

Increasing Lock-in to Facilitate Decision Making:  
A Property Rights Theory of the Firm with Private Information

Niko Matouschek\*  
London School of Economics

First Draft: November 1999  
This Draft: June 2000

Preliminary and Incomplete

**Abstract**

We present an incomplete contract model in which a buyer and a seller first agree on an efficient ownership structure and then bargain over the price of an input. We allow for asymmetric information at the ex post bargaining stage. The ownership structure that the parties agree on ex ante determines the payoff that each of them can realise before reaching an agreement (i.e. the ownership structure determines the inside options). A change in the ownership structure that leads to lower joint inside options has two opposing effects. On the one hand, for a given duration of the delay period, lower inside options constitute an efficiency loss. On the other hand, however, we show that making disagreement more costly accelerates agreement between the parties. This acceleration effect constitutes an efficiency gain. We show that the latter effect dominates the former if the degree of uncertainty is small relative to the gains from trade. In this case the parties optimally choose an ownership distribution that minimises their joint inside options. If the degree of uncertainty is large relative to the gains from trade then the efficiency loss from lowering the inside options outweighs the benefit of faster decision making and the parties optimally choose an ownership distribution that maximises their inside options.

---

\*Centre for Economic Performance, London School of Economics, Houghton Street, London WC2A 2AE. Email: N.B.Matouschek@lse.ac.uk. I would like to thank Antoine Faure-Grimaud, Oliver Hart, Nobu Kiyotaki, David de Meza, John Moore, Andrea Prat and Paolo Ramezzana for many useful comments and suggestions. All remaining errors are, of course, my own. Financial support by the Centre for Economic Performance, the London School of Economics and STICERD is gratefully acknowledged.

# 1 Introduction

In this paper we develop a property rights theory of the firm which focuses exclusively on the role of asset ownership in determining bargaining inefficiencies. We argue that firms at times bargain with their patrons<sup>1</sup> in the presence of private information. Bargaining between the parties may then be inefficient, in the sense that gains from trade are not realised or are only realised after some delay. The firm and its patrons anticipate these bargaining inefficiencies and take them into account when deciding on the ownership distribution of the physical assets. We explicitly model the interdependence between the ownership distribution and the ex post bargaining inefficiencies. This allows us to relate the economic environment to the ownership distribution that minimises the expected bargaining inefficiencies.

To illustrate the structure of the model, consider a situation in which there are two managers and two assets. One manager operates the upstream asset, for instance a plant that produces inputs, and the other manager operates the downstream asset, for instance a plant that produces a final output. Assume further that the two managers are locked-in, in the sense that trade between them is more profitable than trade between either of them and third parties. In the presence of transport costs, the lock-in may, for instance, be due to the close proximity of the two assets. The managers first contract over the ownership distribution of the assets and then bargain over the price of the input. At the ex post bargaining stage only the manager of the downstream firm knows how profitable it is for her to operate in the downstream industry and thus how much she values the input. Due to the presence of ex post private information the managers may spend some time haggling with each other before agreeing on the input price. While the two managers bargain over the input price, and before they reach agreement, each can trade with third parties on the spot market. We refer to the per period payoff that each manager can obtain during the bargaining process (and before reaching agreement with each other) as each manager's 'inside option' or 'pre-agreement payoff'. Each

---

<sup>1</sup>We adopt Hansmann's definition of a firm's patrons as "all persons who transact with a firm either as purchasers of the firm's products or as sellers to the firm of supplies, labor or other factors of production" (Hansmann (1996), p.12).

manager's pre-agreement payoff depends on the ownership distribution on which the parties agree ex ante. To see how the bargaining inefficiencies depend on the ownership distribution, consider a change in the ownership distribution that lowers the managers' *joint pre-agreement payoff*, i.e. the sum of their individual pre-agreement payoffs. On the one hand, for a given duration of the delay period, a reduction in the joint pre-agreement payoff constitutes a direct resource cost. On the other hand, however, the duration of the delay period itself depends on the ownership distribution. In particular, the managers will spend less time haggling over the input price the more dependent they are on each other, i.e. the lower their joint pre-agreement payoff. This *acceleration effect* constitutes an efficiency gain. Before the managers bargain over the input price, and before the downstream manager learns her valuation, they contract over the ownership distribution of the physical assets. We assume that both managers are risk neutral and not liquidity constraint. Thus, they agree on the efficient ownership distribution that minimises the total expected bargaining costs. In the paper we show that the acceleration effect dominates the direct resource cost when uncertainty is small relative to the gains from trade. In this case the efficient ownership distribution *minimises* the managers' joint pre-agreement payoff. When uncertainty is large relative to the gains from trade, the direct resource cost dominates the acceleration effect and the efficient ownership distribution *maximises* the managers' joint pre-agreement payoff.

