Wholesale Markets in Telecommunications*

Marc Bourreau† Johan Hombert‡ Jerome Pouyet§ Nicolas Schutz¶

May 21, 2007

Abstract

In telecommunications some operators have deployed their own networks whereas others have not. The latter firms must purchase wholesale products from the former to be able to compete on the final market. We show that, even when network operators compete in prices and offer homogenous products on the wholesale market, that market may not be perfectly competitive. Based on our theoretical analysis, we derive some policy implications for the broadband and the mobile telephony markets.

Journal of Economic Literature Classification Number: L13, L51.
Keywords: Upstream and downstream markets, vertical integration, telecommunications.

*We gratefully acknowledge France Télécom's and CEPREMAP's intellectual and financial support. We also wish to thank Philippe Février, Bernard Caillaud, Bruno Jullien, Laurent Lamy, Marc Lebourges, Laurent Linnemer, Patrick Rey, Bernard Salanié, Jean Tirole, Timothy Van Zandt and seminar participants for helpful comments on an earlier draft.

†ENST and CREST-LEI. Address: ENST, Department of Economics and Social Sciences, 46 rue Barrault, 75634 Paris Cedex 13, France. E-mail: `marc.bourreau@enst.fr`.

‡CREST-ENSAE and PSE. Address: ENSAE, 3 Avenue Pierre Larousse, 92245, Malakoff, France. E-mail: `johan.hombert@ensae.fr`.

§Ecole Polytechnique and CEPR. Address: Department of Economics, Ecole Polytechnique, 91128 Palaiseau Cedex, France. Phone: +33(0)169332646. Fax: +33(0)169333858. E-mail: `jerome.pouyet@polytechnique.edu`

¶PSE. Address: Paris-Jourdan Sciences Economiques, 48 Boulevard Jourdan, 75014, Paris, France. E-mail: `nicolas.schutz@pse.ens.fr`.

1 Introduction

In telecommunications, most markets are populated with two types of firms. Facility-based firms roll out proprietary networks and rely mainly on their own infrastructures to provide services to end-customers. Service-based firms do not invest in facilities and lease access to the networks of facility-based firms in order to offer services on retail markets.[1] Consequently, markets for access, called wholesale markets, have emerged. For example, in the mobile market, Mobile Virtual Network Operators (MVNOs) do not have a spectrum license nor a mobile network and therefore have to purchase a wholesale mobile service from Mobile Network Operators (MNOs). In the broadband market, Digital Subscriber Line (DSL) operators and cable networks own a broadband infrastructure and compete at the retail level. They can also compete to provide wholesale broadband services to service-based firms.

Potential competition on the markets for access is viewed as a way to boost competition at the retail level. If service-based operators can get cheap access to end-users, so the argument goes, the competitive pressure on final markets should increase, with a direct benefit passed through to end-users in the form of price cuts. Therefore, among practitioners, one question arises recurrently: when facility-based competition is in place, will the wholesale market deliver its promises? There is so far no clear consensus on the answer. Some telecoms regulators question the idea that a wholesale market can be competitive and therefore regulate it on such grounds. Others consider that in an unregulated environment facility-based firms would lease the access to their infrastructures at a competitive price, allowing service-based operators to compete on a level playing field with facility-based firms.[2]

The main objective of this paper is to provide an economic analysis of the functioning of such wholesale markets in telecommunications. Using a stylized model, we show that these wholesale markets may be non-competitive even when all the usual ingredients of Bertrand competition are in place, and that they cannot be studied in isolation of the related retail markets.

In our model, two vertically integrated firms and a pure downstream firm compete in prices with differentiated products. The goods sold to end-users are derived from an intermediate input that the integrated firms can produce in-house. Integrated firms compete, first on the upstream market to provide the input to the pure downstream firm, and second on the downstream market with the pure downstream firm. The upstream market exhibits the usual ingredients of tough competition: integrated firms compete in prices, produce a perfectly homogeneous upstream good and incur the same constant marginal cost.

A first question is whether the perfect competition outcome is an equilibrium. Under

[1] Note that facility-based firms might lease some network elements to other integrated firms, and that service-based firms might have to install some telecommunications equipments. But in a nutshell, a facility-based firm builds more than it leases, and a service-based firm leases more than it builds.

[2] E.g., the UK telecoms regulatory authority, Ofcom, in its review of the wholesale broadband market (Ofcom (2004)) argues that: "Under competitive market conditions, both cable and BT would have an incentive to offer a wholesale product. [...] In a competitive market, cable's and BT's upstream (network) and downstream (retail) divisions would each earn a normal return".

reasonable conditions, there exists indeed an equilibrium in which both integrated firms sell the upstream good at the corresponding marginal cost. The second issue is whether this is the only equilibrium. If one considers the upstream market in isolation of the downstream one, one could be tempted to believe that integrated firms would always have incentives to cut prices to gain wholesale revenues until the marginal cost is attained, as the usual logic underlying Bertrand competition would predict. While attractive, this reasoning misses an important point.

The reason lies in the softening effect: the integrated firm which supplies the upstream market at a strictly positive price-cost margin adopts a soft behavior on the downstream market. Realizing that final customers lost on the downstream market may be recovered indirectly via the upstream market, the upstream supplier is willing to preserve its upstream profit through its downstream pricing. This new effect is robust, as it exists under minimal assumptions on the downstream demand functions. In particular, its existence does not rely on any assumptions regarding the nature of the strategic interactions between downstream prices. It leads to the following trade-off. When an integrated firm undercuts its rival on the upstream market, it earns additional upstream profits at the cost of making its integrated rival more aggressive on the downstream market. When the softening effect is strong enough, the incentives to undercut the upstream market vanish and the Bertrand logic collapses. In particular, consider as a benchmark the situation in which one integrated firm is exogenously given a monopoly position on the upstream market, and assume that the pure downstream firm is not completely foreclosed.[3] Provided that the softening effect is strong enough, competition on the upstream market does not destabilize this outcome. Put differently, the monopoly outcome may persist even under the threat of competition on the upstream market.

The degree of differentiation at the downstream level has an important impact on the strength of the softening effect, hence on the competitiveness of the upstream market. Intuitively, when final products are strongly differentiated, downstream demands are almost independent and the softening effect is consequently weak. As a result, undercutting on the upstream market is always profitable and this market ends up being competitive. Conversely, when downstream products are strong substitutes, the softening effect is strong and the monopoly outcome is an equilibrium. In a nutshell, there is a tension between downstream and upstream competitiveness.

Assuming that downstream prices are strategic complements, we also show that the presence of an additional pure upstream firm restores the competitiveness of the upstream market. Broadly speaking , when an integrated firm undercuts a pure upstream competitor on the upstream market, it does not face the same trade-off as when it undercuts an integrated rival. In particular, in the former case, the integrated firm does not lose the softening effect, i.e., it does not make its integrated rival more aggressive on the downstream market. This implies that integrated firms are always willing to undercut the pure upstream competitor.

[3]Throughout most of the paper, the issue of complete versus partial foreclosure is left aside as it is orthogonal to our focus.

Conversely, the pure upstream competitor is always willing to corner the upstream market, which ends up being competitive. A similar logic applies when one of the integrated firms is vertically separated.

Our paper is related to different strands of the literature. The literature on one-way access pricing deals with situations in which a service-based firm must gain access to the network of a historical incumbent; see Laffont and Tirole (2001), Armstrong (2002) and De Bijl and Peitz (2002). These papers are, by definition, silent on the issue of the competitiveness of wholesale markets, which is central to our analysis.

Our paper is closely related to the literature on vertical foreclosure. The so-called traditional foreclosure theory argues that integrated firms have incentives to raise their rivals' costs through their pricing of the intermediate input. After having been challenged by the Chicago School, this theory has been given firmer theoretical grounds by several authors; see Hart and Tirole (1990), Ordover, Saloner, and Salop (1990), Chen (2001) and Rey and Tirole (2005) for an extensive survey. However, none of these papers deal with the issue of several integrated firms competing to supply pure downstream firms, which is crucial in telecoms.[4] Our analysis unveils that integrated firms do not always have incentives to corner the upstream market and that there is a grain of truth both in the traditional foreclosure theory and in the Chicago School criticism.

In Ordover and Shaffer (2006) and Brito and Pereira (2006), several integrated firms compete to provide access to a pure downstream entrant. Yet, their focus is different from ours, since they are mainly interested in whether a wholesale market will emerge at all, i.e., whether or not the entrant will be completely foreclosed from the downstream market.[5] When the entrant is not completely foreclosed, both papers argue that the perfect competition outcome emerges on the upstream market. Our paper challenges that view and shows that this result has limitations. When a wholesale market has emerged, whether or not it achieves the perfect competition outcome is rooted in the softening effect and cannot be taken for granted. To make our contribution even more transparent, we abstract away from the issue of complete foreclosure throughout most of our analysis.[6]

The paper is organized as follows. Section 2 describes the model. Section 3 presents our main results, namely the possibility of non-competitive equilibria on the upstream market, and illustrates them. Section 4 discusses several extensions and robustness checks of our

[4] Notice that Salinger (1988) and **?** do consider situations in which the upstream market is supplied by several integrated firms, but they do not study tough price competition. The former assumes Cournot competition on both markets, while the latter focuses on tacit collusion on the upstream market.

[5] Ordover and Shaffer (2006) show that complete foreclosure is more likely to occur when the entrant's product cannibalizes the product of its upstream supplier disproportionately. Brito and Pereira (2006) consider asymmetrically localized firms in the Hotelling-Salop circle model of spatial competition and find that the existence of a complete foreclosure equilibrium depends on the firms' position on the circle.

[6] Höffler and Schmidt (2007) take a complementary perspective and study the impact on welfare of having pure downstream firms but assume an exogenous structure on the upstream market (i.e., which integrated firms are upstream suppliers or not). Our results tend to indicate that allowing competition on the upstream market might still leave integrated firms with as much market power as when the market structure is exogenously fixed.

basic framework, including the possibility of complete foreclosure. Section 5 builds on the theoretical analysis to derive some policy implications, in terms of regulation and competition policy, for the telecommunications industry. Finally, section 6 concludes and presents a few avenues for future research.

