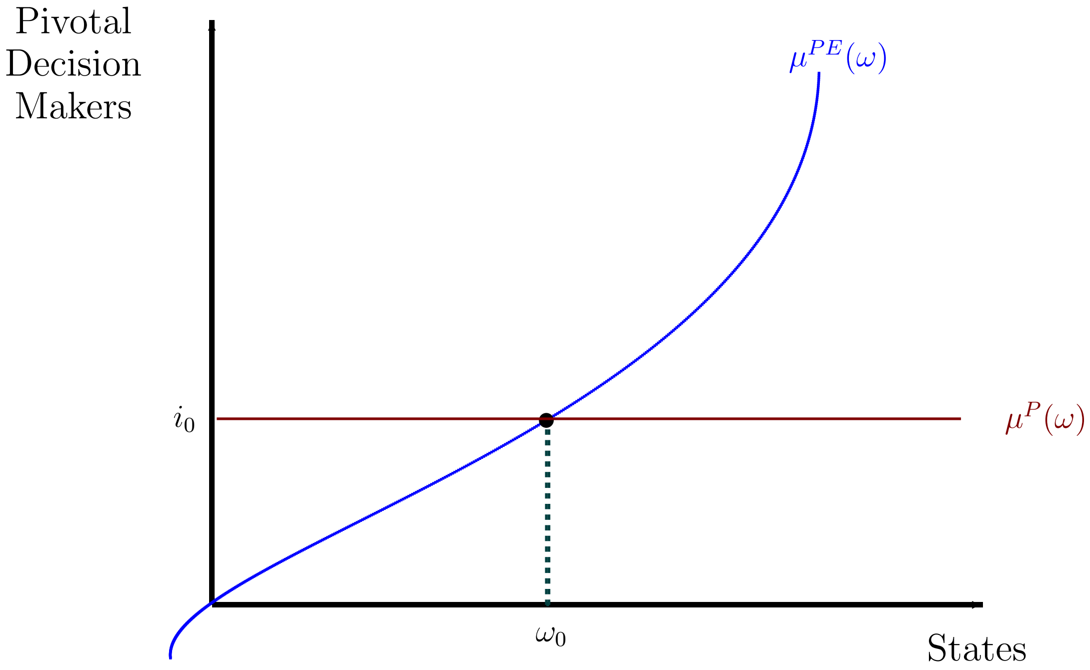

Figure 1: Authority Functions in the Policy-Endogenous and Permanent Power Regimes

in the monotone comparative statics exercise), then there is exactly one such state. Figure 1 displays this case. Proceeding in this way, the difference in equilibrium path comes solely from different political institutions.[9]

The policy-endogenous power regime therefore yields a trajectory of pivotal decision makers given by $i_0, i_1, i_2, \ldots$. Of course, under permanent political power, the identity of i_0 never changes. In terms of specifics, there are two relevant comparisons. First, the incentives of a decision maker i_0 may be compared to the hypothetical case in which the same citizen-type happens to have power in state ω_t. In other words, the comparison is between $\Psi(i_0, \omega_t)$ and $\psi^*(\omega_t)$. This comparison is “hypothetical” in the sense that the PDM i_0 who holds permanent authority will not generally be the one who holds power in state ω_t in the policy-endogenous regime. A second comparison is that between the equilibrium policy path under policy-endogenous power and that of permanent power. That is, between $\Psi^*(\omega_t)$ and $\psi^*(\omega_t)$. Ultimately, the policy “bias” produced by policy-endogenous power comes from this second comparison.

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[9] There are different interpretations of our controlled experiments. From a historical point of view, the comparison can be viewed as a counter-factual thought experiment on institutional change. From the cross-sectional point of view, it can be viewed as an investigation to the effects of institutions on the cross-country difference.

3 A Leading Example

To fix ideas, this Section illustrates the model in a parametric example. The example is representative in two respects. First, it gives a good indication of those environments that the model is intended to explain. Second, the example illustrates how the mechanics of the model work in these environments.

3.1 A Parametric Environment

In this example, society collectively chooses a uniform contribution each period in the form of a lump sum tax or a public service from each of its citizens. The contribution produces a public capital good, ω_t, which is accessible to all citizens, each of whom can use it to augment his own wealth over time.[10] Examples are a property assessment that is used to improve infrastructure or compulsory military service required to enhance national security. Let $a_t \in [\underline{a}, \bar{a}]$ denote the uniform contribution and let $y(i, \omega_t)$ denote citizen i's production function of income in state ω.[11]

To keep things simple, assume full depreciation of the capital good each period, and next period's capital depends linearly on the contribution a_t (note that a_t is both aggregate and per capita contribution). The transition function is therefore given by

$$\omega_{t+1} = Q(\omega_t, a_t) = a_t. \tag{7}$$

Dropping time subscripts, the production function is assumed to take the parametric form

$$y(i, \omega) \ = \ b + i(\omega - 1), \tag{8}$$

where $b \geq 0$ is a given constant. If $b = 0$, then changes in the public capital stock simply rescales the wealth of each individual, and so relative wealth differences are preserved. If $b > 0$, then a change in ω can bring on a change in relative wealth of different citizen-types.

The payoff each period to a citizen i depends positively on wealth $y(i, \omega)$ and negatively on the contribution a in the form

$$u(i, \omega, a) \ = \ \kappa \left((\frac{1}{i} - 1)^2 \frac{1}{i} \right) y\,(i, \omega) \ - \ a^2, \tag{9}$$

where $\kappa > \frac{2}{\delta}$ is a given constant.[12] Use the parametric assumption of wealth $y(i, \omega)$ in (8) we

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[10] In this example, there is no private saving. Therefore, income and wealth are equivalent concepts. Consequently we use income and wealth interchangeably.

[11] Alternatively, $y(i, \omega_t)$ can be interpreted directly as a wealth allocation rule in the society.

[12] The restriction of $\kappa > \frac{2}{\delta}$ is used later to guarantee sustained growth with policy-endogenous political power. If $\kappa < \frac{2}{\delta}$, the model will have a decreasing path with a zero steady state. If $\kappa = \frac{2}{\delta}$, every ω_0 is a steady state.

get

$$u(i,\omega,a) \ = \ \widetilde{u}_1(i) \ + \ \kappa\,(\frac{1}{i}-1)^2\omega \ - \ a^2, \tag{10}$$

where $\widetilde{u}_1\,(i) \, = \, \kappa\frac{1}{i}\left(\frac{1}{i}-1\right)^2$. Because $\widetilde{u}_1(i)$ is an additive constant in the payoff function of each citizen i, it can be dropped without affecting the decisions of PDMs. As a result, the effective payoff function can be expressed (by an abuse of notation) as a linear-quadratic form

$$u(i,\omega,a) \ = \ \kappa\,(\frac{1}{i}-1)^2\omega \ - \ a^2. \tag{11}$$

3.2 Deriving an Authority Function from a Voting Rule

Even though our theoretical analysis models political authority quite abstractly, it does not mean that it cannot be grounded in explicit political institutions. It is instructive to see how an authority function can be derived from a specific voting rule in our parametric example. In addition to grounding the analysis, the derivation will also highlight the exact link between a specific political institution and an authority function.

Consider any voting rule that effectively allocates weights to individual voters. Specifically, the equilibrium level of contribution for state ω is determined by pairwise majority voting on all alternatives, with $\lambda(i,\omega)$ number of votes assigned for each citizen-type i. The effective weights could reflect biases in the explicit (*de jure*) voting procedure itself.[13] The weights could also reflect implicit or “extra-constitutional” biases such that of campaign campaign contributions and/or bribes.

In the case of simple majority voting, $\lambda(i,\omega) = 1$ for each $i \in I$ and $\omega \in \Omega$. A proportional wealth-weighted rule is one which allocates to each citizen i precisely $\lambda(i,\omega) \ = \ y(i,\omega)$ votes. It is also easy to construct intermediate cases of voting rules in between the two extremes. By design, our parametric example falls into a class of utility functions that admit a pivotal decision maker. The following Proposition shows this, and also provides a constructive way to find the authority function.

Proposition 1 *Suppose the transition is deterministic and the payoff function for each citizen i is of the form*

$$u(i,\omega,a) \ = \ h(\omega,a)f(i) + g(\omega,a) \tag{12}$$

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[13] The U.S. Senate was one example given earlier of “structural bias.” Another example is the threshold rule in many parliamentary systems that require smaller parties to gain a minimal threshold percentage of the electorate in order to gain a seat in parliament.

where f is monotone in i. Then any $\lambda(i,\omega)$-weighted majority voting rule admits a pivotal decision maker. The authority function is implicitly defined by

$$\int_{\mu(\omega)}^{1} \lambda(i,\omega)di \;=\; \frac{1}{2}\int \lambda(i,\omega)di \tag{13}$$

The proof is straightforward and is given in the Appendix. The proof proceeds by checking the order-restriction property of the preference. The result then follows from a well known result by Rothstein (1990).