As a further illustration of the basic effects, it is useful to consider the following example. Recently, the US car manufacturer Ford was involved in negotiations about the renewal of a supply contract with its sole supplier of car locks<sup>2</sup>. The firms disagreed about the terms of the contract: while Ford wanted a continuation of the previous conditions, the supplier suspected that Ford's profitability had improved, and therefore insisted on more favorable terms. During the negotiations the supplier suddenly stopped delivering locks to Ford which forced the latter to suspend production. This interruption was very costly for both sides: Ford incurred a production loss of 10,000 cars, costing it turnover of DM200 million, and the supplier's stock price fell sharply. After a

---

<sup>2</sup>The information about this case is taken from the following articles: "Key Position," *Frankfurter Allgemeine Zeitung*, 18.6.1998, p.17; "New Contract Between Ford and Kiekert," *Frankfurter Allgemeine Zeitung*, 14.10.1998, p.8.

few days the supplier started to deliver again, and, soon after that, the companies agreed on a new contract. This contract guaranteed the lock producer a continued position as Ford's exclusive supplier of locks. At the time commentators questioned why, after having experienced such a costly dispute, the two firms did not agree on an ownership distribution that made them less dependent on each other. For instance, it was argued that, if Ford owned the supplier, it could use the upstream assets during a dispute to ensure that at least some cars can be produced. In this paper we argue that ownership structures which maximise the parties' pre-agreement payoffs do indeed reduce the cost of disagreement for a *given disagreement period*. However, we also argue that the duration of the disagreement period itself is negatively related to the cost of disagreement. In terms of our example, this suggests that, while integration might make temporary disagreement less costly for a given disagreement period, it might also increase the duration of the disagreement period itself. It may, therefore, be optimal for Ford and its supplier to make themselves very dependent on each other, since this ensures that disputes are settled quickly.

Our paper is related to the recently developed property rights theory of the firm. This literature was initiated by the seminal papers of Grossman and Hart (1986) and Hart and Moore (1990)<sup>3</sup>, and developed further in a number of papers, including Chiu (1998), de Meza and Lockwood (1998) and Rajan and Zingales (1998). The property rights literature argues that, in the absence of comprehensive contracts, ownership of physical assets is a residual control right. That is to say that the owner of an asset has the right to take all those decisions concerning the use of the asset that have not been specified in an initial contract. The ownership of physical assets is then taken as the defining characteristic of firms. We adopt this basic conceptual framework in our paper. We differ from the existing property rights literature, however, by focusing on ex post rather than ex ante inefficiencies. In the existing property rights literature bargaining is always assumed to be efficient. Thus, once relationship specific investments have been undertaken, the ownership distribution does not influence the size but only the sharing of the overall surplus. Ownership then affects efficiency by determining the agents'

---

<sup>3</sup>For an overview of the literature see Hart (1995).

private incentives to undertake relationship specific investments prior to the bargaining stage. In contrast, in our approach ownership has efficiency implications by determining the bargaining inefficiencies that can arise in the presence of private information.

While recent work on the theory of the firm has emphasised the role of ex ante investment incentives<sup>4</sup>, earlier, and largely informal, contributions stressed the importance of bargaining inefficiencies in determining the boundaries of firms. Coase (1937), for instance, discusses the role of imperfect information and negotiation costs in the emergence of firms and states that: “The most obvious cost of “organizing” production through the price mechanism is that of discovering what the relevant prices are. [...] The costs of negotiation and concluding a separate contract for each exchange transaction which takes place on a market must also be taken into account.”<sup>5</sup> Also, Williamson (1975, 1985) identifies imperfect information as one reason why market transactions can be inefficient. In his view vertical integration allows firms to avoid costly haggling but also increases bureaucracy. Our paper is related to this early literature in that we focus on the interdependence of ownership distributions and ex post bargaining inefficiencies.

A related literature (see, for instance, Arrow (1975) and Riordan (1990)) studies integration in the presence of private information and argues that integration reduces the degree of asymmetric information. We differ from this literature in that we assume that changes in the ownership distribution only affect the payoff agents realise during the bargaining process and have no influence on the degree of the informational asymmetry. We take this approach, not because we think that integration can never reduce informational asymmetries, but because we believe that the impact of ownership changes on agents’ incentives to transmit information is ambiguous and needs to be model explicitly (see, for instance, Aghion and Tirole (1997) and Dessein (1999)). Our analysis shows that a model in which the extent of asymmetric information is not affected by changes in the ownership structure has implications that are consistent with some observed ownership patterns.

The paper proceeds as follows: in the next section we describe a simple model with

---

<sup>4</sup>Next to the above mentioned property rights literature see also Klein, Crawford, and Alchian (1978).

<sup>5</sup>Coase (1937), p.336.

a static bargaining game. In this model the uninformed party simply makes a take-it-or-leave-it offer to the informed party. Because of the static nature of the bargaining game we have to adapt our arguments somewhat. In particular, in the static model, the ownership distribution is assumed to determine the disagreement, rather than the pre-agreement payoffs. Also, a change in the ownership distribution that lowers the disagreement payoffs increases the probability of agreement rather than accelerating agreement. In spite of these differences we can use the static model to illustrate our main arguments. We do so in section 3 by solving the static model. While the static model is straightforward to solve, it has two conceptual problems. First, it does not allow the parties to continue bargaining until the gains from trade are realised and simply assumes that the bargaining process ends after one offer. Second, by assuming that the ownership distribution that is agreed on ex ante determines the disagreement payoffs, it restricts the analysis to irreversible ownership changes. While some ownership changes are indeed irreversible, a model that allows for more general ownership changes is clearly more satisfactory. We present such a model, with a dynamic bargaining game, in section 4 and solve it in section 5. We argue that the static and the dynamic models have very similar implications for the efficient ownership distributions. In section 6 we discuss these implications and show to what extent they are consistent with observed ownership patterns. Section 7 concludes.

## 2 The Static Model

There are two risk neutral players, a buyer  $B$  and a seller  $S$ . Neither the buyer nor the seller is liquidity constrained. The seller can produce an input that can be used by the buyer to produce the final output. To engage in production the buyer and the seller need access to some physical assets. Let  $A$  denote the ownership distribution of these physical assets across the agents.

There are two periods,  $t = -1$  and  $t = 0$ . At  $t = -1$  (ex ante) the parties contract over the ownership distribution  $A$ . We make the important assumption that the particular type of input required by the buyer ex post is uncertain ex ante and

cannot be specified in the initial contract<sup>6</sup>.