2 Model

Firms. There are two vertically integrated firms, denoted by 1 and 2, and one pure downstream firm, denoted by d. Integrated firms are composed of an upstream and a downstream unit, which produce the intermediate input and the final good, respectively. The pure downstream competitor is composed of a downstream unit only. In order to be active on the final market, it must purchase the intermediate input from one of the integrated firms on the upstream market.

Both integrated firms produce the upstream good under constant returns to scale at unit cost c_u. The downstream product is derived from the intermediate input on a one-to-one basis at cost $c_k(.)$ for firm $k \in \{1, 2, d\}$. We assume that integrated firms have the same downstream cost function: $c_1(.) = c_2(.)$.

Markets. All firms compete in prices on the downstream market and provide imperfect substitutes to final customers. Let p_k be the downstream price set by firm $k \in \{1, 2, d\}$ and $p \equiv (p_1, p_2, p_d)$ the vector of final prices. Firm k's demand is denoted by $D_k(p)$; it depends negatively on its price and positively on its competitors' prices: $\partial D_k / \partial p_k < 0$ and $\partial D_k / \partial p_{k'} > 0$ for $k \neq k' \in \{1, 2, d\}$. Symmetry of the integrated firms is assumed again: $D_1(p_1, p_2, p_d) = D_2(p_2, p_1, p_d)$ and $D_d(p_1, p_2, p_d) = D_d(p_2, p_1, p_d)$ for all p.

On the upstream market, integrated firms compete in prices and offer perfectly homogeneous products. We denote by a_i the upstream price set by integrated firm $i \in \{1, 2\}$.[7] The structure of the model is summarized in Figure 1.

Timing. The sequence of decision-making is as follows:

Stage 1 – Upstream competition: The vertically integrated firms simultaneously and non-cooperatively set their prices on the upstream market. Then, the terms of the offers become known to all parties and the pure downstream firm elects at most one upstream provider.

Stage 2 – Downstream competition: All firms simultaneously and non-cooperatively choose their prices on the downstream market.

We focus on pure strategies subgame-perfect equilibria and reason by backward induction.

[7]Throughout the paper, subscripts i and j refer to integrated firms only, whereas subscript k refers either to an integrated firm or to the pure downstream firm.

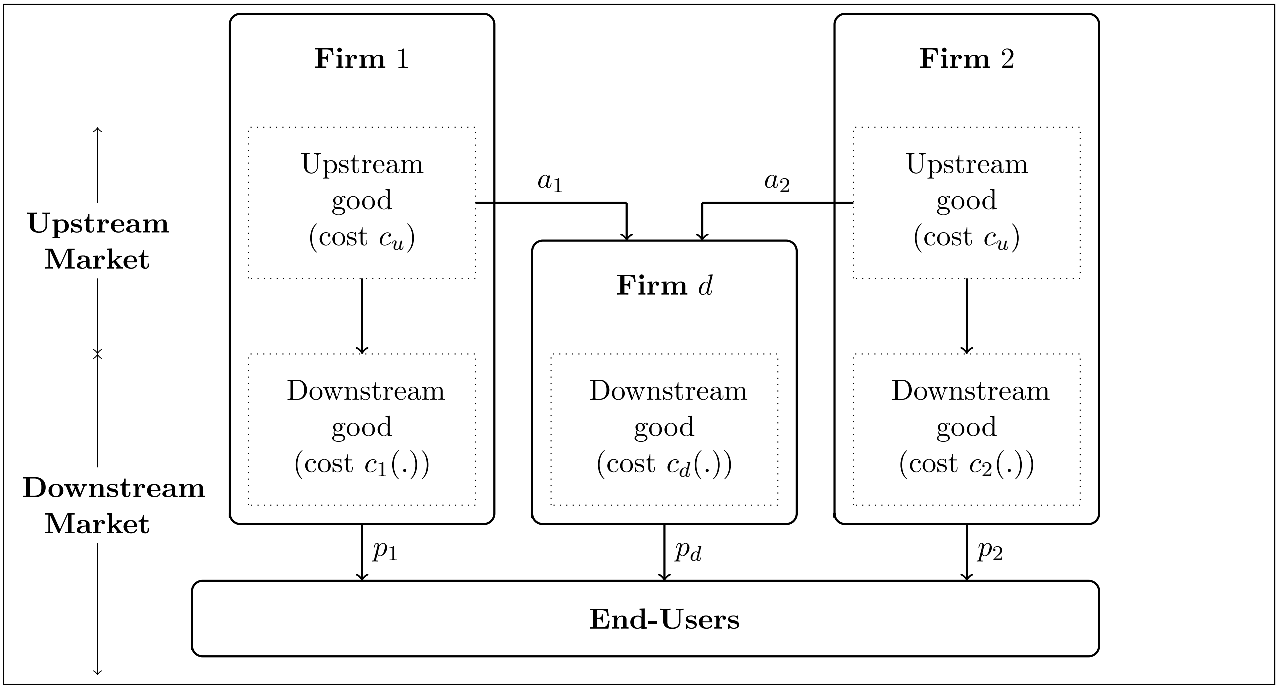

Figure 1: Structure of the model.

Profits. The profit of integrated firm $i \in \{1, 2\}$ which supplies the upstream market at price a_i is:[8]

$$\tilde{\pi}_i^{(i)}(p, a_i) = (p_i - c_u)D_i(p) - c_i(D_i(p)) + (a_i - c_u)D_d(p).$$

The profit of integrated firm $j \neq i \in \{1, 2\}$ which does not supply the upstream market is given by:

$$\tilde{\pi}_j^{(i)}(p, a_i) = (p_j - c_u)D_j(p) - c_j(D_j(p)).$$

The profit of pure downstream firm d is given:

$$\tilde{\pi}_d^{(i)}(p, a_i) = (p_d - a_i)D_d(p) - c_d(D_d(p)).$$

Note that when the upstream price is equal to the upstream unit cost, i.e., $a_i = c_u$, there is no upstream profit and all firms compete on a level playing field. This would be the outcome if the upstream market were perfectly competitive.

3 Main Results

In this section, we develop our main argument: competition on the upstream market (stage 1) does not always yield the perfect competition outcome, even though integrated firms offer homogenous products and compete in prices on that market. We then prove two results of interest. First, the emergence of non-competitive equilibria on the upstream market becomes

[8]Throughout the paper, the superscript in parenthesis indicates the identity of the upstream supplier.

more likely as downstream competition becomes fiercer. Second, provided that downstream prices are strategic complements, the mere presence of a pure upstream competitor restores the competitiveness of the upstream market.

3.1 Preliminaries

Downstream market competition. Consider first the situation in which upstream offers (a_i, a_j) are such that pure downstream firm d is completely foreclosed from the downstream market. We denote by π_{duopoly} the profit earned by each integrated firm in this case.

Conversely, consider situations in which at least one of the upstream offers is acceptable, i.e., allows firm d to be active on the downstream market. Denote by $i \in \{1, 2\}$ the upstream supplier and let $j \neq i$. The best-response in downstream price of firm $k \in \{1, 2, d\}$ is defined by $BR_k^{(i)}(p_{-k}, a_i) = \arg\max_{p_k} \tilde{\pi}_k^{(i)}(p, a_i)$.[9] To streamline the analysis, we make the following standard assumptions: $BR_k^{(i)}(.,.)$ is unique, bounded, characterized by the corresponding first-order condition, and such that $|\partial BR_k^{(i)}/\partial p_{k'}| < 1$, for all $k' \neq k \in \{1, 2, d\}$.[10] We denote by $p_k^{(i)}(a_i)$ the equilibrium price of firm $k \in \{1, 2, d\}$ and by $p^{(i)}(a_i)$ the vector of these downstream prices. At the equilibrium of this subgame, firms' profits are given by functions $\pi_k^{(i)}(a_i) \equiv \tilde{\pi}_k^{(i)}(p^{(i)}(a_i), a_i)$, which are defined over the set of acceptable offers. Note that $p_i^{(i)}(c_u) = p_j^{(i)}(c_u)$ and $\pi_i^{(i)}(c_u) = \pi_j^{(i)}(c_u)$.

Choice of upstream supplier. If only one integrated firm has made an acceptable offer, then it is obviously chosen by the pure downstream firm.

Consider now that both offers are acceptable. If $\pi_d^{(i)}(a_i) > \pi_d^{(j)}(a_j)$, then firm d chooses firm i as its upstream supplier. If both offers lead to the same profit, then firm d chooses any of them. We now make the following economically meaningful assumption:

Assumption 1. $\pi_d^{(i)}(.)$ *is strictly decreasing.*[11]

If firm d preferred to choose the most expensive upstream provider, we would have another, somewhat trivial (and pathological), reason for the existence of non-competitive equilibria on the upstream market. Assumption 1 rules out these cases.

[9] As usual, p_{-k} is the vector obtained by removing p_k from vector p.

[10] This corresponds to the usual stability condition for a duopoly but it is less stringent than the stability requirement in an oligopoly with $n > 2$ firms, which can be stated as $\sum_{k' \neq k} \left| \partial BR_k^{(i)}/\partial p_{k'} \right| < 1$. See Vives (1999).

[11] An increase in firm d's cost has typically two impacts on its profit. First, the price-cost margin is directly reduced, leading unambiguously to a lower profit. Second, the best-response (in downstream price) of firm d shifts upward, which affects the equilibrium of the final market. These two effects are standard in any IO models with price competition and product differentiation. In our context, there is also a third effect since the best-response of the upstream supplier also shifts upward (the softening effect that we explain later on). The overall impact on firm d's profit is a priori ambiguous and depends typically on the strategic interaction on the downstream market. In line with most IO models, Assumption 1 posits that the direct effect outweighs the strategic ones.

Upstream monopoly benchmark. Consider the hypothetical scenario in which the upstream market is monopolized by integrated firm i. Its upstream pricing decision involves a trade-off between partial and complete foreclosure. The elimination of one downstream rival may create discontinuous changes on the demand faced by the remaining competitors. These discontinuities may in turn cause the non-existence of a solution to firm i's optimization problem on the upstream market. To avoid such issues, and to focus on situations in which complete foreclosure does not arise in equilibrium, we make the following assumptions:

Assumption 2. $\pi_i^{(i)}(.)$ *is quasiconcave and admits a unique maximum at* $a_m > c_u$.

Assumption 3. $\pi_i^{(i)}(a_m) > \pi_{duopoly}$.