Proposition 1 provides a simple yet powerful method to find the authority function, as shown in the following examples. The examples also illustrate the crucial role of the wealth production function in translating a specific political institution into an abstract authority function under wealth-weighted majority voting.

Example 1 *Majority Voting. With $\lambda(i,\omega) = 1$, it immediately follows from (13) that $\mu(\omega) = \frac{1}{2}$. In words, the median citizen type $i = \frac{1}{2}$ results as a permanent authority.*

Example 2 *Wealth-Weighted Voting with $b = 0$ (recall that b is the constant in $y(i,\omega)$). With $\lambda(i,\omega) = y(i,\omega)$, the authority function is determined by the equation*

$$\int_{\mu(\omega)}^{1} j(\omega-1)dj = \frac{1}{2}\int_{0}^{1} j(\omega-1)dj. \tag{14}$$

A straightforward calculation yields $\mu(\omega) = \frac{1}{\sqrt{2}}$. Even with wealth-weighted majority voting, a permanent authority still occurs if the wealth function is multiplicatively separable in i and ω.

Example 3 *Wealth-weighted Voting with $b = \frac{1}{2}$. With $\lambda(i,\omega) = y(i,\omega)$ and $b = \frac{1}{2}$, the authority function satisfies*

$$\int_{\mu(\omega)}^{1} \left(\frac{1}{2} + j\,(\omega-1)\right) dj = \frac{1}{2}\int_{0}^{1} \left(\frac{1}{2} + j\,(\omega-1)\right) dj \tag{15}$$

With a little bit of algebra, the authority function can be derived as $\mu(\omega) = \frac{1}{1+\sqrt{\omega}}$. In this case, authority function is policy-endogenous.

Besides demonstrating the map from political rules to authority functions, the examples illustrate at least two additional points. First, different political institutions can give rise to the same authority function, as shown in Example 1 and Example 2. To the extent that political institution affects the economy only through the authority function, this shows the advantage of using authority function in the class of political economy models we are considering. Second, the map from a political institution to an authority function depends on both the economic fundamentals and political rule. A political institution per se does not determine the nature of pivotal decision maker, as shown in Example 2 and Example 3. Consequently, the effect of a political institution can not be understood without resorting to a specific economic environment.

3.3 A Permanent Authority Equilibrium

In this part, we solve a conventional problem for a given permanent authority. With a constant authority function $\mu(\omega) = i_0$ for each $\omega \in \Omega$, the solution can be derived directly from FOC, i.e.,

$$-2\omega_{t+1} + \left(\frac{1}{i_0} - 1\right)^2 \kappa = 0,$$

or

$$\omega_{t+1} = a_t = \psi^*(\omega_t) = \frac{1}{2}\left(\frac{1}{i_0} - 1\right)^2 \kappa,$$

i.e., a constant accumulation each period. Interestingly, the economy reaches a steady state right after one period.

3.4 A Policy-Endogenous Power Equilibrium

Now consider a policy-endogenous authority function derived from Example 3, $\mu(\omega) = \frac{1}{1+\sqrt{\omega}}$. Recall that this authority function is derived from a combination of wealth-weighted majority voting and a specific production function.

To solve for the equilibrium, we have to deal with the extra complexity introduced by the policy-endogenous political power. To understand this point, consider an increase in the contribution a_t. The increase involves two kinds of tradeoff. First, it trades off an immediate disutility of contribution from increased future wealth from improved infrastructure in the form of ω_{t+1}. This first effect is conventional one which is also present with a permanent authority. Second, it will induce a change of the future decision maker to the one specified by $\mu(\omega_{t+1})$. This second effect is absent in permanent political power environment.

It turns out that we can get an analytical solution thanks to the linear-quadratic structure of the problem. In Proposition 2, we present the solution. The proof is given in the appendix.

Proposition 2 *Suppose* $u(i, \omega, a) = \kappa\ (\frac{1}{i} - 1)^2\omega\ - a^2$ *and* $\mu(\omega) = \frac{1}{1+\sqrt{\omega}}$. *The equilibrium policy function is given by*

$$a = \Psi^*(\omega) = K\omega,$$

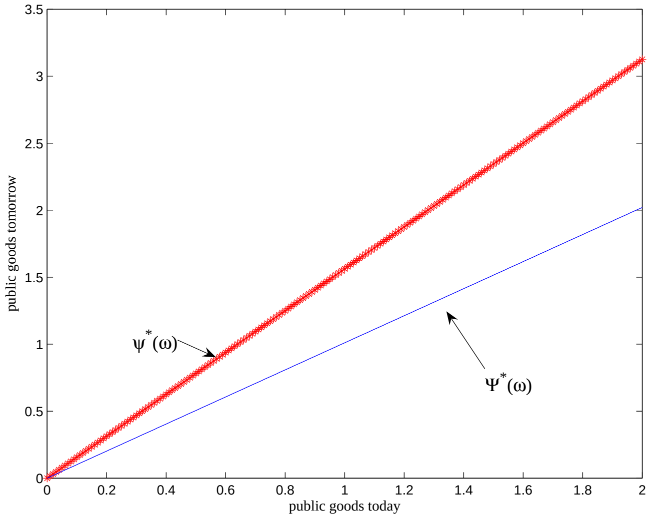

Figure 2: Transition Function in the Policy-Endogenous and Permanent Power Regimes

where

$$K = \frac{-1 + \sqrt{1 + \kappa\delta^2 (\kappa\delta - 2)}}{(\kappa\delta - 2)\,\delta}, \forall \kappa > 0,$$

where it is understood that $K = 1$ *if* $\kappa = \frac{2}{\delta}$ *by applying L'Hospital's law. In addition,* K *is strictly increasing in* κ *and* $0 < K < \frac{1}{\sqrt{\delta}}$ *for every* $\kappa > 0$.

It is interesting to see how the dynamics of the economy depends on the parameter κ. If $\kappa > \frac{2}{\delta}$, $K > 1$ and the economy grows constantly; if $\kappa = \frac{2}{\delta}$, $K = 1$ and the economy reaches steady state immediately; if $\kappa < \frac{2}{\delta}$, $K < 1$ and the economy only admits a steady state of zero.

3.5 Comparison of Permanent and Policy-Endogenous Power

We follow the discussion in Section 2 to compare the equilibrium under permanent and policy-endogenous political power. More specifically, we start with an arbitrary ω_0 and find the corresponding $i_0 = \mu(\omega_0) = \frac{1}{1+\sqrt{\omega_0}}$ under the policy-endogenous political power. Then we compare the equilibrium with endogenous political power with a setup when i_0 holds a permanent power.

To ease the comparison, we restate the equilibrium transition in the following two equations (see also Figure 2).

$$\begin{aligned}\psi^*(\omega_t) &= \frac{1}{2}\left(\frac{1}{i_0}-1\right)^2\kappa \;=\; \frac{1}{2}\kappa\omega_0,\\ \Psi^*(\omega_t) &= K\omega_t \;=\; K^{t+1}\omega_0.\end{aligned}$$

We can draw two conclusions on the equilibrium path. First, an immediate effect of policy-endogenous political power is to induce a more conservative choice and hence lower short-run economic growth. To see this, first notice that for $\kappa > \frac{2}{\delta}$, $\frac{1}{2}\kappa > \frac{1}{\delta}$. Because $\frac{1}{\delta} > \frac{1}{\sqrt{\delta}}$ for $\delta < 1$ and $K < \frac{1}{\sqrt{\delta}}$, we have $\frac{1}{2}\kappa > K$. This verifies the claim and can be seen in Figure 2.

The intuition of a conservative short-run choice goes as follows. The current decision maker i_0 anticipates a growing economy and the corresponding shift of power to lower type in the future. Because a lower type is more aggressive, i.e., has a higher marginal value of investment, the future contribution will be higher than the desired level of current decision maker. A higher contribution will increase future gap between marginal benefit and marginal disutility of i_0. By choosing a lower current contribution, i_0 tends to decrease the future gap of marginal return. This additional benefit induces the current decision maker i_0 to reduce the current choice of contribution (a_0) and shift power to a relatively less aggressive power tomorrow. In the next Section, we identify this as the *preservation effect* since it reflects the desire by the PDM to move policy in the direction of his own preservation of power.