At  $t = 0$  (ex post) uncertainty about the required input is realised and production can take place. At this stage the buyer and the seller bargain over the input price. Once the parties agree on a transfer price the seller produces the relevant input and the buyer uses the input to produce the final output. We normalise the seller's production costs to zero. By producing the final product the buyer generates a payoff of  $\pi$ . Ex ante the value of  $\pi$  is uncertain and both parties only know that it is uniformly distributed on  $[\pi_l, \pi_h]$ , where  $\pi_h = \mu + \alpha$  and  $\pi_l = \mu - \alpha \geq 0$ . At the beginning of the trading period ( $t = 0$ ) the level of  $\pi$  is realised. We assume that only the buyer learns the realization of  $\pi$ .

If trade between the buyer and the seller does not take place, the players realise their respective disagreement payoffs  $b(A)$  and  $s(A)$ . The joint disagreement payoffs are denoted by  $j(A) \equiv b(A) + s(A)$ . We assume that trade between the buyer and the seller is always profitable, i.e.

$$(1) \quad \pi_l \geq j(A), \quad \forall A.$$

Note that only the disagreement payoffs are functions of the ownership distribution. When the buyer and the seller do not transact with each other, and instead trade with third parties on the spot market, the return that each one realises depends on the assets he or she owns. The assumption that disagreement payoffs depend on the ownership distribution that was agreed on ex ante implies that changes in the ownership distribution are irreversible.

Note also that no other variable or parameter depends on the ownership distribution. In particular, the degree of asymmetric information  $\alpha$  does not depend on  $A$ . This captures the idea that the degree of asymmetric information between the players is independent of the ownership distribution. Also, the buyer's valuation of the input  $\pi$  does not depend on the ownership distribution since, when trade between the parties takes place, both players have access to all the assets in the relationship, independent of the ownership distribution.

---

<sup>6</sup>For microfoundations of incomplete contracts see, for instance, Hart and Moore (1999), Segal (1999), and, for the case of one-sided asymmetric information, Reiche (1999).

In this simple model bargaining takes the form of a take-it-or-leave-it offer by the (uninformed) seller. Hence, after the seller makes an offer  $p$ , the buyer decides whether to accept or to reject it. If the buyer accepts the offer, she realises a payoff of  $\pi$  and the seller realises a payoff of  $p$ . If, instead, the buyer rejects the offer, both parties simply realise their disagreement payoffs.

### 3 The Analysis of the Static Model

In this section we analyse the model that was described above. We first solve the bargaining game that takes place at  $t = 0$  and then study the optimal ownership distribution on which the parties agree at  $t = -1$ .

#### 3.1 The Bargaining Game

At  $t = 0$  the uninformed seller makes a take-it-or-leave-it offer  $p$ . The buyer accepts  $p$  if and only if  $\pi - p \geq b(A)$ . Hence,  $p_c(\pi_c) = \pi_c - b(A)$  denotes the price that is accepted by all buyers of type  $\pi \in [\pi_c, \pi_h]$ .

The seller's expected return from making an offer  $p_c(\pi_c)$  is then given by

$$(2) \quad R(\pi_c) = p_c(\pi_c) \frac{\pi_h - \pi_c}{\pi_h - \pi_l} + s(A) \frac{\pi_c - \pi_l}{\pi_h - \pi_l},$$

and his optimal offer solves

$$(3) \quad \max_{\pi_c \in [\pi_l, \pi_h]} R(\pi_c).$$

A cutoff point  $\pi_c^*$  is a maximum of (3) if and only if it satisfies the following first and second order conditions:

$$R'(\pi_c^*) = \begin{cases} 0 & \text{for } \pi_c^* \in (\pi_l, \pi_h) \\ \leq 0 & \text{for } \pi_c^* = \pi_l \\ \geq 0 & \text{for } \pi_c^* = \pi_h, \end{cases}$$

and

$$R''(\pi_c^*) < 0 \text{ for } \pi_c^* \in (\pi_l, \pi_h).$$

Differentiating (2) gives

$$R'(\pi_c) = \frac{1}{(\pi_h - \pi_l)}(\pi_h + j(A) - 2\pi_c)$$

and

$$R''(\pi_c) = -\frac{2}{(\pi_h - \pi_l)}.$$

Note, first, that  $\pi_c^*$  is unique since  $R''(\pi_c) < 0, \forall \pi_c$ . Note, second, that  $R'(\pi_h) < 0$ , so that  $\pi_h$  is not a maximum. Finally, note that  $R'(\frac{1}{2}(\pi_h + j(A))) = 0$  and that  $R'(\pi_l) < 0$  if and only if  $\pi_l > \frac{1}{2}(\pi_h + j(A))$ . Thus, the optimal cutoff type  $\pi_c^*$  that solves (3) is uniquely given by

$$(4) \quad \pi_c^* = \max\left(\frac{1}{2}(\pi_h + j(A)), \pi_l\right).$$

Since  $\pi_c^* < \pi_h$ , there is always a positive probability that the buyer accepts the seller's offer. Note that  $\pi_c^*$  is weakly increasing in  $j(A)$ . Thus, the higher the joint disagreement payoffs  $j(A)$ , the less likely it is that the buyer accepts the seller's offer. Finally, note that any type of buyer  $\pi \in [\pi_l, \pi_h]$  accepts the offer if  $\pi_c^* = \pi_l$ .