To summarize, if the upstream market were exogenously monopolized, the pure downstream firm would not be completely foreclosed; as an aside, this means equivalently that complete foreclosure never arises when integrated firms compete on the upstream market. We discuss this assumption in section 4. Besides, monopoly market power on the upstream market leads to a strictly positive mark-up on the price of the upstream good, hence to partial foreclosure under an exogenously monopolized upstream market. a_m is referred to as the monopoly upstream price.

3.2 Persistence of the monopoly outcome

We now study the first stage of our game in which integrated firms compete on the upstream market, and establish the main result of the paper. We show that the usual mechanism of Bertrand competition may be flawed and that non-competitive equilibria may exist.

Assume that integrated firm i has made an acceptable upstream offer to firm d, $a_i > c_u$, and let us see whether integrated firm j is willing to corner the upstream market, as it is the case with standard (single-market) Bertrand competition.

The integrated firms' best-responses on the downstream market are characterized by the following first-order conditions:

$$\frac{\partial \tilde{\pi}_i^{(i)}}{\partial p_i}(p, a_i) = D_i + (p_i - c_i'(D_i) - c_u)\frac{\partial D_i}{\partial p_i} + (a_i - c_u)\frac{\partial D_d}{\partial p_i} = 0, \quad (1)$$

$$\frac{\partial \tilde{\pi}_j^{(i)}}{\partial p_j}(p, a_i) = D_j + (p_j - c_j'(D_j) - c_u)\frac{\partial D_j}{\partial p_j} = 0. \quad (2)$$

The comparison between (1) and (2) shows that the upstream supplier has more incentives to raise its downstream price than its integrated rival. It internalizes the fact that, when it increases its downstream price, some of the customers it loses will purchase from the pure downstream firm, thereby increasing its upstream revenues. As formally shown in Appendix, this mechanism, together with our stability assumption, implies that the upstream supplier charges a higher downstream price than its integrated rival.[12]

[12]This holds whatever the nature of the strategic interaction between downstream prices.

Lemma 1. *Let $a_i > c_u$ be an acceptable offer. Then the upstream supplier charges a strictly higher downstream price than its integrated rival:*

$$p_i^{(i)}(a_i) > p_j^{(i)}(a_i).$$

Proof. See Appendix A.1. □

The literature on vertical foreclosure has long emphasized that an integrated firm may have incentives to preserve its downstream profit through its upstream offer. Lemma 1 points out that the reverse mechanism also exists: the integrated firm which supplies the upstream market has incentives to preserve its upstream profit through its downstream pricing. Realizing that final customers lost on the downstream market may be recovered via the upstream market, the upstream supplier is less aggressive on the downstream market in order not to jeopardize its upstream profit. We shall refer to that mechanism as the 'softening effect'.[13]

This effect implies that the upstream supplier is a soft competitor on the downstream market. This favors the other integrated firm which, by a revealed preference argument, earns more downstream profit than the upstream supplier.

Lemma 2. *Let $a_i > c_u$ be an acceptable offer. Then, the upstream supplier earns strictly smaller downstream profits than its integrated rival:*

$$\left[p_i^{(i)}(a_i) - c_u\right] D_i(p^{(i)}(a_i)) - c_i\left(D_i(p^{(i)}(a_i))\right) < \left[p_j^{(i)}(a_i) - c_u\right] D_j(p^{(i)}(a_i)) - c_j\left(D_j(p^{(i)}(a_i))\right).$$

Proof. See Appendix A.2. □

A key consequence of that result is that we cannot tell unambiguously which of the integrated firms earns more total profits. On the one hand, the upstream supplier extracts revenues from the upstream market. On the other hand, its integrated rival benefits from larger downstream profits, owing to the softening effect. It may well be the case that the integrated firm which does not supply the upstream market earns larger total profits, if the additional downstream profits outweigh the foregone upstream revenues. Hence, the usual logic of Bertrand competition may not work anymore. An integrated firm may not always want to undercut its integrated rival on the upstream market, even though the upstream price is above the marginal cost. This potentially opens the door to non-competitive equilibria on the upstream market, in which the intermediate input would be priced above its marginal cost. In particular, the monopoly outcome can be an equilibrium, as illustrated by the following proposition.

Proposition 1. *Under Assumptions 1-3, there exists a monopoly-like equilibrium, i.e., a subgame-perfect equilibrium in which the upstream market is supplied by an integrated firm*

[13] This result bears similarities with that of Chen (2001), who shows that an integrated firm sets a higher downstream price when it supplies the upstream market than when a pure upstream competitor does.

at price a_m if and only if $\pi_j^{(i)}(a_m) \geq \pi_i^{(i)}(a_m)$.[14]

Proof. Assume that $\pi_j^{(i)}(a_j) \leq \pi_i^{(i)}(a_m)$. Suppose firm i offers $a_i = a_m$ and firm j makes an unacceptable offer. Then, firm j has no incentives to undercut firm i and, by Assumption 3, firm i has no incentives to deviate. Conversely, assume that $\pi_j^{(i)}(a_m) < \pi_i^{(i)}(a_m)$. If firm i supplies the upstream market at price a_m, then it is strictly profitable for firm j to propose $a_m - \epsilon$, an offer which firm d would accept, by Assumption 1.[15] □

Because losers on the upstream market become winners on the downstream market, the usual competitive forces may collapse. This does not hinge on any commitment device for the integrated firms to exit the upstream market;[16] nor does this rely on any kind of overt or tacit collusion.

As Proposition 1 highlights, the existence of monopoly-like equilibria depends on a comparison between profit levels. Whether the upstream supplier or its integrated rival earns more profits at a_m depends on the strength of the softening effect as compared to the size of upstream profits. Subsection 3.4 shows that the intensity of downstream competition plays an important role in this comparison.

Our result sheds light on the old debate between the traditional vertical foreclosure theory and the Chicago School. The traditional foreclosure theory points out that competition on the upstream market may be flawed since buyers and sellers interact on the downstream market, so that integrated firms have incentives to raise the pure downstream firms' costs. The Chicago School has forcefully criticized the validity of this argument on the following grounds: even if integrated firms have incentives to raise their rivals' costs, these incentives are not strong enough to offset the competitive forces on the upstream market. In other words, as long as the upstream price remains above the marginal cost, an integrated firm can always undercut its integrated rival by a small amount, thereby stealing the upstream profit without altering the downstream outcome. Our results highlight the fact that an important point is missing in that reasoning: the softening effect implies that undercutting on the upstream market does have an important adverse impact on the downstream outcome. This may lead to equilibrium partial foreclosure.

3.3 Other equilibria

In this section, we give a complete characterization of the subgame-perfect equilibria of our game.

[14]Notice that different strategies can be used to support a monopoly-like equilibrium: $a_i = a_m$ and a_j unacceptable, or $a_i = a_m$ and a_j acceptable such that $\pi_j^{(i)}(a_j) \leq \pi_i^{(i)}(a_m)$.

[15]Notice that the result can be proven under a much weaker assumption. If $\pi_d^{(i)}(.)$ does not reach a local maximum in a_m, which seems reasonable, then firm j can propose a deviation (either downward of upward) which would be accepted by firm d.

[16]In our model, firms cannot commit not to enter the upstream market; however, endogenously, the incentives to corner the upstream market may disappear.

Proposition 2. *Under Assumptions 1-2, there exists a subgame-perfect equilibrium, in which* $a_1 = a_2 = a_*$ *if and only if* $a_* \leq a_m$ *and* $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$. *These equilibria are referred to as matching-like equilibria.*

Proof. See Appendix A.3. □

In a matching-like equilibrium, both integrated firms offer the same upstream price and are indifferent between supplying the upstream market and not supplying it. A particular matching-like equilibrium corresponds to the case $a_* = c_u$, in which integrated firms offer an upstream price equal to their upstream marginal cost. As the intuition suggests, the perfect competition outcome on the upstream market is an equilibrium.

Corollary 1. *Under Assumptions 1-2, the perfect competition outcome is a subgame-perfect equilibrium.*

Proof. Immediate. □

However, nothing precludes the existence of other matching-like equilibria featuring either a supra-competitive upstream market ($a_* > c_u$) or a super-competitive upstream market ($a_* < c_u$). The existence of these equilibria also hinges on the softening effect. For $a_* > c_u$, the integrated firm which does not supply the upstream market benefits from the softening effect and does not want to undercut. For $a_* < c_u$, the softening effect is reversed. The upstream supplier offers an aggressive downstream price to reduce the upstream demand, which hurts its integrated rival. Even though the upstream supplier makes losses on the upstream market, it does not want to exit that market since it would then suffer from an adverse softening effect.

We conclude this paragraph with the following result:

Proposition 3. *Under Assumptions 1-3:*

- *Only monopoly-like and matching-like outcomes can arise in equilibrium.*
- *From the viewpoint of the integrated firms, any monopoly-like equilibrium Pareto-dominates any matching-like equilibrium.*

Proof. See Appendix A.4. □

Propositions 1, 2 and 3 provide a characterization of all the possible equilibria of our game. Moreover, the monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria from the integrated firms' standpoint. Therefore there is a strong presumption that these equilibria will actually be played when they exist.

3.4 The dilemma between upstream and downstream competitiveness

A key ingredient of the persistence of the monopoly outcome is the degree of differentiation of the pure downstream firm. Suppose that the entrant is on a niche market, in the sense that its demand does not depend on the prices set by the rival downstream firms and vice-versa.[17] In that situation, the wholesale profit of the upstream supplier is fully disconnected from its retail behavior and the softening effect disappears. Hence, with a pure downstream firm on a niche market, the perfect competition outcome always emerges in equilibrium.

In order to refine this intuition, consider the following illustration. Symmetry is assumed on the downstream market: the demand that addresses to firm $k \in \{1, 2, d\}$ is given by $D_k(p) = D - p_k - \gamma(p_k - \bar{p})$, where $\bar{p}$ is the average of downstream prices and $\gamma \geq 0$ traduces the intensity of downstream competition or the degree of differentiation between downstream products. All firms incur the same downstream costs, which we assume to be linear: $c'_k(.) = c$.[18] With that specification, profit functions satisfy all the assumptions we made so far.