On the other hand, in the long run the policy-endogenous political power promotes economic growth relative to the permanent authority. This can be seen from a simple observation: because $K > 1$, $\omega_t^{PE} > \omega_t^{P}$ (where "PE" and "P" correspond to "policy-endogenous" and "permanent," resp.) if t is large enough. This may be seen in Figure 3. At the first glance, this seems to contradict the result of a short-run conservative choice under policy-endogenous political power. To reconcile these two apparently contradicting facts, first notice the following fact: as economy grows, the power shifts to the more aggressive types. We later identify this in the next Section as the *reformation effect*. More aggressive types i_t at date t large enough tend to make more contribution than type i_0, even though he is more conservative compared to the hypothetical permanent authority of his own type (i_t) at the same date t.

4 A Monotone Model

This Section returns to the general framework. Very little can be said without further restrictions. Specifically, meaningful comparisons across different parameter configurations requires that we employ the following monotonicity assumptions throughout the remainder of the analysis.

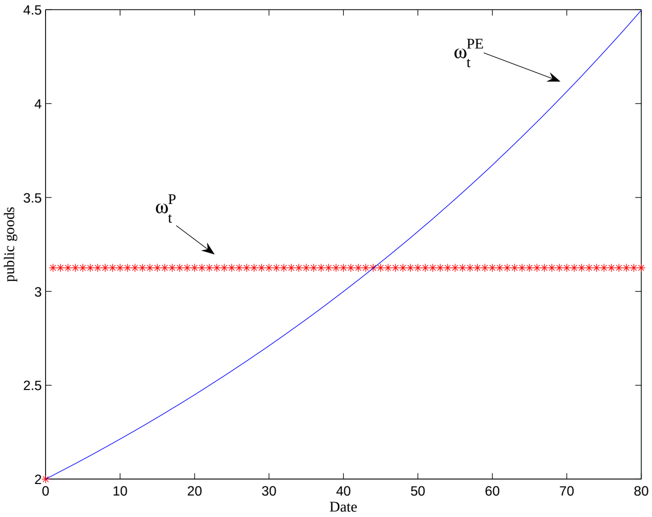

Figure 3: Equilibrium Path in the Policy-Endogenous and Permanent Power Regimes

(A1). (Monotonicity) Q is increasing in ω_t and strictly increasing in a_t. u is increasing in ω_t and strictly decreasing in a_t.

(A2). (Increasing Differences) Q has increasing differences in the pair (a, ω).[14] u has increasing differences in the pairs (i, a) and (ω, a).

(A3). (Monotonicity of Authority Function) The authority function μ is weakly increasing in the state ω.

Assumption (A2) is a standard monotone comparative statics assumption. It is used to establish monotonicity of equilibrium decision rules in parameters and state variables.[15] In particular, we seek monotone comparisons between policy-endogenous and permanent political power regimes.

(A3) is used to show the monotonicity of equilibrium path $\{\omega_t\}$. Since we can reorder the individual index if necessary, (A3) will also be satisfied if the authority function is weakly decreasing in ω. However, because it is not necessarily satisfied by every conceivable political institution, (A3) imposes a restriction on the political rules we consider. The examples illustrate instances in which a standard voting rules give rise to monotone μ.

4.1 A Decomposition Result

We use Assumptions (A1) and (A2), together with smoothness and concavity properties on u and Q assumed earlier, to characterize properties of smooth, interior Markov Perfect equilibria. These are MPE that are smooth functions of the state and such that the chosen policy always lies in the interior of the feasible set. Differentiability plays a crucial role in our characterization. We use it to examine properties of the Euler equations, roughly following an approach dating back to Basar and Olsder (1982) for dynamic stochastic games.[16]

The main result of this Section establishes a natural decomposition of the Euler equation in which two distinct effects: the "political preservation effect" and the "reformation effect" can be identified and used to compare the equilibrium trajectory of policy and power with that

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[14] A real valued function f has increasing differences in x and y if for any $\hat{x} > x$ and $\hat{y} > y$,

$$f(\hat{x}, y) - f(x, y) < f(\hat{x}, \hat{y}) - f(x, \hat{y}). \tag{16}$$

[15] See Roberts (1998,1999) for a similar use of monotone comparative statics assumptions. He studies a dynamic game of club admissions in which the pivotal voter in the club at date t admits a new club member in $t+1$. Attributes of new club members appear directly in the preferences of current voters. A single crossing condition guarantees that process of change is monotone in some well ordered space of member characteristics.

[16] More recently this approach has been adapted to dynamic policy problems by Klein, Krusell, and Ros Rull (2002), Krusell, and Krusell, Kuruscu, and Smith (2002), and Judd (2004), and to dynamic political games by Jack and Lagunoff (2006a - unpublished version).

of the permanent power regime. The question of existence of such an equilibrium is deferred until Section 6. The proofs of all results are in the Appendix.

Consider a Markov Perfect equilibrium Ψ, and its corresponding Markov strategy $\Psi^*(\omega_t) = \Psi(\mu(\omega_t), \omega_t)$, under policy-endogenous power. Let ψ^* be a solution under permanent political power. We restrict attention to Ψ^* and ψ^* that satisfy (i) Ψ^* and ψ^* are differentiable in the state, and (ii) the resulting policies and $\Psi^*(\omega_t)$ and $\psi^*(\omega_t)$ lie in the interior of the feasible policy space, A. The first-order conditions in the two models are given side by side by

$$D_{a_t}u(\mu(\omega_t), \omega_t, \Psi^*(\omega_t)) \;+\; \delta\; D_{a_t}Q(\omega_t, \Psi^*(\omega_t)) \cdot \int D_{\omega_{t+1}}U(\mu(\omega_t), \omega_{t+1};\; \Psi^*)\; \pi(d\nu_{t+1}) \;=\; 0 \tag{17}$$

$$D_{a_t}u(i_0, \omega_t, \psi^*(\omega_t)) \;+\; \delta\; D_{a_t}Q(\omega_t, \psi^*(\omega_t)) \cdot \int D_{\omega_{t+1}}U(i_0, \omega_{t+1};\; \psi^*)\; \pi(d\nu_{t+1}) \;=\; 0 \tag{18}$$

with $\omega_{t+1} = \nu_{t+1} + z_t$. In these equations, $D_{a_t}u$ is a $\ell \times 1$ gradient vector of partial derivatives in a_t, $D_{\omega_{t+1}}U$ a $\ell \times 1$ gradient vector of partial derivatives in ω_{t+1}, and $D_{a_t}Q$ a $\ell \times \ell$ gradient matrix of partial derivatives in a_t.

For an arbitrary citizen-type i, define this citizen-type's *distortion function* $\Delta(i, \omega_t; \Psi^*)$ in the policy-endogenous regime by

$$D_{a_t}u(i, \omega_t, \Psi^*(\omega_t)) \;+\; \delta\; D_{a_t}Q(\omega_t, \Psi^*(\omega_t)) \cdot \int D_{\omega_{t+1}}U(i, \omega_{t+1};\; \Psi^*)\; \pi(d\nu_{t+1}) \;=\; \Delta(i, \omega_t; \Psi^*) \tag{19}$$

Note that $\Delta(i, \omega_t; \Psi^*)$ is a $\ell \times 1$ gradient vector. The distortion function describes the marginal payoff deviation from i's his own preferred policy, due to the fact that i is possibly not the PDM in the state ω_t. When i *is* the PDM, i.e., when $i = i_t = \mu(\omega_t)$, then $\Delta(i_t, \omega_t; \Psi^*) = 0$, i.e., the distortion is zero. Of course, by definition, it also follows that $\Delta(i_0, \omega_t; \psi^*) = 0$ since there is no distortion when i_0 holds power forever.