The following lemma summarises the agents' bargaining strategies:

**Lemma 1** *At  $t = 0$  the seller makes a take-it-or-leave-it offer of  $p_c(\pi_c^*)$ . The offer is accepted if the buyer is of type  $\pi \in [\pi_c^*, \pi_h]$ , and rejected if the buyer is of type  $\pi \in [\pi_l, \pi_c^*)$ .*

### 3.2 The Optimal Ownership Distribution

At  $t = -1$  the parties contract over the ownership distribution. Since the agents are risk neutral, forward looking, and not wealth constrained, they always agree on the jointly efficient ownership distribution. Clearly, the ex ante division of surplus depends on the relative bargaining powers of the players. However, since the division of the surplus at the ex ante stage does not affect the analysis of the optimal ownership structure, we make no explicit assumptions about the relative bargaining powers at the ex ante stage.

Trade takes place if the buyer is of type  $\pi \in [\pi_c^*(A), \pi_h]$ . In this case the players generate a social surplus of  $\pi$ . In contrast, trade does not take place if the buyer is of type  $\pi \in [\pi_l, \pi_c^*(A))$ . The players then realise their joint disagreement payoffs  $j(A)$ . Hence, at  $t = -1$ , expected social surplus  $W(A)$  is given by

$$(5) \quad W(A) = \frac{1}{\pi_h - \pi_l} \left( \int_{\pi_c^*(A)}^{\pi_h} \pi d\pi + \int_{\pi_l}^{\pi_c^*(A)} j(A) d\pi \right).$$

Let  $\bar{A}$  and  $\underline{A}$  denote the ownership distributions that, respectively, maximise and minimise  $j(A)$ . To find the optimal ownership distribution, we first analyse the efficiency implications of moving from  $\bar{A}$  to  $\underline{A}$ . It follows from (4) and (5) that

$$(6) \quad W(\underline{A}) - W(\bar{A}) = \frac{1}{\pi_h - \pi_l} \left( \int_{\pi_c^*(\underline{A})}^{\pi_c^*(\bar{A})} \pi - j(\bar{A}) d\pi - \int_{\pi_l}^{\pi_c^*(\underline{A})} j(\bar{A}) - j(\underline{A}) d\pi \right).$$

This is the key equation in the model. We know from equation (4) that the seller's offer is (weakly) increasing in  $j(A)$ . Hence, the lower the joint disagreement payoffs, the more likely it is that the buyer accepts the seller's offer. The first integral on the RHS of (6) represents the corresponding efficiency gain. Clearly, in the case of disagreement, a reduction in the joint disagreement payoff constitutes a direct resource cost. This effect is represented by the second integral on the RHS of (6).

By moving from  $\bar{A}$  to  $\underline{A}$  the parties therefore increase the probability of trade but also make disagreement more costly. We now study the conditions under which either effect dominates. To do so, we prove the following lemma:

**Lemma 2** *In the permissible parameter range the function  $\Delta W(\underline{A}, \bar{A}) = W(\underline{A}) - W(\bar{A})$  has the following properties:*

$$\Delta W(\underline{A}, \bar{A}) = \begin{cases} 0 & \text{if } \alpha \in [0, \frac{1}{3}(\mu - j(\bar{A}))] \\ > 0 & \text{if } \alpha \in (\frac{1}{3}(\mu - j(\underline{A})), \frac{3}{5}(\mu - \frac{1}{2}(j(\bar{A}) + j(\underline{A})))) \\ \leq 0 & \text{if } \alpha \in [\frac{3}{5}(\mu - \frac{1}{2}(j(\bar{A}) + j(\underline{A}))), \mu - j(\bar{A})] \end{cases}$$

and

$$\frac{\partial \Delta W(\underline{A}, \bar{A})}{\partial \alpha} = \begin{cases} 0 & \text{if } \alpha \in [0, \frac{1}{3}(\mu - j(\bar{A}))] \\ > 0 & \text{if } \alpha \in (\frac{1}{3}(\mu - j(\bar{A})), \frac{1}{3}(\mu - j(\underline{A}))) \\ < 0 & \text{if } \alpha \in (\frac{1}{3}(\mu - j(\underline{A})), \mu - j(\bar{A})]. \end{cases}$$

**Proof:** see appendix.

Lemma 2 is illustrated in figure 1 and its intuition is straightforward. When the degree of uncertainty is small relative to the gains from trade, i.e.  $\alpha \in [0, \frac{1}{3}(\mu - j(\bar{A}))]$ , the seller's offer is always accepted by the buyer, independent of the ownership distribution. This can be seen by noting that  $\pi_c^*(\underline{A}) = \pi_c^*(\bar{A}) = \pi_l$ . Thus, for these parameter values, moving from  $\bar{A}$  to  $\underline{A}$  neither increases the probability of agreement (since the parties agree immediately even when  $A = \bar{A}$ ) nor does it lead to any efficiency losses (since the players never realise the disagreement payoffs). As a result, changes in the ownership distribution have no efficiency implications.

Consider now the effect of increasing the degree of uncertainty relative to the gains from trade so that  $\alpha \in [\frac{1}{3}(\mu - j(\bar{A})), \frac{1}{3}(\mu - j(\underline{A}))]$ . In this case some buyer types reject the seller's offer when the disagreement payoffs are high ( $\pi_c^*(\bar{A}) > \pi_l$ ), while all buyer types accept the offer when the disagreement payoffs are low ( $\pi_c^*(\underline{A}) = \pi_l$ ). Thus, moving from  $\bar{A}$  to  $\underline{A}$  increases the probability of agreement. Note that, in equilibrium, the disagreement payoffs are never realised when  $A = \underline{A}$  since, in this case, the buyer always accepts the seller's offer. As a result, there is no efficiency cost to moving from  $\bar{A}$  to  $\underline{A}$  so that, in this parameter range,  $\Delta W(\underline{A}, \bar{A})$  is positive.