Figure 2 offers a graphical representation of the profit functions $\pi_i^{(i)}(.)$, $\pi_j^{(i)}(.)$ and $\pi_d^{(i)}(.)$. As discussed in Subsection 3.2, when $a_i > c_u$, two opposite effects are at work. On the one hand, the upstream supplier derives profit from the upstream market; on the other hand, its integrated rival benefits from the softening effect on the downstream market. When the upstream price is not too high, the upstream profit effect dominates and $\pi_i^{(i)}(a_i) > \pi_j^{(i)}(a_i)$. When the upstream price is high enough, upstream revenues shrink, the softening effect is strengthened and $\pi_i^{(i)}(a_i) < \pi_j^{(i)}(a_i)$. Notice also that, conditionally on firm j not serving the upstream market, firm i does not want to completely foreclose the pure downstream firm.

We then obtain the following proposition.

Proposition 4. *Consider the linear demands case. There exists $\bar{\gamma} > 0$ such that:*
If $\gamma \geq \bar{\gamma}$, then there exist four subgame-perfect equilibria:[19]

- *the* perfect competition *outcome;*
- *a supra-competitive* matching-like *outcome;*
- *two* monopoly-like *outcomes.*

Otherwise, i.e., when $\gamma < \bar{\gamma}$, the perfect competition outcome is the only subgame-perfect equilibrium.

Proof. See Appendix A.5. □

[17] Formally, $\partial D_d / \partial p_i = \partial D_i / \partial p_d = 0$ for $i \in \{1, 2\}$.

[18] We assume that the total unit cost $c_u + c$ is strictly smaller than the intercept of the demand functions D, otherwise, it would not be profitable to be active on the final market.

[19] The perfect competition and monopoly-like equilibrium are stable; the matching-like is unstable.

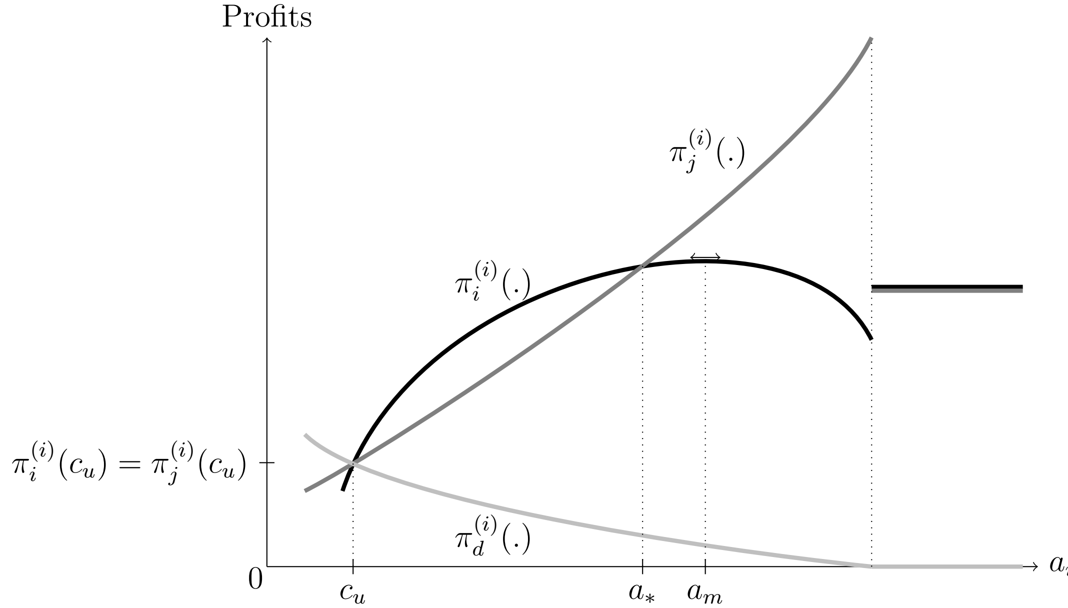

Figure 2: Profits in the linear case ($\gamma \geq \bar{\gamma}$).

To grasp the intuition of the proposition, suppose that the upstream market is supplied at the monopoly upstream price. When the substitutability between final products is strong, the integrated firm which supplies the upstream market is reluctant to set too low a downstream price since this would strongly contract its upstream profit. The other integrated firm benefits from a substantial softening effect and, as a result, is not willing to corner the upstream market. There exists a monopoly-like equilibrium when downstream products are sufficient substitutes. By the reverse token, only the perfect competition outcome emerges when the competition on the downstream market is sufficiently weak. In other words, tension exists between competitiveness on the downstream market and competitiveness on the upstream market. Intuitively, the same downstream interactions which strengthen the competitive pressure on the downstream market, are those which soften the competitive pressure on the upstream market.

This tension is revealed in downstream prices, which turn out to be non-monotonic in the substitutability parameter (provided that a monopoly-like equilibrium is selected when it exists). The level of downstream prices results indeed from two combined forces: the level of upstream prices on the one hand, and the intensity of downstream competition / substitutability on the other hand. Paradoxically, this implies that downstream prices can increase after an exogenous increase in the substitutability between downstream products.

3.5 Pure upstream competitor

So far, we have assumed that the upstream market could only be supplied by integrated firms. We now assume that a pure upstream competitor, firm u, is able to produce the intermediate input at constant marginal cost c_u. For $k \in 1, 2, d, u$, we denote by $\pi_k^{(u)}(a_u)$ the profit earned by firm k when the upstream market is supplied by firm u at price a_u. We make the following assumption:

Assumption 4. $\pi_d^{(u)}(.)$ *is strictly decreasing.*

Together with Assumption 1, this implies that, whatever the upstream supplier, the profit of the pure downstream firm decreases in the upstream price.

It is natural to wonder whether the mere presence of firm u is sufficient to break the interactions between the upstream and the downstream markets, and to ensure that the perfect competition outcome emerges. The following proposition states that this is indeed the case provided that downstream prices are strategic complements.

Proposition 5. *Under Assumptions 1-4, if downstream prices are strategic complements, then there is no subgame-perfect equilibrium in which the upstream market is supplied at a price strictly above the marginal cost.*

Proof. See Appendix A.6. □

Let us briefly state the intuition underlying that proposition. Assume that firm u supplies the upstream market at price $a_u > c_u$. We claim that both firm i and firm d become strictly better off if firm i matches firm u's price, and firm d elects firm i as its upstream supplier. If firm d agrees to purchase the input from firm i at price a_u, this creates a softening effect: firm i raises its downstream price and, by strategic complementarity, firms j and d react by increasing their prices as well. By a revealed preference argument, these price increases benefit firms i and d. Moreover, when firm i matches firm u, it also earns upstream profits, which provides additional incentives to match.

Conversely, if the upstream market is supplied by integrated firm i at a price $a_i > c_u$, then firm u always wants to undercut, for its sole source of profit comes from the upstream market. Moreover, firm u is always able to attract firm d, provided that it offers a sufficiently low upstream price.[20]

Having said that, it becomes clear that the upstream market cannot be supplied at a supra-competitive price in equilibrium. One may then wonder which outcome will arise on the upstream market. As noted above, in our basic setting, Proposition 2 implies that the perfect competition outcome is always a subgame-perfect equilibrium. Obviously, adding a pure upstream competitor does not affect this result. However, nothing precludes a priori the existence of super-competitive equilibria.

A similar logic applies if the pure upstream competitor comes from the vertical separation of one of the integrated firm.[21]

---

[20] $\pi_d^{(u)}(.)$ and $\pi_d^{(i)}(.)$ are decreasing and $\pi_d^{(u)}(c_u) = \pi_d^{(i)}(c_u)$.

[21] The difference with the previous case is that there are two pure downstream firms now; see Hombert, Pouyet, and Schutz (2007) for the complete analysis.

4 Extensions and Discussions

We now discuss some extensions and robustness checks.

Complete foreclosure. Let us first relax Assumption 3: assume that $\pi_i^{(i)}(a_m) \leq \pi_{\text{duopoly}}$, i.e., a hypothetical upstream monopolist would prefer foreclosing completely the entrant to allowing it to be active in the downstream market. Then, for trivial reasons there exists an equilibrium in which the pure downstream firm is completely foreclosed.

Whether or not such an equilibrium exists, i.e., whether or not Assumption 3 is satisfied, is orthogonal to our analysis. To see why, it is worth rephrasing our main result. When an integrated firm undercuts its integrated rival which supplies the upstream market, it steals the upstream profits at the cost of losing the softening effect. By contrast, when it starts supplying a pure downstream firm which was previously completely foreclosed, an integrated firm does not lose the softening effect. On the other hand, it modifies profoundly the pattern of downstream demands. Analyzing how downstream demands are affected is obviously beyond the scope of this paper; readers interested in this issue should refer to Brito and Pereira (2006) and Ordover and Shaffer (2006).

Two-part tariffs. We have assumed so far that the upstream pricing was constrained to be linear. We now show that the possibility for non-competitive equilibria carries over to the case of two-part tariffs $\{(a_i, T_i), (a_j, T_j)\}$ on the upstream market. The equilibrium upstream variable part maximizes the joint profit of the upstream supplier and the pure downstream firm, i.e., $a_{tp} = \arg\max_{a_i} \pi_i^{(i)}(a_i) + \pi_d^{(i)}(a_i)$.[22] The fixed fee T_i must be such that, first, firm j does not want to undercut, and, second, firm d is willing to accept that tariff, or: $T_i \leq \min\{\pi_j^{(i)}(a_{tp}) - \pi_i^{(i)}(a_{tp}); \pi_d^{(i)}(a_{tp})\}$.[23] Having said that, the characterization of the subgame-perfect equilibria of the game with two-part tariffs becomes straightforward.

If $\pi_i^{(i)}(a_{tp}) + \pi_d^{(i)}(a_{tp}) \leq \pi_j^{(i)}(a_{tp})$, then the only two equilibria are such that one of the integrated firm charges the variable part a_{tp} and a fixed fee equal to $\pi_d^{(i)}(a_{tp})$, which extracts all the rent from the downstream firm, while the other integrated firm makes no upstream offer. Under two-part tariff competition, this is a monopoly-like equilibrium.

If $\pi_i^{(i)}(a_{tp}) + \pi_d^{(i)}(a_{tp}) \geq \pi_j^{(i)}(a_{tp})$, then the only equilibrium features both integrated firms charging the variable part a_{tp} and a fixed fee equal to $\pi_j^{(i)}(a_{tp}) - \pi_i^{(i)}(a_{tp})$, which makes them indifferent between supplying the upstream demand or not. This is a matching-like equilibrium.