To save on notation, we use the abbreviated notation $u_i = u(i, \cdot)$, and $U_i = U(i, \cdot)$ to denote the stage and dynamic payoffs, respectively, of a type $i \in I$. Given a pivotal decision maker $i_t = \mu(\omega_t)$ in state ω_t, if $D_{a_t}Q$ is invertible then i_t's first order condition in (17) can be rewritten as

$$\delta \int D_{\omega_{t+1}}U_{i_t}\; \pi(d\nu_{t+1}) \;=\; -\,[D_{a_t}Q]^{-1} \cdot D_{a_t}u_{i_t} \tag{20}$$

For any other type i, the distortion equation (19) can be written as

$$\delta \int D_{\omega_{t+1}}U_i\; \pi(d\nu_{t+1}) \;=\; \quad [D_{a_t}Q]^{-1} \cdot [\Delta(i, \omega_t; \Psi^*) \;-\; D_{a_t}u_i] \tag{21}$$

Differentiating the value function $U_i(i, \omega_{t+1};\; \Psi^*)$ with respect to ω_{t+1} yields

$$D_{\omega_{t+1}}U_i \;=\; D_{\omega_{t+1}}u_i + D_{\omega_{t+1}}\Psi^* \cdot D_{a_{t+1}}u_i + \delta \left[D_{\omega_{t+1}}Q \;+\; D_{\omega_{t+1}}\Psi^* \;\cdot\; D_{a_{t+1}}Q\right] \cdot \int D_{\omega_{t+2}}U_i\, \pi(d\nu_{t+1}) \tag{22}$$

Iterating (21) forward one period while holding indexed i fixed, and then substituting it into (22) yields

$$\begin{aligned} D_{\omega_{t+1}} U_i \;&=\; D_{\omega_{t+1}} u_i \;+\; D_{\omega_{t+1}} Q \cdot \left[D_{a_{t+1}} Q\right]^{-1} \cdot \left[-D_{a_{t+1}} u_i\right] \\ &\quad + \left[D_{\omega_{t+1}} Q \cdot \left[D_{a_{t+1}} Q\right]^{-1} \cdot [\Delta(i, \omega_{t+1}; \Psi^*)]\right] \;+\; D_{\omega_{t+1}} \Psi^* \;\cdot\; \Delta(i, \omega_{t+1}; \Psi^*) \end{aligned} \tag{23}$$

This expression can be further simplified. Recall the $z = Q(\omega, a)$ is strictly increasing in a by (A1). Define the inverse $a \;=\; Q_\omega^{-1}(z) \;\equiv\; L(\omega, z)$. By the invertibility of $D_{a_t} Q$, there exist functions $\Theta(i, \omega)$ and $\Theta^*(\omega)$ defined implicitly by

$$\Psi(i, \omega) \;=\; L(\omega, \Theta(i, \omega)) \qquad \text{and} \qquad \Psi^*(\omega) \;=\; L(\omega, \Theta^*(\omega)) \tag{24}$$

The function $\Theta^*(\omega_t) = \omega_{t+1}$ is the equilibrium transition function that maps current states into future ones. Totally differentiating the expression (24) with the respect to ω_t gives

$$\begin{aligned} D_{\omega_t} \Psi^* \;&=\; D_{\omega_t} L \;+\; D_{z_t} L \;\cdot\; D_{\omega_t} \Theta^* \\ &=\; -D_{\omega_{t+1}} Q \cdot \left[D_{a_{t+1}} Q\right]^{-1} \;+\; D_{z_t} L \;\cdot\; D_{\omega_t} \Theta^* \end{aligned} \tag{25}$$

Substituting (25) into (23) and then the resulting expression into the first order condition (17) of date t PDM, i_t, yields the following *Distortion-adjusted Euler equation*

$$D_{a_t} u_{i_t} \;+\; \delta\, D_{a_t} Q \;\cdot\; \int [R(i_t, \omega_{t+1};\; \Psi^*) \;+\; P(i_t, \omega_{t+1};\; \Psi^*)]\; \pi(d\nu_{t+1}) \;=\; 0 \tag{26}$$

where $\omega_{t+1} = \nu_{t+1} + z_t$, and

$$R(i_t, \omega_{t+1};\; \Psi^*) \;\equiv\; D_{\omega_{t+1}} u_{i_t} \;+\; D_{\omega_{t+1}} L \;\cdot\; D_{a_{t+1}} u_{i_t} \tag{26.a}$$

and

$$P(i_t, \omega_{t+1};\; \Psi^*) \;\equiv\; D_{z_{t+1}} L \;\cdot\; D_{\omega_{t+1}} \Theta^* \;\cdot\; \Delta(i_t, \omega_{t+1}; \Psi^*) \tag{26.b}$$

The Distortion-adjusted Euler Equation illustrates a basic decomposition of motives of any pivotal decision maker when power is policy-endogenous. Each PDM weighs the marginal cost $D_{a_t} u_{i_t}$ against two types of marginal gains/losses. One such effect is obvious. A marginal effect on i_t's payoff is brought about when the policy induces a different, and clearly less desirable, political authority in the future. This effect is given above by $P(i_t, \omega_{t+1}; \Psi^*)$. We will refer to it as the *Preservation Effect.* Intuitively, the preservation effect induces the current PDM to choose "more conservatively" than under permanent authority, in the sense that it induces him to choose policies that decrease the rate of political change. We show this more precisely below.

The preservation effect vanishes in the permanent power model due to the Envelope Theorem. Indeed, the Euler equation under permanent political power, assuming i_0 is the permanent authority, is given by

$$D_{a_t} u_{i_0} \ + \ \delta \ D_{a_t} Q \ \cdot \ \int R(i_0, \omega_{t+1}; \psi^*) \pi(d\nu_{t+1}) \ = \ 0 \tag{27}$$

where $R(i_0, \omega_{t+1}; \cdot)$ is defined by (26.a) and, in this case, is evaluated by ψ^* rather than Ψ^*.

For an arbitrary citizen-type i, the function $R(i, \omega_{t+1}; \Psi^*)$ produces a marginal outcome which is distinct from the preservation effect. R produces both a direct gain in i's payoff next period and an indirect gain from increased productivity of future policies. This effect is present, though differing in magnitude, in both the permanent and policy-endogenous regimes. Using the Example of the previous section for intuition, when power shifts from lower to higher productivity types, the change represents a commitment to more aggressive policies than would be the case if the PDM himself were making the same decisions in those future dates. The upshot is that in the Example, the PDM can count on more aggressive policies in the future, thus the marginal productivity of the present policy is higher. This intuition is, in fact, true generally as indicated by the Lemma.

Lemma 1 *For each citizen-type i and each state ω_t, $R(i, \omega_t; \Psi^*) > R(i, \omega_t; \psi^*)$ if and only if $\Psi^*(\omega_t) > \psi^*(\omega_t)$.*

We refer to the difference $R(i, \omega_t; \Psi^*) - R(i, \omega_t; \psi^*)$ as the *Reformation Effect* because it reflects the distortion in marginal payoffs, relative to the permanent authority regime, due to permanent changes in the state created by the changes in policy as political power shifts. This distortion leads to a distortion in incentives of the particular decision maker. Thus, according to the Lemma, the reformation effect is positive whenever policy-endogenous power produces a more aggressive policy decision than in the permanent authority regime. Significantly, the Lemma implies that *only* the reformation effect matters in the overall comparison between policy-endogenous power and permanent power.

However, the net effect on continuation payoffs of policy-endogenous power relative to permanent power for an arbitrary type i is

$$\overbrace{R(i,\omega_t;\ \Psi^*)\ -\ R(i,\omega_t;\psi^*)}^{\text{Reformation Effect}}\ \ +\ \ \overbrace{P(i,\omega_{t+1};\ \Psi^*)}^{\text{Preservation Effect}} \tag{28}$$

Using the permanent PDM i_0 as the hypothetical decision maker in a given state, the following result shows how the sign of this expression (once shocks are averaged out) matter when comparing an individual's choices across the two regimes.

Lemma 2 *For any state ω_t, $\Psi(i_0,\omega_t)\ >\ \psi^*(\omega_t)$ if and only if*

$$\int [R(i_0,\omega_{t+1};\ \Psi^*)\ -\ R(i_0,\omega_{t+1};\psi^*)\ \ +\ P(i_0,\omega_{t+1};\ \Psi^*)]\ \pi(d\nu_{t+1})\ >\ 0 \tag{29}$$

Under two additional assumptions a stronger characterization can be obtained.

(A4). For all i, ω and a, $D_iD_au\ \ >\ \ \dfrac{\delta}{1-\delta}\ D_aQ\ \cdot\ \dfrac{[\sup(D_iu)\ -\ \inf(D_iu)]}{2}$.

(A5). Let ω_0 denote the initial state at date 0. Then $Q(\omega_0,\underline{a})\ \geq\ \omega_0$.

Assumption (A4) is a strong form of increasing differences. It states basically that the increasing differences of u in i and a, assumed in (A2), dominates the maximal average variation in marginal payoffs in citizen type. (A5) is more standard. It implies that all steady states of a smooth Markov Perfect equilibrium are bounded away from the initial state. Given Assumptions (A1)- (A5), the main result of the Section can now be stated.