Finally, consider the effect of further increasing the degree of uncertainty relative to the gains from trade so that  $\alpha \in (\frac{1}{3}(\mu - j(\underline{A})), \mu - j(\bar{A})]$ . In this parameter region, some buyer types reject the seller's offer even when  $A = \underline{A}$  (this can be seen by noting that  $\pi_c^*(\bar{A}) > \pi_c^*(\underline{A}) > \pi_l$ ). Moving from  $\bar{A}$  to  $\underline{A}$  then still increases the probability of agreement but also makes disagreement more costly. The larger  $\alpha$ , the stronger is the former effect relative to the latter. Hence, in this region, the expected efficiency gain of moving from  $\bar{A}$  to  $\underline{A}$  is decreasing in  $\alpha$ . It can be shown that, for  $\alpha \in (\frac{1}{3}(\mu - j(\underline{A})), \frac{3}{5}(\mu - \frac{1}{2}(j(\bar{A}) + j(\underline{A})))$ , moving from  $\bar{A}$  to  $\underline{A}$  reduces expected efficiency, while, for  $\alpha \in (\frac{3}{5}(\mu - \frac{1}{2}(j(\bar{A}) + j(\underline{A}))), \mu - j(\bar{A})]$ , it increases expected efficiency. Note that  $\frac{3}{5}(\mu - \frac{1}{2}(j(\bar{A}) + j(\underline{A})))$  is increasing in  $\mu$  and decreasing in  $j(\bar{A})$  and  $j(\underline{A})$ . Thus, the parameter space in which it is (weakly) optimal to reduce the disagreement payoffs is increasing in the  $\mu$  and decreasing in  $j(\bar{A})$  and  $j(\underline{A})$ .

So far we have only considered two possible ownership allocations, namely  $\bar{A}$  and  $\underline{A}$ .

The following lemma extends the analysis to all other possible ownership allocations.

**Lemma 3** *Consider any  $\hat{A}$  such that  $j(\hat{A}) \in [j(\underline{A}), j(\overline{A})]$ . It can be shown that*

$$\max[W(\overline{A}), W(\underline{A})] \geq W(\hat{A}).$$

**Proof:** see appendix.

Together lemmas (2) and (3) establish the following proposition.

**Proposition 1** *For  $\alpha \in [0, \frac{3}{5}(\mu - \frac{1}{2}(j(\overline{A}) + j(\underline{A})))]$  it is weakly optimal for the buyer and the seller to minimise their disagreement payoffs by choosing the ownership distribution  $\underline{A}$ . For  $\alpha \in (\frac{3}{5}(\mu - \frac{1}{2}(j(\overline{A}) + j(\underline{A}))), \mu - j(\overline{A})]$  it is optimal for the parties to maximise their disagreement payoffs by choosing the ownership distribution  $\overline{A}$ .*

Proposition 1 shows that the efficient ownership distribution maximises the joint disagreement payoffs if the gains from trade are small relative to the degree of uncertainty. If the gains from trade are large relative to the degree of uncertainty, the efficient ownership allocation minimises the joint disagreement payoffs.

## 4 The Dynamic Model

The static model that was presented in the previous section is very simple and tractable. There are, however, two conceptual problems with this model. Firstly, it assumes that the seller can only make one offer. If that offer is not accepted, trade does not take place. However, it is not clear why the parties cannot continue to bargain over the price of the input after the initial offer has been rejected. After all, there are still gains from trade that the parties could realise. Secondly, even if the parties have to stop bargaining over the input price after only one offer, it is not evident why, in the case of disagreement over the input price, they do not renegotiate the ownership distribution (since all relevant information is common knowledge we would expect such renegotiations to be efficient). Only if changes in the ownership distribution are irreversible would the parties not be able to engage in such renegotiations. While some ownership changes may indeed be irreversible (see section (6) for an example), a model that allows for

reversible ownership changes is clearly more satisfactory. In this section we present a version of the model which addresses both of these issues. In particular, we now allow for a dynamic bargaining game in which the parties continue to bargain until agreement is reached and the gains from trade are realised. We will argue that the dynamic model leads to results that are very similar to those presented above.

The bargaining game that we consider here is related to Admati and Perry (1987). They study a Rubinstein infinite-horizon, alternating-offers model with one-sided asymmetric information in which players can delay making offers. We adopt their basic framework but extend the analysis in two ways: first, while they allow for only two types of buyers, we allow for a continuum of such types<sup>7</sup>. Secondly, and more importantly, in our model the players can realise non-zero payoffs during the delay period (i.e. we allow for non-zero ‘inside options’ or ‘pre-agreement payoffs’). In Admati and Perry (1987) the inside options are assumed to be zero.

We now turn to the formal description of the model. The ex ante period ( $t = -1$ ) is exactly as described in section 2.

Ex post the parties bargain over the price of the input. As in the static model, the seller’s cost of producing the input is zero and the buyer’s return from using the input in the final good production is given by  $\pi$ , where  $\pi$  is distributed uniformly on  $[\pi_l, \pi_h]$ . The buyer learns the realisation of  $\pi$  at the beginning of the bargaining period ( $t = 0$ ). Before they reach an agreement over the input price the buyer and the seller can each trade with third parties on the spot market. By doing so the seller can realise an inside option<sup>8</sup> of  $s(A)$  per instant of time and the buyer can realise an inside option of  $b(A)$  per instant of time. We continue to assume that trade is always profitable, i.e.

$$(7) \quad \pi_l \geq \frac{j(A)}{r}, \quad \forall A,$$

where  $r$  is the positive discount rate.