Proposition 6. *Under two-part tariff competition on the upstream market, the equilibrium is either monopoly-like or matching-like.*

Proof. Immediate. □

[22] See Bonanno and Vickers (1988).
[23] Note that the equilibrium fixed part T_i may be negative.

Once again, upstream competition does not modify the outcome with respect to the monopoly benchmark. If the upstream equilibrium is monopoly-like, both the fixed and the variable part of the tariffs remain the same; obviously, downstream prices are not affected either. If the equilibrium is matching-like, competition modifies the fixed part only, without affecting any downstream price. In other words, the only impact of competition is to redistribute some profits from the integrated firms to the pure downstream firm. Besides, provided that $a_{tp} > c_u$, it is straightforward to show that the upstream profit is strictly positive.[24] In this sense, we claim that the upstream market remains non-competitive under two-part tariff competition.

Quantity competition. The softening effect exists if the upstream supplier can enhance its upstream profits by behaving softly on the downstream market. As discussed previously, this requires that it actually interacts with the pure downstream firm. One may wonder whether the softening effect hinges on the assumption of price competition on the downstream market, for if the downstream strategic variables are quantities and all firms play simultaneously, then the upstream supplier can no longer impact its upstream profit through its downstream behavior. However, if for instance integrated firms are Stackelberg leaders on the downstream market, then the upstream supplier's quantity choice modifies its upstream profit, and the softening effect is still at work. To summarize, the question is not whether firms compete in prices or in quantities, but whether the strategic choice of a firm can affect its rivals' quantities.[25]

Several upstream suppliers. Throughout the paper, we have assumed that the upstream market could be supplied by one integrated firm only. Consider now that, when upstream offers are identical, the upstream demand is split equally between integrated firms. This would be a reasonable assumption if there were several downstream firms. In that case, we can still think about the upstream market in terms of softening effect and upstream profit effect. When the integrated firms share the upstream market, they both obtain some upstream profits, and they both benefit from a softening effect, since they both have incentives to protect their upstream revenues. Behaviors on the upstream market still trade-off the softening effect and the upstream profit effect.

It becomes clear that the possibility for non-competitive equilibria remains. Monopoly-like equilibria feature the same outcome as in the basic model. Matching-like equilibria can also exist, in which the upstream market is shared between integrated firms. [26]

[24] If the equilibrium is monopoly-like, this is obvious. If it is matching-like, then the upstream profit is equal to $[a_{tp} - c_u]\, D_d(p^{(i)}(a_{tp})) + \pi_j^{(i)}(a_{tp}) - \pi_i^{(i)}(a_{tp}) = \left[p_j^{(i)}(a_{tp}) - c_u\right] D_j(p^{(i)}(a_{tp})) - c_j\left(D_j(p^{(i)}(a_{tp}))\right) - \left[p_i^{(i)}(a_{tp}) - c_u\right] D_i(p^{(i)}(a_{tp})) + c_i\left(D_i(p^{(i)}(a_{tp}))\right)$, which is strictly positive by Lemma 2.

[25] With a linear demand function and quantity competition, if integrated firms are Stackelberg leaders on the downstream market, then a monopoly-like equilibrium always exists (proof available upon request).

[26] For instance, solving the linear demand model with a high substitutability parameter and constant marginal costs, and assuming that the pure downstream firm splits equally its demand when upstream prices

Downstream strategic interaction. In Bourreau, Hombert, Pouyet, and Schutz (2007), we provide another example, in which downstream prices can be either strategic substitutes or strategic complements. We obtain two results of interest. First, as downstream prices become more strategic complements, the softening effect weakens and the incentives to undercut on the upstream market are reinforced; therefore, the nature of the strategic interaction on the downstream sector may give some hints on the potential competitiveness of the upstream market. Second, with high strategic substitutability and assuming that there exists a pure upstream supplier, there exists an equilibrium in which both integrated firms are inactive on the upstream market and the pure upstream firm sets its monopoly price on that market; in that case, vertical separation is not the ideal remedy to a poorly competitive upstream market.

5 Application to Telecommunications

In the telecommunications industry, most markets have a two-tier structure, with a wholesale level and a retail level. Over the last few years, facility-based competition has developed. One consequence has been the emergence of wholesale markets, where facility-based firms compete to provide wholesale services to service-based firms. Service-based firms do not own network infrastructures, and use wholesale services to design their retail offers and compete in the retail market.

This trend has been observed in the broadband market and, more markedly, in the mobile telephony market, with the advent of mobile virtual network operators (MVNOs).[27] In what follows, we first apply our setting to draw some predictions and evidence on the competitiveness of wholesale markets in the broadband and mobile industries. We then analyze which regulatory tools might be used to stimulate competition in wholesale markets.

5.1 Predictions and evidence

The competitiveness of wholesale markets. Facility-based competition has clear benefits; as facility-based firms do not rely on historical incumbents (or to a limited degree only) to provide services to end consumers, the regulatory burden can be lifted to some extent. However, as our analysis indicates, it seems unlikely that facility-based competition will also always lead to a competitive wholesale market and thus stimulate the development of service-based competition.

One good example is the broadband market. Facility-based competition has grown, with

are identical, we obtain the following subgame-perfect equilibria: the two monopoly-like outcomes, the perfect competition outcome, and a continuum of non-competitive equilibria in which both integrated firms set the same price and share the upstream demand (proof available upon request).

[27] MVNOs are service-based competitors. They do not have a spectrum license nor a mobile network. Some MVNOs are merely resellers, hence do not have any infrastructure. Others (e.g., "full" MVNOs) own some mobile network elements, which gives them higher possibilities of differentiation.

the development of cable modem networks and local loop unbundling (LLU) operators.[28] Therefore, potential competition in the wholesale broadband market exists.

In some European countries a competitive wholesale broadband market (also known as the "bitstream" access market) has emerged. For instance, in 2005 in France, two LLU operators, Cegetel and Neuf Telecom, owned between 30 and 50 percent of the national wholesale broadband market, while the incumbent historical operator (France Telecom) served the rest.[29] In the UK, LLU operators also provide wholesale broadband services in densely populated areas,[30] and compete with the incumbent, BT, in the wholesale broadband market.

However, competition in the wholesale market is not always observed. In most European countries, the broadband wholesale market is served by only one firm.[31] In some countries, like the Netherlands and the US, there is even no wholesale broadband market.

In most mobile national markets, a wholesale market has emerged, with multiple upstream providers and many MVNOs. According to Hazlett (2005), in 2005 there were around 20 MVNOs in the US.[32] In July 2006, in the European Union there was a total of 290 MVNOs in 15 Member States out of 25 (European Commission, 2007). The prices for the wholesale product are typically set on a retail minus basis. Therefore, even though several mobile network operators (MNOs) offer wholesale mobile services, competition has not driven wholesale prices to costs.

To summarize, evidence on the broadband and mobile markets suggests that wholesale markets are rarely competitive. This is consistent with our analysis, which has highlighted that competitive outcomes are possible, but unlikely.

The impact of downstream differentiation. We have also suggested that the nature of competition in the retail market, and in particular, the degree of differentiation between firms in that market, can affect the outcome in the wholesale market. More precisely, in our illustration in Section 3.4, we showed that if the degree of differentiation in the retail market was sufficiently high, the wholesale market was competitive, and that otherwise, non-competitive outcomes were also possible.

Our model therefore suggests that strongly differentiated service-based firms are more likely to enter in the market, not only because entrants have incentives to differentiate to avoid head-to-head competition, but also because they are more likely to benefit from attractive wholesale offers by facility-based firms. In the mobile market, the evidence seems to confirm

[28]Different technologies can deliver broadband, but two of them dominate local broadband markets worldwide: the cable modem platform (which is the most widely used in the US) and the copper-based digital subscriber line (DSL) platform (which is the dominant technology in the EU).

[29]See DGCCRF, Decision C2005-44 related to the merger between Neuf Telecom and Cegetel. The merger was accepted; one condition though required the merged company to continue to provide its wholesale services.

[30]See Ofcom, "Review of the wholesale broadband access markets 2006/07," 21 November 2006.

[31]Typically, this is the incumbent operator, whose offer is often (but not always) regulated. However, in Sweden, the monopoly supplier of the wholesale market is an alternative operator (See: European Commission (2007)).

[32]The two main MVNOs were TracFone which served over 3.5 million customers and Virgin Mobile USA, which served over 3 million customers.

this intuition. Indeed, many MVNOs target specific niche market segments (e.g., teenagers for Virgin Mobile in the UK or NRJ Mobile in France). Hence, in the mobile industry we might expect a high degree of product diversity and moderate price competition on the downstream market.

5.2 Policy implications

Price cap and asymmetric regulation. Imposing a price cap on upstream offers can be an appropriate remedy to enhance competition in wholesale markets. If the regulator is able to set an upstream price cap equal to the upstream marginal cost, c_u, she can trivially achieve the perfectly competitive outcome on the upstream market. More realistically, the regulator cannot assess c_u with certainty or is not willing to implement a very tight regulation. To fix ideas, consider the illustration developed in Section 3.4, with a high substitutability index. If the regulator can impose a price cap between c_u and a_*, then all non-competitive equilibria vanish and the upstream price goes down to the upstream marginal cost.

We would like to emphasize that this is not a mere mechanical effect. Of course, imposing a price cap reduces the upstream price mechanically. But, more fundamentally, a price cap can also initiate a process by which integrated firms will undercut each other, leading to tough competition in the wholesale market. Interestingly, a price cap can influence the outcome of the market even though the regulatory constraint does not bind (i.e., the upstream price is strictly smaller than the price cap) in equilibrium.

Note also that it is sufficient to impose a price cap on one of the integrated firms only to fuel competition in the wholesale market. In some European countries, this kind of asymmetric regulation has been implemented in the wholesale broadband market.[33] In those countries, only the offers of the ILECs are regulated while those of competing local exchange carriers (CLECs) are left unregulated. Our model provides a rationale for such policies.