Theorem 1 *Given the smooth Markov Perfect equilibrium Ψ (and associated Ψ^*) in the policy-endogenous power regime, and given the solution ψ in the permanent authority regime, the following hold.*

1. *Almost every realized equilibrium path of states $\{\omega_t\}$ and pivotal decision makers $\{i_t\}$ under Ψ is increasing:*

$$\omega_{t+1} > \omega_t \ \ \text{and} \ \ i_{t+1} > i_t, \qquad \forall t. \tag{30}$$

2. *The political preservation effect under Ψ is always negative. For each state ω_t and any state ω_{t+1} realized in equilibrium the equilibrium following ω_t,*[17]

$$P(i,\omega_{t+1};\ \Psi^*)\ <\ 0 \qquad \forall\ i < \mu(\omega_{t+1}). \tag{31}$$

In particular, (31) holds for the pivotal decision maker $i_t = \mu(\omega_t)$.

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[17] A state ω_{t+1} is *realized in the equilibrium continuation following* ω_t if $\omega_{t+1} = \nu_{t+1} + Q(\omega_t, \Psi^*(\omega_t))$ where ν_{t+1} lies in the support of density π.

3. For each $t > 0$,

(a) $\int [R(i_0, \omega_{t+1};\ \Psi^*) \ - \ R(i_0, \omega_{t+1}; \psi^*))] \ \pi(d\nu_{t+1}) \quad > \ -\int P(i_0, \omega_{t+1};\ \Psi^*) \ \pi(d\nu_{t+1})$
implies $\Psi^*(\omega_t) > \Psi(i_0, \omega_t) > \psi^*(\omega_t)$*.*

(b) $0 \ < \ \int [R(i_0, \omega_{t+1};\ \Psi^*) \ - \ R(i_0, \omega_{t+1}; \psi^*))] \ \pi(d\nu_{t+1}) \ < \ -\int P(i_0, \omega_{t+1};\ \Psi^*) \ \pi(d\nu_{t+1})$
implies $\Psi^*(\omega_t) > \psi^*(\omega_t) > \Psi(i_0, \omega_t)$

(c) $0 \ > \ \int [R(i, \omega_{t+1};\ \Psi^*) \ - \ R(i, \omega_{t+1}; \psi^*))] \ \pi(d\nu_{t+1})$
implies $\psi^*(\omega_t) > \Psi^*(\omega_t) > \Psi(i_0, \omega_t)$

Part (1) of the Theorem shows that the equilibrium is monotone in states and decision makers - though not necessarily monotone in policies. Part (2) shows the preservation effect is always negative. Looking at incentives under this effect in isolation from others, it means that political actors who hold power act less aggressively or more conservatively than if power is permanent. The reason is that more aggressive action leads to loss of power to more objectionable political types. The preservation effect may be counter-balanced, of course, by the reformation effect, as indicated in Part (3) which provides an exhaustive (modulo our avoiding the case of equalities) comparison of the possibilities.

In Case (3a), for instance, the reformation effect outweighs the preservation effect. Case 3b is illustrated in Figure 4. Consider decision problem of the PDM i_0 who initially holds power initially under both the policy-endogenous and permanent power regimes. When power is permanent, this decision maker's authority is perpetual, and so his policy choice in a subsequent state ω_t is $\psi(i_0, \omega_t)$. This decision balances the upward sloping marginal disutility $-D_a u_{i_0}$ in current payoff with the downward sloping marginal benefit $\delta D_a Q \cdot \int R_{i_0} \pi(d\nu)$ in terms of the discounted continuation value.

Consider the (hypothetical) decision problem of this same decision maker when power is policy-endogenous. He chooses policy $\Psi(i_0, \omega_t)$. This same decision maker balances the same marginal disutility with a smaller marginal benefit due to the negative preservation effect P_{i_0}. In other words, the pivotal decision maker is *individually* less aggressive when power is policy-endogenous, than when his power is permanent.

Of course, the policy $\Psi(i_0, \omega_t)$ is hypothetical since individual i_0 does not actually hold power in state ω_t in equilibrium. That honor belongs to $i_t = \mu(\omega_t)$. In fact, the equilibrium policy $\Psi^*(\omega_t) = \Psi(i_t, \omega_t)$ is *more*, not less aggressive than under permanent power. Why? The reason is that power evolves toward more aggressive types of decision makers. i_t's marginal disutility $-D_a u_{i_t}$ is lower, and his marginal continuation value $\delta D_a Q \cdot \int [R_{i_t} + P_{i_t}] \pi(d\nu)$ higher than the individual i_0, and this fact more than compensates for the increased timidity of each decision maker individually.

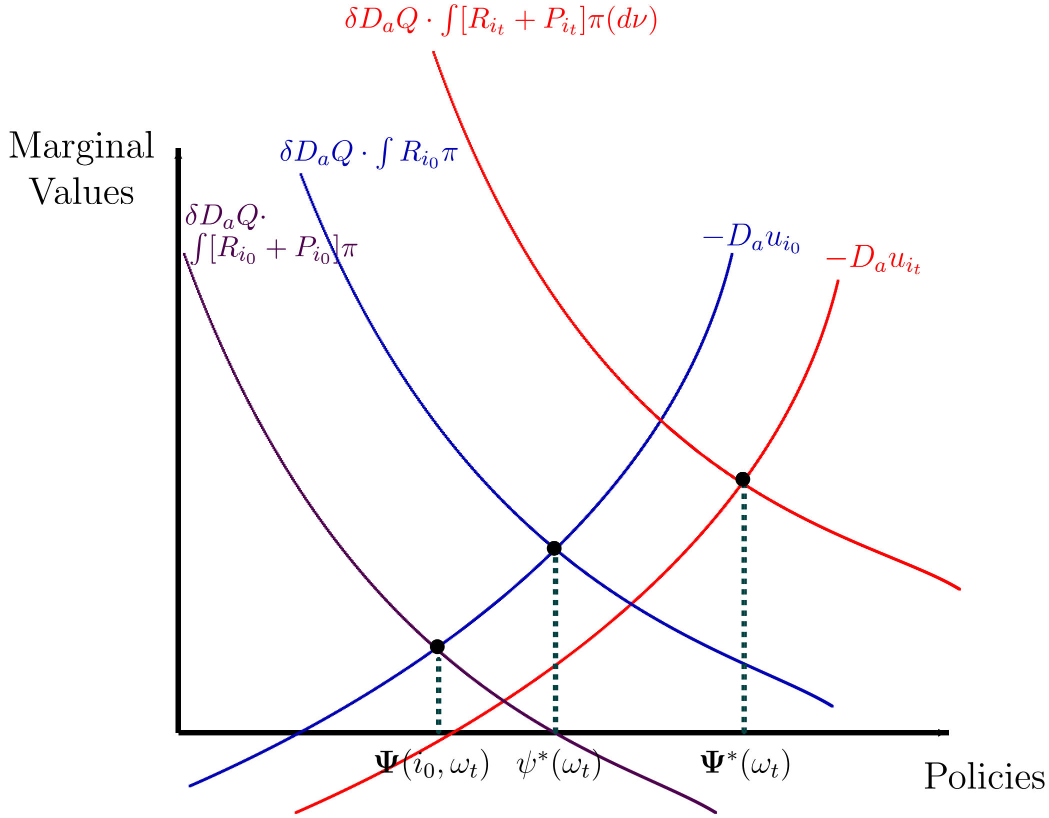

Figure 4: Case (3b) in Theorem 1. Pivotal Decision Makers are Individually Less Aggressive than under Fixed Power, but Power Evolves toward More Aggressive Types

This Section derived results presupposing the existence of a smooth Markov Perfect equilibrium. We take up the existence question in the next Section, identifying conditions under which such a equilibria exist.

5 Equilibrium Existence under Policy-Endogenous Power

TO BE COMPLETED

6 Conclusion

Throughout the analysis, we keep the political institution exogenous in the analysis. This allows us to isolate the effect of changing political institution and focus only on the de facto evolution of the political power within a stable de jure political institution. By construction, the framework does not answer the question of why a certain political institution is chosen and what determines the evolution of the de jure political institution. In the future work, it is interesting to investigate the interaction of the policy-endogenous political power and policy-endogenous political institutions.

TO BE COMPLETED

7 Appendix

Proof of Proposition 1

We first show that in any state ω and for an arbitrary strategy profile Ψ^*, each i's dynamic payoff, $H = u(i, \omega, a) + \delta U(i, \omega'^*)$ is of the form,

$$E(\omega, a;\ \Psi^*)F(i) + G(\omega, a;\ \Psi^*)$$

for some monotone function F. If this is true, then it can be easily verified that H satisfies Order Restrictedness (a generalized single crossing condition), in which case, Rothstein's (1990) Theorem applies to show that majority vote admits a solution, and the solution is the policy that maximizes the payoff of the citizen whose index i is the weighted median. In our case, the weighted median index is given by (13).