---

<sup>7</sup>Crampton (1992) studies a model in which he allows for a continuum of types and two-sided asymmetric information.

<sup>8</sup>Note the difference between inside and outside options. Inside options are the payoffs that the parties realise while they disagree temporarily. Outside options, in contrast, are the payoffs that the parties realise if they terminate bargaining. For full information models with positive inside options see, for instance, Binmore, Rubinstein, and Wolinsky (1986) and Muthoo (1999).

The seller can make the first offer at any time  $t \geq 0$ . Thereafter the parties alternate making offers. The minimum time between offers is  $\bar{t} = -\frac{1}{r} \log \delta$ , where  $\delta \in [0, 1]$  is the discount factor from one period delay. After a player has received an offer he can take either of two actions: accept the offer, in which case trade takes place and the game ends, or reject the offer and make a counteroffer. Either action can be taken at any time after the minimum delay period  $\bar{t}$ . After a player has made an offer he can take no further action until the other player has either accepted the offer or made a counteroffer.

Let  $(t, p)$  denote an outcome of the game in which the parties agree at time  $t$  to trade at price  $p$ . For any  $(t, p)$  the seller's payoff is given by  $e^{-rt}p + (1 - e^{-rt})\frac{s(A)}{r}$  and the buyer's payoff is given by  $e^{-rt}(\pi - p) + (1 - e^{-rt})\frac{b(A)}{r}$ . Note that these preferences imply that the players are risk neutral and that they are impatient, in the sense that they prefer agreement today to the same agreement at any later date. Note also that the buyer is more impatient the higher the gains from trade.

Let  $n \in \{1, 2, \dots, \infty\}$  denote the number of offers that have been made and let  $\tau_n$  denote the time after the minimum delay of  $\bar{t}$  after which offer  $p_n$  has been made. After  $n$  rounds have been played the history is given by  $h^n = \{p_i, \tau_i\}_{i=1}^n$ . Throughout the paper, when we state that a player replies "immediately", we mean that he replies without any further delay after the minimum time  $\bar{t}$ . Similarly, when we say that an offer  $p_n$  has been "delayed", we mean that  $\tau_n > 0$ .

A pure strategy  $\sigma_S$  for the seller and  $\sigma_B(\pi)$  for the buyer specifies, for any history  $h^n$  after which it is the player's turn to move, the delay period  $\tau_{n+1}$ , whether  $p_n$  is accepted, and, if not accepted, the counteroffer  $p_{n+1}$ . Let  $\lambda = \{F(\cdot/h^n)\}$  denote the system of beliefs that the seller has about the distribution of  $\pi$  after any history  $h^n$ .

Below we study a sequential equilibrium of this game. A sequential equilibrium specifies the strategies  $\sigma_S$  and  $\sigma_B$  and a system of beliefs  $\lambda$  such that each strategy is optimal given the other strategy and beliefs, and the beliefs are consistent with Bayes' rule (when possible).

## 5 The Analysis of the Dynamic Model

We proceed as in the static model: we first describe the equilibrium of the bargaining game and then turn to the optimal ownership distribution.

### 5.1 The Bargaining game

In this section we construct a sequential equilibrium in pure strategies for the dynamic bargaining game. We believe that the equilibrium we study is appealing because of its simplicity<sup>9</sup>. However, in general, there may be a large number of equilibria and we do not attempt to characterise the conditions under which the equilibrium is unique. Thus, we do not show how bargaining must proceed and only study one form it might take.

Intuitively, the equilibrium can be described as follows: the seller makes an offer at  $t = 0$ . The buyer then either accepts the offer or rejects it. Acceptance takes place at  $t = \bar{t}$ . A buyer who rejects the offer delays her counteroffer so long as to distinguish herself from the buyer with a valuation that is just higher than her own. She then makes an offer that corresponds to the complete information equilibrium offer. This offer is immediately accepted by the seller. Below we show that an equilibrium of this form does indeed exist. We first describe the complete information equilibrium offer. We then show by how long the buyer has to delay her counteroffer to credibly signal her type. Finally, we derive the seller's optimal first offer and describe the players' strategies and the beliefs that support such an equilibrium.

To economise on notation, we suppress the ownership distribution  $A$  from all the functions that are applied and derived in this section. We analyse the impact of changes of  $A$  on the described equilibrium in the section 5.2.

**The Full Information Game** Rubinstein (1982) has shown that a full information alternating offers game with fixed time between offers has a unique subgame perfect equilibrium. It is straightforward to extend this analysis to the case of non-zero inside

---

<sup>9</sup>It is also closely related to the equilibria studied by Admati and Perry (1987) and Crampton (1992).

options and to show that such a game also has a unique subgame perfect equilibrium. The equilibrium depends on who makes the first offer. Let  $p^B(\pi)$  denote the equilibrium price if a buyer of type  $\pi$  makes the first offer, and let  $p^S(\pi)$  be the equilibrium price if the seller makes the first offer to a buyer of type  $\pi$ . It is straightforward to show (see for example Muthoo (1999, pp.138-43)) that

$$\begin{aligned} p^B(\pi) &= \frac{1}{1+\delta}(\delta\pi - \delta\frac{b}{r} + \frac{s}{r}) \\ p^S(\pi) &= \frac{1}{1+\delta}(\pi - \frac{b}{r} + \delta\frac{s}{r}). \end{aligned}$$