Vertical separation. It is sometimes advocated that vertical separation of the incumbent operator, into two independent upstream and downstream divisions, can also promote competition in the telecommunication industry. For instance, in Europe, Viviane Reding, Member of the European Commission responsible for Information Society and Media, has many times argued that structural separation of the dominant operator was a policy option.[34] Vertical

[33] In France, bitstream access has been regulated since 1999. In 2007, the regulator, ARCEP, removed the obligation to provide bitstream access at the national level (See: Decision n°2007-0089, 30 January 2007), but an obligation to provide a cost-based bitstream access offer at the regional level remains (See: Decision n°05-0280, 19 May 2005). In the UK, BT has an obligation to provide a wholesale broadband offer on a retail minus basis. The Italian and Spanish regulators recently introduced an obligation to offer a cost-based bitstream access offer. Finally, in Germany, there is no obligation for the incumbent to provide a wholesale broadband (bitstream access) offer (See: European Commission (2007)).

[34] In May 2007, in a speech, she declared: "I believe that functional separation (...) could indeed serve to make competition more effective in a service-based competition environment where infrastructure-based competition is not expected to develop in a reasonable period. It may be a useful remedy in specific cases. It is certainly not a panacea." (Viviane Reding, "How Europe can Bridge the Broadband Gap", Brussels, 14

separation is perceived as a means to eliminate the incentives of the dominant integrated firm to foreclose its competitors in downstream markets.[35] In particular, vertical separation has been considered to stimulate competition in the broadband market. Two types of separation could be implemented. First, the local access unit of the ILEC could be separated from its downstream unit. Second, the Internet service provider unit of the ILEC could be separated from the upstream unit.[36]

If vertical separation is implemented, the upstream division of the ILEC sets the price of its wholesale offer, without taking into account the impact on the downstream division, and reciprocally. In Section 4, we have analyzed how the competitiveness of the wholesale market is affected when a pure upstream competitor is introduced. We have shown that if downstream prices are strategic complements, the introduction of a pure upstream firm leads to a competitive wholesale market. Putting aside the costs of vertical separation, this is another argument in favor of it.

Pure upstream competitors. There are two other types of situations in which a pure upstream unit can operate. Municipalities can invest in broadband networks and offer wholesale services to service-based operators. We argue that this can stimulate competition in wholesale markets more surely than entry of integrated firms. However, the burden of the financing of these investments is likely to be born by the taxpayers as tough competition on the upstream market will erode the upstream profits.

Second, some private companies can decide to enter as pure upstream providers. For instance, in the broadband market, firms like Covad or Northpoint in the US, or Mangoosta in France, adopted this strategy. In the mobile market, so-called mobile virtual enablers (MVNEs) are also pure upstream firms.[37] Our model suggests that this type of strategy is likely not to be viable.

Possibilities of bypass. So far we have ignored that a service-based operator may decide to build its own infrastructure if the upstream offers are prohibitively costly. In such a case, it would compete on a level playing field with the integrated firms in the downstream market. Because integrated firms would prefer service-based competition with a (relatively) high upstream price to this situation, they may decide to lower their upstream prices sufficiently to avoid bypass. This effect is stronger the lower the cost of bypassing the upstream market.[38]

May 2007).

[35]For instance, Viviane Reding from the European Commission declared: "Many of the countries that are behind in broadband coverage and take-up have endemic problems of discriminatory behaviour by the incumbent: favouring its service providers over competitors." (Viviane Reding, "How Europe can Bridge the Broadband Gap", Brussels, 14 May 2007).

[36]This type of situation has been observed in some countries. For instance, in France, some years ago, the ILEC's Internet service provider, Wanadoo, was a subsidiary of its parent company, France Télécom.

[37]Examples of MVNEs are Transatel in France and Belgium, Effortel in Belgium, Versent Mobile and Visage Mobile in the US.

[38]See Bourreau and al. (2007) for the complete analysis.

In particular, if the cost of bypass is sufficiently low, then only the perfect competition equilibrium emerges.

This result has interesting policy implications. In the mobile industry, it means that favorable terms for spectrum licences (e.g., terms for ungranted mobile licences, or for Wimax licences) might increase MNOs' incentives to set low wholesale prices for MVNOs. In the broadband market, it implies that favorable conditions for local loop unbundling investments (e.g., low rates for colocation in the historical operator's premises) might stimulate the development of the wholesale broadband market.

6 Conclusion

Our analysis has focused on the links between vertically-related markets, when the upstream good is an essential input to the downstream product, and when the competitors on the upstream market are also rivals on the downstream one. Such a theoretical framework clearly depicts wholesale markets in telecommunications.

One of the main insights conveyed in the paper is that these upstream markets might not be competitive. Put differently, the monopoly outcome might persist even when competition in that market is possible. The main reason lies in the softening effect, according to which an integrated firm supplying the upstream market tends to be a soft competitor on the downstream market. This implies in turn that the rival integrated firm might not be willing to compete at all on the upstream market. An example has been proposed to illustrate this mechanism and one of its determinants: product differentiation at the downstream level. Roughly speaking, product differentiation affects the strength of the softening effect and therefore the potential competitiveness of the upstream market.

The theoretical analysis has been the basis of several policy implications and robustness checks. Let us reemphasize that, from a competition policy perspective, our analysis suggests that analyzing upstream markets in isolation of the related downstream markets is bad economics. Taken on a stand-alone basis, our upstream market has all the features which traditionally lead to the perfect competition outcome. Once the downstream market is added to the analysis, the picture is much more mixed. Overall, the potential competitiveness of upstream markets hinges as much on the economic fundamentals pertaining to that market as on the fundamentals of the related downstream markets.

Various extensions would certainly be worth studying. From a more industry-specific perspective, and as regards the broadband market, it would be worth investigating the role of the local loop in our setting. Most facility-based firms rely to some extent on the local loop owned by the incumbent to offer broadband services to final customers. It would be interesting to understand how the competition on the wholesale market is affected by the regulation of access to the local loop. Likewise, as regards the mobile telephony market, interconnection flows between MNOs, and their regulation, may impact on competition between MNOs to attract MVNOs.

From a more theoretical viewpoint, the role of the market structures remains to be studied. A first set of questions relates to the impact of the number of vertically integrated firms and of pure downstream firms on the competitiveness of the wholesale market. A second set of questions relates to the entry process in this industry: are vertically integrated firms more likely to enter than pure downstream ones?

Finally, our upstream price competition game can be viewed as an auction run by the pure downstream firm in which integrated firms bid to supply the upstream market. Given that the outcome of this auction determines the efficiency of the pure downstream firms, this auction is characterized by externalities which depend on the price paid by the pure downstream firm, as studied by Ettinger (2002). As our analysis indicates, these externalities are non-monotonic with respect to the price, which explains the multiplicity of equilibria. Under asymmetric information, it would be worth studying how different auction formats impact the outcome of our game, as in Jehiel and Moldovanu (2000). In the same vein, if upstream suppliers offer differentiated inputs which affect differently the outcome on the downstream market, then identity-dependent externalities emerge; this may be another force which affects the integrated firms' incentives to participate to the upstream market, as in Jehiel and Moldovanu (1996).

All these questions are left for future research.

A Appendix

A.1 Proof of Lemma 1

To begin with, we prove the following fixed point lemma:

Lemma 0. *Let $f : [0, \infty) \to [0, \infty)$ be a $\mathcal{C}^1$ function. Assume that f is bounded and $|f'(.)| < 1$.*
Then, f admits a unique fixed point x^*. Besides, $f(x) > x$ if, and only if, $x < x^*$; and $f(x) < x$ if, and only if, $x > x^*$.

Proof. Define $g(x) \equiv f(x) - x$. Computing its first derivative, we see that $g(.)$ is strictly decreasing, since $f'(.) < 1$. $g(0) = f(0) \geq 0$, and $\lim_{x\to\infty} g(x) = -\infty$, since $f(.)$ is bounded. By continuity, there exists a unique $x^* \geq 0$ such that $g(x) = 0$. Since $g(.)$ is strictly decreasing, $g(x) > 0$ if, and only if, $x < x^*$. And $g(x) < 0$ if, and only if, $x > x^*$. This concludes the proof. □

Assume that integrated firm $i \in \{1, 2\}$ is the upstream supplier at price $a_i > c_u$, and let us show that $p_i^{(i)}(a_i) > p_j^{(i)}(a_i)$. By definition of the unique downstream equilibrium,

$$p_i^{(i)}(a_i) \;\; = \;\; BR_i^{(i)}\left(BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), a_i\right), p_d^{(i)}(a_i), a_i\right).$$

We see from first-order condition (2) that firm j's best-response function does not depend on a_j: $BR_j^{(i)}(.,., a_i) = BR_j^{(i)}(.,., c_u)$. By contrast, (1) implies that firm i's best-response is strictly increasing in the upstream price: $BR_i^{(i)}(.,.,a_i) > BR_i^{(i)}(.,.,c_u)$. Hence:

$$p_i^{(i)}(a_i) > BR_i^{(i)}\left(BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u\right), p_d^{(i)}(a_i), c_u\right). \quad (3)$$

By assumption, $|\partial BR_i^{(i)}/\partial p_j| < 1$ and $BR_i^{(i)}$ is bounded. We can apply Lemma 0 to function $BR_i^{(i)}(., p_d^{(i)}(a_i), c_u)$: There exists a unique p^* such that $BR_i^{(i)}(p^*, p_d^{(i)}(a_i), c_u) = p^*$. Besides, since $BR_i^{(i)}(.,.,c_u) = BR_j^{(i)}(.,.,c_u)$, we also have that $BR_j^{(i)}(p^*, p_d^{(i)}(a_i), c_u) = p^*$. p^* is the downstream price set by integrated firms in the hypothetical situation in which firm i would supply the upstream market at c_u and firm d's price would be exogenously fixed at $p_d^{(i)}(a_i)$.

Define the following function:

$$f : x \mapsto BR_i^{(i)}\left(BR_j^{(i)}\left(x, p_d^{(i)}(a_i), c_u\right), p_d^{(i)}(a_i), c_u\right).$$

Note that $f(p^*) = p^*$, by definition of p^*. Obviously, f is bounded and $|f'(.)| < 1$. Thus, it admits a unique fixed point: p^*. Besides, $f(x) < x \Leftrightarrow x > p^*$. We deduce from (3) that $p_i^{(i)}(a_i) > p^*$.