So, suppose then that the stage game utility function u satisfies (12). We now show that given a strategy profile Ψ^* the function H satisfies this decomposition. At time t,

$$\begin{aligned}
H &= u(i, \omega_t, a_t) + \delta U(i, \omega_{t+1};\ \Psi^*) \\
&= h(\omega_t, a) f(i) + g(\omega_t, a_t) \\
&\quad + \delta \left(\sum_{\tau=t+1}^{\infty} \delta^{\tau-(t+1)} \left[h(\omega_\tau, \Psi^*(\omega_\tau)) f(i) + g(\omega_\tau, \Psi^*(\omega_\tau)) \right] \right) \\
&= \left[h(\omega_t, a_t) + \delta \sum_{\tau=t+1}^{\infty} \delta^{\tau-(t+1)} h(\omega_\tau, \Psi^*(\omega_\tau)) \right] f(i) \\
&\quad + \left[g(\omega_t, a_t) + \delta \sum_{\tau=t+1}^{\infty} \delta^{\tau-(t+1)} g(\omega_\tau, \Psi^*(\omega_\tau)) \right] \\
&\equiv E(\omega_t, a_t, \Psi^*) F(i) + G(\omega_t, a_t, \Psi^*)
\end{aligned}$$

where $F(i) \equiv f(i)$. This gives the desired result. ∎

Proof of Proposition 2

The proof uses a standard method of undetermined coefficients. To start with, conjecture a solution of the form

$$a = \Psi(\omega) = K\omega,$$

where $K > 0$ is the coefficient to be determined. Notice that the solution implies $\omega_{t+1} = K^{t+1}\omega_0$.

At any point in time, the continuation utility for an arbitrary i would be

$$\begin{aligned}
U(i, \omega_t) &= \sum_{s=0}^{\infty} \delta^s u(i, \omega_{t+s}, a_{t+s}), \\
&= \sum_{s=0}^{\infty} \delta^s \left[\kappa \left(\frac{1}{i} - 1 \right)^2 \omega_{t+s} - a_{t+s}^2 \right], \\
&= \sum_{s=0}^{\infty} \delta^s \left[\kappa \left(\frac{1}{i} - 1 \right)^2 K^s \omega_t - K^{2(s+1)} \omega_t^2 \right] \\
&= \kappa \left(\frac{1}{i} - 1 \right)^2 \frac{1}{1 - \delta K} \omega_t - \frac{K^2}{1 - \delta K^2} \omega_t^2,
\end{aligned}$$

where in the last step we implicitly assume that $|K| < \frac{1}{\delta}$ and $K^2 < \frac{1}{\delta}$. Because $0 < \delta < 1$, it suffices to impose $0 < K < \frac{1}{\sqrt{\delta}}$.

The optimization problem for any i is

$$\begin{aligned} v\left(i,\omega\right) &= \max_{\omega'}\left\{u\left(i,\omega,\omega'\right)+\delta U\left(i,\omega'\right)\right\} \\ &= \max_{\omega'}\left\{\kappa\left(\frac{1}{i}-1\right)^2\omega-\left(\omega'\right)^2+\delta\left[\kappa\left(\frac{1}{i}-1\right)^2\frac{1}{1-\delta K}\omega'-\frac{K^2}{1-\delta K^2}\left(\omega'\right)^2\right]\right\}. \end{aligned}$$

The first-order conditions read

$$-2\omega'+\delta\left[\kappa\left(\frac{1}{i}-1\right)^2\frac{1}{1-\delta K}-2\frac{K^2}{1-\delta K^2}\omega'\right]=0,$$

which leads to

$$\omega'=\frac{\delta\kappa}{2}\frac{1-\delta K^2}{1-\delta K}\left(\frac{1}{i}-1\right)^2.$$

Now use the fact $i=\mu\left(\omega\right)=\frac{1}{1+\sqrt{\omega}}$ to get

$$\omega'=\frac{\delta\kappa}{2}\frac{1-\delta K^2}{1-\delta K}\omega.$$

Comparing with the conjectured solution, we have

$$K=\frac{\delta\kappa}{2}\frac{1-\delta K^2}{1-\delta K},$$

which implies

$$\left(\kappa\delta-2\right)\delta K^2+2K-\kappa\delta=0.$$

Solution to the Equation

If $\kappa\delta-2=0$, we have $K=\frac{\kappa\delta}{2}=1$. Notice that $K=1<\frac{1}{\sqrt{\delta}}$. If $\kappa\delta-2\neq 0$, we have two roots

$$\begin{aligned} K_1 &= \frac{-1+\sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)}}{\left(\kappa\delta-2\right)\delta}, \\ K_2 &= \frac{-1-\sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)}}{\left(\kappa\delta-2\right)\delta}. \end{aligned}$$

To make sure that the roots are real, we need to check the condition that $g\left(\kappa\right)=1+\kappa\delta^2\left(\kappa\delta-2\right)>0$. To see this, notice that $g\left(\kappa\right)$ is a quadratic curve with the lowest point $g\left(\frac{1}{\delta}\right)=1-\delta>0$. Therefore, it must be true that $g\left(\kappa\right)>0$ globally.

Now we show that only K_1 satisfies the condition that $0<K<\frac{1}{\sqrt{\delta}}$. We discuss two possible cases.

First, $\kappa\delta - 2 > 0$. Under this condition, we know that $K_1 > 0$ and $K_2 < 0$. This immediately rules out K_2 as a solution. To check that $K_1 < \frac{1}{\sqrt{\delta}}$, we follow the following equivalent steps.

$$\begin{aligned}
& K_1 < \frac{1}{\sqrt{\delta}} \\
\iff\ & \frac{-1+\sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)}}{\left(\kappa\delta-2\right)\delta} < \frac{1}{\sqrt{\delta}} \\
\iff\ & 1+\left(\kappa\delta-2\right)\sqrt{\delta} > \sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)} \\
\iff\ & \left(1+\left(\kappa\delta-2\right)\sqrt{\delta}\right)^2 > 1+\kappa\delta^2\left(\kappa\delta-2\right) \\
\iff\ & 1+2\left(\kappa\delta-2\right)\sqrt{\delta}+\left(\kappa\delta-2\right)^2\delta > 1+\kappa\delta^2\left(\kappa\delta-2\right) \\
\iff\ & 1 > \sqrt{\delta}
\end{aligned}$$

Second, $\kappa\delta - 2 < 0$. Under this condition, $K_1 > 0$ and $K_2 > 0$. We will show that use $K_1 < \frac{1}{\sqrt{\delta}}$ and $K_2 > \frac{1}{\sqrt{\delta}}$. For K_1, we have

$$\begin{aligned}
& K_1 < \frac{1}{\sqrt{\delta}} \\
\iff\ & \frac{-1+\sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)}}{\left(\kappa\delta-2\right)\delta} < \frac{1}{\sqrt{\delta}} \\
\iff\ & 1+\left(\kappa\delta-2\right)\sqrt{\delta} < \sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)} \\
\iff\ & \left(1+\left(\kappa\delta-2\right)\sqrt{\delta}\right)^2 < 1+\kappa\delta^2\left(\kappa\delta-2\right) \\
\iff\ & 1+2\left(\kappa\delta-2\right)\sqrt{\delta}+\left(\kappa\delta-2\right)^2\delta < 1+\kappa\delta^2\left(\kappa\delta-2\right) \\
\iff\ & 1 > \sqrt{\delta}
\end{aligned}$$

For K_2, we have

$$\begin{aligned}
& K_2 < \frac{1}{\sqrt{\delta}} \\
\iff\ & \frac{-1-\sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)}}{\left(\kappa\delta-2\right)\delta} < \frac{1}{\sqrt{\delta}} \\
\iff\ & \frac{1+\sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)}}{\left(2-\kappa\delta\right)\delta} < \frac{1}{\sqrt{\delta}} \\
\iff\ & \sqrt{1+\kappa\delta^2\left(\kappa\delta-2\right)} < \left(2-\kappa\delta\right)\sqrt{\delta}-1 \\
\iff\ & 1+\kappa\delta^2\left(\kappa\delta-2\right) < \left(1+\left(\kappa\delta-2\right)\sqrt{\delta}\right)^2 \\
\iff\ & 1+2\left(\kappa\delta-2\right)\sqrt{\delta}+\left(\kappa\delta-2\right)^2\delta > 1+\kappa\delta^2\left(\kappa\delta-2\right) \\
\iff\ & 1 < \sqrt{\delta},
\end{aligned}$$

which can not be true if $\delta < 1$.