These prices have the property that each player is indifferent between trading at the other player's offer immediately and trading at the player's own offer next period, i.e.

$$(8) \quad \begin{aligned} \pi - p^S(\pi) &= \delta(\pi - p^B(\pi)) + (1-\delta)\frac{b}{r} \\ p^B(\pi) &= \delta p^S(\pi) + (1-\delta)\frac{s}{r}. \end{aligned}$$

**Delay** Let  $\pi_c$  denote the type of buyer who is indifferent between accepting an offer  $p_c$  and counteroffering  $p^B(\pi_c)$  in the knowledge that the seller immediately accepts such a counteroffer. Then  $\pi_c$  is defined by

$$(9) \quad \pi_c - p_c = \delta(\pi_c - p^B(\pi_c)) + (1-\delta)\frac{b}{r}.$$

Suppose the seller infers that the buyer is of type  $\pi_c$  if, in response to an offer of  $p_c$  by the seller, the buyer makes a counteroffer after the minimum delay  $\bar{t}$ . Suppose further that the seller immediately accepts an offer  $p^B(\pi_c)$  if he thinks that the buyer is of type  $\pi_c$ . It then follows from (9) that any  $\pi \in (\pi_c, \pi_h]$  prefers accepting  $p_c$  to pretending to be of type  $\pi_c$  by offering  $p^B(\pi_c, A)$  after  $\bar{t}$ . This is the case since

$$\pi - p_c > \delta(\pi_c - p^B(\pi_c)) + (1-\delta)\frac{b}{r}, \text{ for } \pi \in (\pi_c, \pi_h].$$

Suppose next that the seller offers  $p_c$  and believes that any  $\pi \in [\pi_c, \pi_h]$  accepts this offer. Suppose further that the seller believes that the buyer is of type  $\pi(\tau, p_c)$  if, in

response to being offered  $p_c$ , the buyer counteroffers  $p^B(\pi)$  after a delay of  $\tau$ . Then the present discounted value of a buyer of type  $\pi(\tau, p_c)$  from pretending to be of type  $\pi(\tilde{\tau}, p_c)$  is given by

$$(10) \quad U(\tilde{\tau}) = \delta e^{-r\tilde{\tau}}(\pi(\tau, p_c) - p^B(\pi(\tilde{\tau}, p_c))) + (1 - \delta e^{-r\tilde{\tau}})\frac{b}{r}.$$

Substituting (8) into (10) and differentiating with respect to  $\tilde{\tau}$  gives the following first order condition

$$(11) \quad U'(\tilde{\tau}) = -r[\pi(\tau, p_c) - p^B(\pi(\tilde{\tau}, p_c))] - \frac{\delta}{1 + \delta} \frac{\partial \pi(\tilde{\tau}, p_c)}{\partial \tau} + b = 0.$$

If (11) describes a maximum, and we check the second order condition below, then the function  $\pi(\tau, p_c)$  has to satisfy the first order condition (11) for  $\tau = \tilde{\tau}$ . Thus, we get

$$(12) \quad \delta \frac{\partial \pi(\tau, p_c)}{\partial \tau} + r\pi(\tau, p_c) - j = 0.$$

Note that the buyer does not have to signal her type if she has the highest possible valuation, and thus  $\pi(0) = \pi_c$ . The differential equation (12) can then be solved:

$$(13) \quad \pi(\tau, p_c) = \pi_c(p_c)e^{-r\tau} + (1 - e^{-r\tau})\frac{j}{r}.$$

We can now use (13) to confirm that the second order condition for the maximisation problem is satisfied. Differentiating (10) twice with respect to  $\tilde{\tau}$  gives

$$U''(\tilde{\tau}) = \frac{\delta}{1 + \delta} \left( r \frac{\partial \pi(\tilde{\tau}, p_c)}{\partial \tau} - \frac{\partial^2 \pi(\tilde{\tau}, p_c)}{\partial \tau^2} \right) = \frac{-2r^2 \delta e^{-r\tilde{\tau}}}{1 + \delta} (\pi_c - j) < 0,$$

so that the second order condition is indeed satisfied.

Let  $\tau(\pi, p_c)$  denote the inverse of  $\pi(\tau, p_c)$  with respect to  $\tau$ . Hence, after rejecting an offer of  $p_c$ , a buyer of type  $\pi$  can credibly signal her type by delaying her offer by  $\tau(\pi, p_c)$ . Rearranging (13) gives

$$(14) \quad e^{-r\tau(\pi, p_c)} = \left( \frac{r\pi - j}{r\pi_c - j} \right)^\delta.$$

Note, first, that  $\tau(\pi, p_c)$  is decreasing in  $\pi$ . Thus, the time needed by the buyer to signal her type is longer the lower the her profits. This feature of the model is, of course, due to the particular preferences we described above which implicitly assumed that a buyer is more patient the lower  $\pi$ . Note, also, that  $\tau(\pi, p_c)$  is increasing in the level of the inside options  $j$ <sup>10</sup>. Thus, the higher the inside options, the longer it takes the buyer to signal her type. This feature will be important in the discussion of the optimal ownership structure below.