We can now apply the mean value inequality to function $BR_j^{(i)}(., p_d^{(i)}(a_i), c_u)$ between p^* and $p_i^{(i)}(a_i)$:

$$\begin{aligned}\left|BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u\right) - BR_j^{(i)}\left(p^*, p_d^{(i)}(a_i), c_u\right)\right| \\ \leq \sup_{p_i \in \left[p^*, p_i^{(i)}(a_i)\right]} \left|\frac{\partial BR_j^{(i)}}{\partial p_i}\left(p_i, p_d^{(i)}(a_i), c_u\right)\right| \left|p_i^{(i)}(a_i) - p^*\right|.\end{aligned}$$

Since the upper bound is taken over a compact set, it is strictly lower than 1. Besides, since firm j's best-response does not depend on the upstream price, and by definition of $p_j^{(i)}$, $BR_j^{(i)}(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u) = p_j^{(i)}(a_i)$. By definition of p^*, $BR_j^{(i)}(p^*, p_d^{(i)}(a_i), c_u) = p^*$. Using the fact that $p^* < p_i^{(i)}(a_i)$, the mean value inequality can then be rewritten as:

$$\left|p_j^{(i)}(a_i) - p^*\right| < p_i^{(i)}(a_i) - p^*.$$

In particular, $p_j^{(i)}(a_i) - p^* < p_i^{(i)}(a_i) - p^*$, hence, $p_j^{(i)}(a_i) < p_i^{(i)}(a_i)$.

A.2 Proof of Lemma 2

Let integrated firm $i \in \{1, 2\}$ be the upstream supplier at price $a_i > c_u$. Its downstream profit is given by:

$$(p_i^{(i)}(a_i) - c_u) D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) - c_i\left(D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i))\right), \quad (4)$$

with $p_i^{(i)}(a_i) > p_j^{(i)}(a_i)$ by Lemma 1. Define $\hat{p} > p_i^{(i)}(a_i)$ such that:

$$D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i)) = D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)).$$

Downstream profit (4) is strictly smaller than:

$$(\hat{p} - c_u) D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i)) - c_i \left(D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i)) \right).$$

By symmetry between integrated firms, this can be rewritten as:

$$(\hat{p} - c_u) D_j(p_i^{(i)}(a_i), \hat{p}, p_d^{(i)}(a_i)) - c_j \left(D_j(p_i^{(i)}(a_i), \hat{p}, p_d^{(i)}(a_i)) \right).$$

By revealed preferences, this profit is smaller than the downstream profit of firm j:

$$(p_j^{(i)}(a_i) - c_u) D_j(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) - c_j \left(D_j(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) \right),$$

which concludes the proof.

A.3 Proof of Proposition 2

Suppose first that both integrated firms offer the same upstream price $a_* \leq a_m$, such that $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$. The pure downstream firm chooses indifferently one of them as its upstream supplier, and both integrated firms earn the same profit.

Consider an upward deviation of integrated firm i, be it the upstream supplier or not. Now, by Assumption 1, firm d strictly prefers buying the upstream good from firm j at price a_*, and firm i's profit is unchanged. Consider now a downward deviation: $a_i < a_*$. Firm d strictly prefers to buy from firm i, which then earns $\pi_i^{(i)}(a_i)$. By Assumption 2, since $a_* \leq a_m$, this profit is smaller than $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$. That situation is therefore an equilibrium.

Conversely, consider that both integrated firms offer the same upstream price a_*. Suppose first that $a_* > a_m$. The upstream supplier then has a strictly profitable deviation: propose a_m.

If $\pi_i^{(i)}(a_*) < \pi_j^{(i)}(a_*)$, then the upstream supplier would rather set an upstream price above a_* to earn $\pi_j^{(i)}(a_*)$.

If $\pi_i^{(i)}(a_*) > \pi_j^{(i)}(a_*)$, then the integrated firm which does not supply the upstream market would rather set an upstream price slightly smaller than a_* to earn a profit almost equal to $\pi_i^{(i)}(a_*)$.

A.4 Proof of Proposition 3

Consider by contradiction an equilibrium configuration in which $a_i < a_j$ and $a_i \neq a_m$. By Assumption 1, the upstream supplier is firm i.

If $a_j > a_m$, it is a strictly profitable deviation for firm i to offer a_m. If $a_j \leq a_m$, firm 1 would rather charge any upstream price in (a_i, a_j), since $\pi_i^{(i)}(.)$ is increasing in this interval by Assumption 2.

Let us now show that a monopoly-like equilibrium Pareto-dominates any matching-like equilibrium, from the viewpoint of integrated firms. We have $\pi_j^{(i)}(a_m) \geq \pi_i^{(i)}(a_m)$ by Proposition 1. Consider a matching-like equilibrium at upstream price a_*. By definition of a_m, $\pi_i^{(i)}(a_m) \geq \pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$. This concludes the proof.

A.5 Proof of Proposition 4

The proof proceeds in several steps. To begin with, we show that, w.l.o.g., the intercepts of the linear demands can be normalized to 1 and all upstream and downstream costs can be set at 0. We compute the downstream equilibrium, and check that all the assumptions we need are satisfied. We can then compute the monopoly benchmark and make some comparisons between integrated firms' profits. This allows us to apply Propositions 1, 2 and 3, and obtain all existing subgame-perfect equilibria.

Simplifications. Assume that integrated firm i supplies the upstream market at price a_i, and denote its integrated rival by j. Define $\tilde{a}_i \equiv \frac{a_i - c_u}{D-c-c_u}$, and $\tilde{p}_k \equiv \frac{p_k - c - c_u}{D-c-c_u}$ for all $k \in \{1,2,d\}$. Then, $\tilde{\overline{p}} = \frac{\overline{p}-c-c_u}{D-c-c_u}$, and it is straightforward to show that:

$$\begin{aligned}
\tilde{\pi}_i^{(i)}(p, a_i) &= (D-c-c_u)^2 \left[\tilde{p}_i \left(1 - \tilde{p}_i - \gamma(\tilde{p}_i - \tilde{\overline{p}})\right) + \tilde{a}_i \left(1 - \tilde{p}_d - \gamma(\tilde{p}_d - \tilde{\overline{p}})\right)\right], \\
\tilde{\pi}_j^{(i)}(p, a_i) &= (D-c-c_u)^2 \left[\tilde{p}_j \left(1 - \tilde{p}_j - \gamma(\tilde{p}_j - \tilde{\overline{p}})\right)\right], \\
\tilde{\pi}_d^{(i)}(p, a_i) &= (D-c-c_u)^2 \left[(\tilde{p}_d - \tilde{a}_i) \left(1 - \tilde{p}_d - \gamma(\tilde{p}_d - \tilde{\overline{p}})\right)\right].
\end{aligned}$$

As a result, we can impose the following normalizations without loss of generality: $D = 1$ and $c = c_u = 0$.

Downstream equilibrium. For all downstream and upstream prices, we have:

$$\frac{\partial^2 \tilde{\pi}_k^{(i)}}{\partial p_k^2} = -2(1 + \frac{2}{3}\gamma) < 0, \quad \forall k \in \{1, 2, d\}.$$

This ensures that the best-response functions are uniquely defined. They are equal to:[39]

$$
\begin{aligned}
BR_i^{(i)}(p_j, p_d, a_i) &= \frac{3 + a_i\gamma + \gamma(p_j + p_d)}{6 + 4\gamma}, \\
BR_j^{(i)}(p_i, p_d, a_i) &= \frac{3 + \gamma(p_i + p_d)}{6 + 4\gamma}, \\
BR_d^{(i)}(p_i, p_j, a_i) &= \frac{3 + 3a_i + 2a_i\gamma + \gamma(p_i + p_j)}{6 + 4\gamma}.
\end{aligned}
$$

The stability condition is satisfied, since, for all $k \neq k'$, we have:

$$
\left| \frac{\partial BR_k^{(i)}}{\partial p_{k'}} \right| = \frac{\gamma}{6 + 4\gamma} < 1.
$$

There is a unique downstream equilibrium, which can be calculated by solving the set of first-order conditions. We get:

$$
\begin{aligned}
p_i^{(i)}(a_i) &= \frac{18 + 15\gamma + 9a_i\gamma + 5a_i\gamma^2}{36 + 42\gamma + 10\gamma^2}, \\
p_j^{(i)}(a_i) &= \frac{18 + 15\gamma + 3a_i\gamma + 3a_i\gamma^2}{36 + 42\gamma + 10\gamma^2}, \\
p_d^{(i)}(a_i) &= \frac{18 + 18a_i + 15\gamma + 21a_i\gamma + 7a_i\gamma^2}{36 + 42\gamma + 10\gamma^2}.
\end{aligned}
$$

They are well-defined if the equilibrium quantity served by downstream firm d is positive, which is equivalent to:

$$
a_i \leq a_{\max}(\gamma) \equiv \frac{6 + 5\gamma}{6 + 7\gamma + \gamma^2} > 0.
$$

Assumption 1 is satisfied, since:

$$
\pi_d^{(i)}(a_i) = \frac{3(1+\gamma)^2(6+\gamma)^2(3+2\gamma)}{4(3+\gamma)^2(6+5\gamma)^2}[a_i - a_{\max}(\gamma)]^2,
$$

thus $\pi_d^{(i)}(.)$ is decreasing for $a_i \leq a_{\max}(\gamma)$.

[39] At first sight, these best-response functions seem to be unbounded, which would violate one of our assumptions. If downstream demands are defined more carefully, as $D_k = \max\{1 - p_k - \gamma(p_k - \bar{p}), 0\}$, then it can be shown that downstream prices have to lie below a certain threshold for the three firms to be active. We abstract from these considerations in the following, since we are only interested in configurations in which all firms supply a positive quantity.

Monopoly benchmark. The profit of the upstream supplier is concave:

$$\frac{d^2\pi_i^{(i)}}{da_i^2} = -\frac{648 + 1944\gamma + 2205\gamma^2 + 1158\gamma^3 + 269\gamma^4 + 20\gamma^5}{2(18 + 21\gamma + 5\gamma^2)^2} < 0.$$

In particular, $\pi_i^{(i)}(.)$ is quasiconcave. Firm i's maximum is reached for:

$$a_i = a_m(\gamma) \quad \equiv \quad \frac{324 + 594\gamma + 360\gamma^2 + 75\gamma^3}{648 + 1296\gamma + 909\gamma^2 + 249\gamma^3 + 20\gamma^4}.$$

Since $a_m \in (0, a_{\max}(\gamma))$, Assumption 2 is satisfied.