To summarize, for any $\kappa \neq \frac{2}{\delta}$, the solution is

$$K(\kappa) = \frac{-1 + \sqrt{1 + \kappa\delta^2 (\kappa\delta - 2)}}{(\kappa\delta - 2)\,\delta}.$$

Notice that $K\left(\frac{2}{\delta}\right) = 1$ by L'Hospital's Law. Therefore, $K(\kappa)$ defines the solution for any $\kappa > 0$.

The monotonicity of K

After some messy algebra, the derivative of $K(\kappa)$ can be derived as

$$\frac{dK}{d\kappa} = \frac{-\delta(\kappa\delta - 2) - 1 + \sqrt{1 + \kappa\delta^2 (\kappa\delta - 2)}}{(\kappa\delta - 2)^2 \sqrt{1 + \kappa\delta^2 (\kappa\delta - 2)}}$$

To prove $\frac{dK}{d\kappa} > 0$, we discuss two cases. First, if $\delta(\kappa\delta - 2) + 1 < 0$, then obvious $\frac{dK}{d\kappa} > 0$. Next, suppose $\delta(\kappa\delta - 2) + 1 \geq 0$, the conclusion can be reached from the following equivalent steps.

$$\begin{aligned} & -\delta(\kappa\delta - 2) - 1 + \sqrt{1 + \kappa\delta^2 (\kappa\delta - 2)} > 0 \\ \Longleftrightarrow\ & \sqrt{1 + \kappa\delta^2 (\kappa\delta - 2)} > 1 + \delta(\kappa\delta - 2) \\ \Longleftrightarrow\ & 1 + \kappa\delta^2 (\kappa\delta - 2) > (1 + \delta(\kappa\delta - 2))^2 \\ \Longleftrightarrow\ & 1 > \delta. \end{aligned}$$

This concludes the proof. ∎

Proof of Lemma 1

To prove the result, we first the establish the following Claim.

Claim 1 *The function L ($a = L(\omega, z)$) is smooth, decreasing in ω, increasing in z, and exhibits* decreasing differences *in ω and z.*

Proof of the claim. The fact that $L(\omega, z)$ is increasing in z follows directly from Q strictly increasing in a. Now observe $Q(\omega, a) > Q(\tilde{\omega}, a)$ for any pair $\omega > \tilde{\omega}$. Consequently, given a pair a and $\tilde{a}$, $Q(\omega, \tilde{a}) = Q(\tilde{\omega}, a)$ implies $\tilde{a} < a$. In turn, this implies that a is decreasing in ω, holding fixed the value of z. Observe that L is smooth, and so cross differentiating $Q(\omega, L(\omega, z)) = z$ gives

$$D_\omega D_a Q \cdot D_z L \ + \ D_a^2 Q \cdot D_z L \cdot D_\omega L \ + \ D_a Q \cdot D_\omega D_z L \ = \ 0 \tag{32}$$

By increasing differences of Q (from Assumption (A2)) and the fact that L is increasing in z, the first term is positive. The second term is positive due to concavity of Q and our earlier monotonicity results on L. In order to satisfy the equality, we therefore require $D_\omega D_z L < 0$. Consequently, L exhibits decreasing differences in ω and z. We have therefore proved the Claim. ▮

Now, recall from (26.a) that

$$R(i, \omega_{t+1};\ \Psi^*) \ \equiv\ D_{\omega_{t+1}} u(i, \omega_{t+1}, \Psi^*(\omega_{t+1})) \ +\ D_{\omega_{t+1}} L(\omega, \Theta^*(\omega_{t+1})) \cdot D_{a_{t+1}} u(i, \omega_{t+1}, \Psi^*(\omega_{t+1})) \tag{33}$$

Define $\theta^*(\omega_t)$ implicitly by $\psi^*(\omega_t) = L(\omega_t, \theta^*(\omega_t))$. By extension of the definition of R in (26.a) we have

$$R(i, \omega_{t+1};\ \psi^*) \ \equiv\ D_{\omega_{t+1}} u(i, \omega_{t+1}, \psi^*(\omega_{t+1})) \ +\ D_{\omega_{t+1}} L(\omega, \theta^*(\omega_{t+1})) \ \cdot\ D_{a_{t+1}} u(i, \omega_{t+1}, \psi^*(\omega_{t+1})) \tag{34}$$

It follows directly from (A2) that $\Psi^*(\omega_t) > \psi^*(\omega_t)$ holds whenever $D_{\omega_{t+1}} u(i, \omega_{t+1}, \Psi^*(\omega_{t+1})) > D_{\omega_{t+1}} u(i, \omega_{t+1}, \psi^*(\omega_{t+1}))$. Also by strict concavity of u,

$$D_{a_{t+1}} u(i, \omega_{t+1}, \psi^*(\omega_{t+1})) > D_{a_{t+1}} u(i, \omega_{t+1}, \Psi^*(\omega_{t+1}))$$

whenever $\Psi^*(\omega_t) > \psi^*(\omega_t)$. But since $D_{a_{t+1}} u < 0$ by (A1), the Lemma is established if we can show both $D_{\omega_{t+1}} L < 0$ and $D_{\omega_{t+1}} L(\omega, \theta(i, \omega_{t+1})) > D_{\omega_{t+1}} L(\omega, \Theta^*(\omega_{t+1}))$ whenever $\psi^*(\omega_{t+1}) < \Psi^*(\omega_{t+1})$.

By definition of $\Psi^*(\omega_t) = L(\omega_t, \Theta^*(\omega_t))$ and $\psi^*(\omega_t) = L(\omega_t, \theta^*(\omega_t))$. Consequently, by the monotonicity of L in z, $\Psi^*(\omega_t) > \psi^*(\omega_t)$ whenever $\Theta^*(\omega_t) > \theta^*(\omega_t)$. The application of Claim 1 therefore gives $D_{\omega_{t+1}} L < 0$ and $D_{\omega_{t+1}} L(\omega, \theta^*(\omega_{t+1})) > D_{\omega_{t+1}} L(\omega, \Theta^*(\omega_{t+1}))$. Hence, we have shown

$$\begin{aligned} & D_{\omega_{t+1}} u(i, \omega_{t+1}, \Psi^*(\omega_{t+1})) \ +\ D_{\omega_{t+1}} L(\omega, \Theta^*(\omega_{t+1})) \ \cdot\ D_{a_{t+1}} u(i, \omega_{t+1}, \Psi^*(\omega_{t+1})) \\ > \ & D_{\omega_{t+1}} u(i, \omega_{t+1}, \psi^*(\omega_{t+1})) \ +\ D_{\omega_{t+1}} L(\omega, \theta^*(\omega_{t+1})) \ \cdot\ D_{a_{t+1}} u(i, \omega_{t+1}, \psi^*(\omega_{t+1})) \end{aligned} \tag{35}$$

whenever $\Psi^*(\omega_t) > \psi^*(\omega_t)$. We conclude the proof of the Lemma. ▮

Proof of Lemma 2 Using the inverse function $L = Q^{-1}$, consider the Euler equations in each regime when i_0 is the (possibly hypothetical) decision maker in state ω_t. They are given by

$$\begin{aligned} D_{z_{t+1}}L(\omega_t,\Theta(i_0,\omega_t))\cdot D_{a_t}u(i_0,\omega_t,\Psi(i_0,\omega_t))+\delta\int[R(i_0,\omega_{t+1};\ \Psi^*)\ +\ P(i_0,\omega_{t+1};\ \Psi^*)]\ \pi(d\nu_{t+1}) &= 0 \\ D_{z_{t+1}}L(\omega_t,\theta^*(\omega_t))\cdot D_{a_t}u(i_0,\omega_t,\psi^*(\omega_t))+\delta\int[R(i_0,\omega_{t+1};\ \psi^*)]\ \pi(d\nu_{t+1}) &= 0 \end{aligned} \tag{36}$$

(where, recall that θ is defined in the proof of the previous Lemma).

By concavity of u and Q and by Claim 1, $\Psi(i_0,\omega_t) > \psi^*(\omega_t)$ whenever $D_{a_t}u(i_0,\omega_t,\Psi(i_0,\omega_t)) < D_{a_t}u(i_0,\omega_t,\psi^*(\omega_t))$ and whenever $D_{z_{t+1}}L(\omega_t,\Theta(i,\omega_t)) < D_{z_{t+1}}L(\omega_t,\theta^*(\omega_t))$. Consequently,

$$\int[R(i_0,\omega_{t+1};\ \Psi^*)\ +\ P(i_0,\omega_{t+1};\ \Psi^*)]\ \pi(d\nu_{t+1})\ >\ \int[R(i_0,\omega_{t+1};\ \psi)]\ \pi(d\nu_{t+1}) \tag{37}$$

which concludes the proof of the Lemma. ▮

Proof of Theorem 1

Define the objective function of a pivotal decision maker i in state ω_t in the policy-endogenous power regime by

$$H(i,\omega_t,a_t,U) = u(i,\omega_t,a_t)\ +\ \delta\int U(i,\nu'+Q(\omega_t,a_t);\ \Psi^*)d\pi(\nu') \tag{38}$$

Using the inverse function L, one can rewrite the objective function H defined in (38) as

$$H(i,\omega_t,L(\omega_t,z_t),\ U)\ =\ u(i,\omega_t,L(\omega_t,z_t))\ +\ \delta\int U(\nu'+z_t;\ \Psi^*)\pi(\nu')d\nu' \tag{39}$$

The following Claim is will be used in the Proof of the Theorem.

Claim 2 *The objective function $H(i,\omega_t,L(\omega_t,z_t),\ U)$ defined in (39) has increasing differences in the pair ω_t and z_t and in the pair i and z_t.*

Proof of Claim 2 For the proof, it will be convenient to drop time subscripts. Notice, the continuation value U does not vary with the current state ω. Hence, to show increasing differences in ω and z, it suffices to show that $u(i,\omega,L(\omega,z))$ has increasing differences in ω and z. Totally differentiating u with respect to ω yields

$$D_\omega u\cdot D_a u\cdot D_\omega L \tag{40}$$

Differentiating this expression with respect to z gives

$$D_\omega D_z u \;=\; D_\omega D_a \cdot D_z L \;+\; D_a^2 u \cdot D_z L \cdot D_\omega L \;+\; D_a u \cdot D_\omega D_z L \tag{41}$$

By increasing differences of u and by Claim 1 (in the proof of Lemma 2), the first term is positive. By concavity of u and Claim 1, the second term is positive. Finally, by the fact that u is decreasing in a and once again by Claim 1, the last term is positive. Hence, $D_\omega D_z u > 0$ term-by-term, and so H has increasing differences in z and ω.

Next, we show that $H(i, \omega, L(\omega, z),\ U)$ has increasing differences in i and z. By a simple change of variables, we rewrite (39) as

$$H(i, \omega, L(\omega, z),\ U) \;=\; u(i, \omega, L(\omega, z)) \;+\; \delta \int U(i, \omega')\ \pi(\omega' - z) d\omega' \tag{42}$$

For H to have increasing differences in i and z, we establish, term by term, that $D_i D_z H > 0$. Observe,

$$D_i D_z H \;=\; D_i D_a u \cdot D_z L \;-\; \delta \int D_i U \cdot D_{\omega'} \pi(\omega' - z) d\omega' \tag{43}$$

Notice that $\int D_{\omega'} \pi(\omega' - z) d\omega' = 0$. Letting $\bar{\omega}(z)$ denote the median value of next period's state given z, it follows that $\int_{\omega < \bar{\omega}(z)} D_{\omega'} \pi(\omega' - z) d\omega' = 1/2$ and $\int_{\omega \geq \bar{\omega}(z)} D_{\omega'} \pi(\omega' - z) d\omega' = -1/2$. Notice also that (A4)) can be written as

$$D_i D_a u \cdot D_z L \;>\; \frac{\delta}{2(1-\delta)} \left[\sup(D_i u) \;-\; \inf(D_i u)\right] \tag{44}$$

Using these two facts, it follows that

$$\begin{aligned}
D_i D_a u \cdot D_z L \;&>\; \frac{\delta}{2(1-\delta)} \left[\sup(D_i u) \;-\; \inf(D_i u)\right] \\
&\geq\; \frac{\delta}{2} \left[\sup(D_i U) \;-\; \inf(D_i U)\right] \\
&=\; \delta \left[\sup(D_i U) \cdot \int_{\omega' < \bar{\omega}(z)} D_{\omega'} \pi(\omega' - z) d\omega' \;+\; \inf(D_i U) \cdot \int_{\omega' \geq \bar{\omega}(z)} D_{\omega'} \pi(\omega' - z) d\omega'\right] \\
&\geq\; \delta \left[\int_{\omega' < \bar{\omega}(z)} D_i U \cdot D_{\omega'} \pi(\omega' - z) d\omega' \;+\; \int_{\omega' \geq \bar{\omega}(z)} D_i U \cdot D_{\omega'} \pi(\omega' - z) d\omega'\right] \\
&=\; \delta \int D_i U(i, \omega') \cdot D_{\omega'} \pi(\omega' - z) d\omega'
\end{aligned} \tag{45}$$

Hence $D_i D_z H > 0$ as required. We conclude the proof of the Claim. ∎

Proof of Part (1). Observe $\Theta(i, \omega_t) = \arg\max_z H(i, \omega_t, L(\omega_t, z_t),\ U)$ is the state transition function as a function of the PDM's identity. By Topkis' Monotonicity Theorem and Claim 2, $D_i\Theta > 0$ and $D_{\omega_t}\Theta > 0$. By definition, $\Theta^*(\omega_t) = \Theta(\mu(\omega_t), \omega_t)$. Applying the Chain Rule to Θ^*, $D_{\omega_t}\Theta^* = D_{\omega_t}\Theta + D_i\Theta D_{\omega_t}\mu$. Hence, term-by-term, all partial derivatives on the right-hand side are positive. Note that by definition, $\Theta^*(\omega_0) = \Theta(i_0, \omega_0)$. By (A1) and (A5), $Q(\omega_0, a) > \omega_0$ for all a. Using this fact, the monotonicity of Θ^*, and the fact Q is concave, it must hold that $\Theta^*(\omega_0) = Q(\omega_0, \Psi^*(\omega_0)) > \omega_0$. Given that the support of π lies above 1, we also have $\omega_1 = \nu_1 + Q(\omega_0, \Psi^*(\omega_0)) > \omega_0$, and so $i_1 = \mu(\omega_1) > \mu(\omega_0) = i_0$. Continuing in this fashion, we establish $\omega_{t+1} > \omega_t$ and $i_{t+1} > i_t$ until either a steady state is reached, or until $i_t = 1$.

Proof of Part (2). Combining the definition of H in (38) and (39) and that of the distortion function Δ in (19), we have

$$\begin{aligned} D_{z_{t+1}} H(i_t, \omega_{t+1}, L(\omega_{t+1}, \Theta^*(\omega_{t+1}))\ ) &= D_{a_t} H(i_t, \omega_{t+1}, \Psi^*(\omega_{t+1})) \cdot D_{z_{t+1}} L(\omega_{t+1}, \Theta^*(\omega_{t+1})) \\ &= \Delta(i_t, \omega_{t+1}; \Psi^*) \cdot D_{z_{t+1}} L(\omega_{t+1}, \Theta^*(\omega_{t+1})) \end{aligned} \tag{46}$$

By Claim 2, $D_{z_{t+1}} H(i_t, \omega_{t+1}, L(\omega_{t+1}, \Theta^*(\omega_{t+1}))\ )$ is increasing in i_t, and so $\Delta(i_t, \omega_{t+1}; \Psi^*)$ must be increasing in i_t as well. Since $i_t < i_{t+1} = \mu(\omega_{t+1})$ by Part (1), it follows that

$$\Delta(i_t, \omega_{t+1}; \Psi^*) \ < \ \Delta(i_{t+1}, \omega_{t+1}; \Psi^*) \ = \ 0 \tag{47}$$

where the latter equality follows from the type i_{t+1}'s first order condition in state ω_{t+1}. Indeed, this inequality holds for any $i < i_{t+1}$, not just i_t. Now recall that the preservation effect is given by

$$P(i_t, \omega_{t+1};\ \Psi^*) \ \equiv \ D_{z_{t+1}} L \ \cdot \ D_{\omega_{t+1}} \Theta^* \ \cdot \ \Delta(i_t, \omega_{t+1}; \Psi^*) \tag{48}$$

By Claim 1, $D_{z_{t+1}} L > 0$ and from our earlier argument in Part (1), $D_{\omega_{t+1}}\Theta^* > 0$. Therefore, $P(i_t, \omega_{t+1};\ \Psi^*) < 0$ follows from the fact that $\Delta(i_t, \omega_{t+1}; \Psi^*) < 0$.

Proof of Part (3). The argument in Part 1 established $\Theta^*(\omega_t) > \Theta(i_0, \omega_t)$ for all $t > 1$. By definition,

$$\Psi^*(\omega_t) = L(\omega_t, \Theta^*(\omega_t)) \ > \ L(\omega_t, \Theta(i_0, \omega_t)) = \Psi(i_0, \omega_t) \tag{49}$$

which is asserted in all the subcases (a)-(c). The remainder of Part (3) follows directly from Lemma 1 and Lemma 2.

We conclude the proof of the Theorem. ∎

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