The profit earned by an integrated firm when the pure downstream firm is completely foreclosed can be computed easily: $\pi_{\text{duopoly}} = \frac{2(2+\gamma)}{(4+\gamma)^2}$. On the other hand, when firm i supplies the upstream market at price a_m, it earns:

$$\pi_i^{(i)}(a_m) = \frac{972 + 2052\gamma + 1449\gamma^2 + 345\gamma^3}{2592 + 5184\gamma + 3636\gamma^2 + 996\gamma^3 + 80\gamma^4}.$$

Simple calculations show that π_{duopoly} is always smaller than $\pi_i^{(i)}(a_m)$, implying that Assumption 3 is satisfied.

Comparison of integrated firms' profits. $\pi_i^{(i)}(.)$ and $\pi_j^{(i)}(.)$ are parabolas, they cross each other twice, in $a_i = c_u$ and in:

$$a_i = a_*(\gamma) \quad \equiv \quad \frac{9(12 + 16\gamma + 5\gamma^2)}{108 + 180\gamma + 93\gamma^2 + 13\gamma^3}.$$

$\pi_i^{(i)}(.)$ is concave and $\pi_j^{(i)}(.)$ is convex since:

$$\frac{d^2\pi_j^{(i)}}{da_i^2} = \frac{3(3 + 2\gamma)\gamma^2(1 + \gamma)^2}{2(3 + \gamma)^2(6 + 5\gamma)^2} > 0.$$

Hence, we have:

$$\pi_i^{(i)}(a_i) \geq \pi_j^{(i)}(a_i) \quad \Leftrightarrow \quad a_i \in [0, a_*(\gamma)]. \tag{5}$$

Let us now check whether or not $a_m(\gamma) \in [0, a_*(\gamma)]$:

$$a_m(\gamma) - a_*(\gamma) = \frac{3(3 + \gamma)(6 + 5\gamma)(-648 - 1296\gamma - 864\gamma^2 - 183\gamma^3 + 5\gamma^4)}{(108 + 180\gamma + 93\gamma^2 + 13\gamma^3)(648 + 1296\gamma + 909\gamma^2 + 249\gamma^3 + 20\gamma^4)}.$$

Analyzing the above function, we establish that there exists $\overline{\gamma} > 0$, such that:

$$a_m(\gamma) \geq a_*(\gamma) \quad \Leftrightarrow \quad \gamma \geq \overline{\gamma}.$$

Upstream equilibrium. Since $\pi_i^{(i)}(0) = \pi_j^{(i)}(0)$ and $0 \leq a_m(\gamma)$, Proposition 2 implies that the perfect competition outcome is always an equilibrium.

If $\gamma < \overline{\gamma}$, then $0 < a_m(\gamma) < a_*(\gamma)$. By Proposition 1, (5) implies that there is no monopoly-like equilibrium. Moreover $a_*(\gamma) > a_m(\gamma)$ implies by Proposition 2 that there is no other matching-like equilibrium than the perfect competition outcome.

Similarly, if $\gamma \geq \overline{\gamma}$, then $a_m(\gamma) \geq a_*(\gamma)$ and there exist monopoly-like equilibria. This is also a necessary and sufficient condition for the matching-like equilibrium with upstream price a_*.

A.6 Proof of Proposition 5

The proof is made easier by noting that our game exhibits some supermodular features. More precisely, assume that firms 1 and u propose the same upstream price $a_1 = a_u = a$, while firm 2 makes no upstream offer. The game $(\mathbb{R}; (p_k, p_{-k}, i) \mapsto \tilde{\pi}_k^{(i)}(p_k, p_{-k}, a), i = u, 1; k = 1, 2, d)$ is strictly supermodular (with the order relation $u < 1$).[40] For all k, $\tilde{\pi}_k^{(i)}(p_k, p_{-k}, a)$ has increasing differences in (p_k, i), and $\tilde{\pi}_1^{(i)}(p_1, p_{-1}, a)$ has strictly increasing differences in (p_k, i). Besides, the downstream equilibrium is, by assumption, unique. Supermodularity theory (see Vives (1999), p. 35) tells us that the equilibrium of that game is strictly increasing in i, i.e., $p_k^{(u)}(a) < p_k^{(1)}(a)$ for $k = 1, 2, d$.

Let us now show by contradiction that the upstream market cannot be supplied at an upstream price strictly larger than c_u.

Suppose that integrated firm 1 supplies the upstream market at $a_1 > c_u$. Pure upstream firm u earns zero profit. However, it is able to corner the upstream market and earn a positive profit by offering $a_u > c_u$. Since $\pi_d^{(u)}(c_u) = \pi_d^{(1)}(c_u) > \pi_d^{(1)}(a_1)$, the pure upstream firm can undercut with an upstream price close enough to c_u.[41]

Suppose now that the upstream market is supplied by pure upstream firm u at $a_u = a > c_u$. Let us show that, if it offers $a_1 = a_u = a$, integrated firm 1 corners the upstream market and enhances its profit. First, firm d strictly prefers to purchase from firm 1. When it purchases from firm u, it earns:

$$\pi_d^{(u)}(a) = (p_d^{(u)}(a) - a) D_d(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)) - c_d \left(D_d(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)) \right).$$

Since $p_k^{(u)}(a) < p_k^{(1)}(a)$ for $k = 1, 2$, there exists $\hat{p} > p_d^{(u)}(a)$ such that:

$$D_d(p_1^{(1)}(a), p_2^{(1)}(a), \hat{p}) = D_d(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)).$$

[40] The new notations are similar to the previous ones: $\tilde{\pi}_k^{(u)}(.,.,a)$ denotes the out-of-equilibrium profit of firm k when the upstream market is supplied by firm u at price a, while $p_k^{(u)}(a)$ denotes its downstream price at downstream equilibrium.

[41] It cannot undercut with $a_1 - \varepsilon$ since, as we shall see below, firm d would still prefer to buy from firm 1 in that case.

Then,

$$\pi_d^{(u)}(a) < (\hat{p} - a)D_d(p_1^{(1)}(a), p_2^{(1)}(a), \hat{p}) - c_d\left(D_d(p_1^{(1)}(a), p_2^{(1)}(a), \hat{p})\right),$$

and, by revealed preference,

$$\pi_d^{(u)}(a) < (p_d^{(1)}(a) - a)D_d(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a)) - c_d\left(D_d(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a))\right),$$

which is equal to $\pi_d^{(1)}(a)$. Therefore, firm d prefers to purchase from firm 1. It remains to be shown that firm 1's profit is larger when it supplies the upstream market. When u supplies the upstream market firm 1 earns:

$$\pi_1^{(u)}(a) = D_1(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)) - c_1\left(D_1(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a))\right).$$

Using that $p_k^{(u)}(a) < p_k^{(1)}(a)$ for $k = 2, d$, there exists $\tilde{p} > p_1^{(u)}(a)$ such that:

$$D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a)) = D_1(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)).$$

Then,

$$\pi_1^{(u)}(a) < (\tilde{p} - c_u)D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a)) - c_1\left(D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a))\right),$$

which is smaller than:

$$(\tilde{p} - c_u)D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a)) - c_1\left(D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a))\right) + (a - c_u)D_d(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a))$$

since $a > c_u$. Finally we find by revealed preference that:

$$\begin{aligned}\pi_1^{(u)}(a) < (p_1^{(1)}(a) - c_u)D_1(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a)) - c_1\left(D_1(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a))\right)\\ + (a - c_u)D_d(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a)),\end{aligned}$$

which is equal to $\pi_1^{(1)}(a)$. This concludes the proof.

References

Armstrong, M. (2002): *Hanbook of telecommunications economics*chap. The theory of access pricing and interconnection. North-Holland, Amsterdam.

Bonanno, G., and J. Vickers (1988): "Vertical Separation," *Journal of Industrial Economics*, 36(3), 257–65.

Bourreau, M., J. Hombert, J. Pouyet, and N. Schutz (2007): "Wholesale markets in telecommunications," Discussion paper, Ecole Polytechnique.

Brito, D., and P. Pereira (2006): "Access to Bottleneck Inputs under Oligopoly: A Prisoners' Dilemma?," Portuguese Competition Authority Working Paper 16.

Chen, Y. (2001): "On Vertical Mergers and Their Competitive Effects," *RAND Journal of Economics*, 32(4), 667–85.

De Bijl, P., and M. Peitz (2002): *Regulation and entry into telecommunications markets*. Cambridge University Press.

Ettinger, D. (2002): "Bidding Among Friends and Enemies," Working Paper Théma-Université de Cergy-Pontoise.

Hart, O., and J. Tirole (1990): "Vertical integration and market foreclosure," *Brookings Papers on Economic Activity*, 1990, 205–276.

Höffler, F., and K. Schmidt (2007): "Two Tales on Resale," GESY Working Paper.

Hombert, J., J. Pouyet, and N. Schutz (2007): "Anticompetitive Vertical Mergers Waves," Unpublished work.

Jehiel, P., and B. Moldovanu (1996): "Strategic Nonparticipation," *The RAND Journal of Economics*, 27(1), 84–98.

——— (2000): "Auctions with Downstream Interaction Among Buyers," *The RAND Journal of Economics*, 31(4), 768–791.

Laffont, J.-J., and J. Tirole (2001): *Competition in telecommunications*. The MIT Press, Cambridge, Massachusetts.

Ofcom (2004): "Review of the Wholesale Broadband Access Markets, Final Explanatory Statement and Notification, 13 May 2004," .

Ordover, J., and G. Shaffer (2006): "Wholesale Access in Multi-Firm Markets: when is it profitable to supply a competitor?," The Bradley Policy Research Center, Financial Research and Policy WP FR 06-08.

Ordover, J. A., G. Saloner, and S. C. Salop (1990): "Equilibrium vertical foreclosure," *American Economic Review*, 80, 127–142.

Rey, P., and J. Tirole (2005): *Handbook of industrial organization, vol. III* chap. A Primer on Foreclosure, pp. 456–789. North Holland.

Salinger, M. A. (1988): "Vertical mergers and market foreclosure," *Quarterly Journal of Economics*, 103, 345–356.

Vives, X. (1999): *Oligopoly pricing: Old ideas and new tools*. The MIT Press, Cambridge, Massachusetts.