**The First Offer** Suppose the seller believes that, in response to an offer of  $p_c$ , any buyer of type  $\pi \in [\pi_c, \pi_h]$  accepts immediately, while any buyer of type  $\pi \in [\pi_l, \pi_c)$  rejects the offer and instead counteroffers  $p^B(\pi)$  after  $\tau(\pi, p_c)$ . Suppose further that the seller accepts  $p^B(\pi)$  if he believes that the buyer is of type  $\pi$ . Let  $R(\pi_c)$  denote the seller's expected return from making an offer of  $p_c(\pi_c)$ . Then

$$R(\pi_c) = \delta p_c(\pi_c) \frac{\pi_h - \pi_c}{\pi_h - \pi_l} + \int_{\pi_l}^{\pi_c} [\delta^2 e^{-rt(\pi, p_c(\pi_c))} p^B(\pi) + (1 - \delta^2 e^{-rt(\pi, p_c(\pi_c))}) \frac{s(A)}{r}] \frac{1}{\pi_h - \pi_l} d\pi.$$

The optimal first offer is given by  $p_c(\pi_c^*)$ , where

$$(15) \quad \pi_c^* = \arg \max_{\pi_c \in [\pi_l, \pi_h]} R(\pi_c).$$

The next lemma describes the solution to (15) when  $\delta \rightarrow 1$  (the case for  $\delta < 1$  is described in the appendix).

**Lemma 4** *For  $\delta \rightarrow 1$  the maximisation problem (15) has a unique solution  $\pi_c^* \in (\pi_l, \pi_h)$  which solves*

$$(16) \quad 3r(\pi_h - \pi_c^*)(r\pi_c^* - j)^2 - (r\pi_c^* - j)^3 + (r\pi_l - j)^3 = 0.$$

*It can be shown that*

$$\frac{\partial \pi_c^*}{\partial j} > 0.$$

---

<sup>10</sup>Differentiating (14) gives  $\frac{\partial \tau(\pi, p_c)}{\partial j} = \frac{\delta(\pi_c - \pi)}{(r\pi - j)(r\pi_c - j)}$ . Hence,  $\frac{\partial \tau(\pi, p_c)}{\partial j} \geq 0$ , for  $\pi \in [\pi_l, \pi_c]$ .

**Proof:** see appendix.

It is now straightforward to prove the following proposition:

**Proposition 2** *The following strategies and beliefs form a sequential equilibrium of the dynamic bargaining game:*

- *The seller's beliefs:* If, in response to an offer  $p$  by the seller, the buyer makes an offer after a delay of  $\tau(\pi, p)$ , then  $\lambda(\pi) = 1$ . Otherwise  $\lambda(\pi_l) = 1$ .
- *The seller's strategy:* the seller starts by offering  $p_c(\pi_c^*)$ . If  $\lambda(\pi) = 1$ , then the seller immediately accepts any offer  $p \geq p^B(\pi)$  and, in response to an offer  $p < p^B(\pi)$ , the seller immediately counteroffers  $p^S(\pi)$ .
- *The buyer's strategy:* A buyer of type  $\pi$  accepts any offer  $p \leq p^S(\pi)$ . In response to an offer  $p > p^S(\pi)$  a buyer of type  $\pi$  counteroffers  $p^B(\pi)$  after a delay of  $\tau(\pi, p)$ .

**Proof:** see appendix.

## 5.2 The Optimal Ownership Distribution

In this section we analyse the optimal ownership structure when the minimum time between offers gets arbitrarily small, i.e.  $\delta \rightarrow 1$ . It is one of the appealing features of the Admati and Perry (1987) analysis, which continues to hold in this model, that delay occurs with positive probability even if parties can reply to offers instantaneously.

At  $t = -1$  the players contract over the ownership distribution. Since the agents are risk neutral, forward looking, and not wealth constrained, they always agree on the jointly efficient ownership distribution. The players know that, ex post, the seller makes an offer which is instantly accepted by any buyer of type  $\pi \in [\pi_c^*(A), \pi_h]$ . If the buyer is of such a type, then the parties immediately realise a joint surplus of  $\pi$ . Any buyer of type  $\pi \in [\pi_l, \pi_c^*(A))$  does not accept the seller's initial offer. Instead she waits  $\tau(\pi, p_c, A)$  before making an offer that the seller accepts immediately. Thus, for any buyer of type  $\pi \in [\pi_l, \pi_c^*(A))$  the joint payoff is given by

$$\pi e^{-r\tau(\pi, p_c, A)} + \frac{j(A)}{r}(1 - e^{-r\tau(\pi, p_c, A)}).$$

Expected social surplus is then given by

$$(17) \quad W(A) = \frac{1}{(\pi_h - \pi_l)} \left[ \int_{\pi_c^*(A)}^{\pi_h} \pi d\pi + \int_{\pi_l}^{\pi_c^*(A)} \pi e^{-r\tau(\pi, p_c, A)} + \frac{j(A)}{r} (1 - e^{-r\tau(\pi, p_c, A)}) d\pi \right].$$

Recall that  $\bar{A}$  and  $\underline{A}$  denote the ownership distributions that, respectively, maximise and minimise  $j(A)$ . Changing the ownership distribution from  $\bar{A}$  to  $\underline{A}$  affects expected social surplus in two opposing ways: on the one hand, such a change accelerates agreement but, on the other hand, it also makes temporary disagreement more costly. To understand the acceleration effect, consider how  $\pi_c^*(A)$  and  $\tau(\pi, p_c, A)$  depend on the joint inside options. In lemma 4 we have shown that  $\pi_c^*(A)$  is increasing in  $j(A)$ . Hence, the higher the joint inside options, the less likely it is that the buyer will accept the seller's first offer. We have also noted above that the delay period  $\tau(\pi, p_c, A)$  is increasing in the inside options (see equation (14)). Together these two effects constitute the *acceleration effect*: the lower the joint inside options, the faster the parties can be expected to reach agreement. While lower inside options accelerate agreement, they also make temporary disagreement more costly. The last term on the RHS of (17) gives the joint payoff during the delay period which, for a given  $\tau(\pi, p_c, A)$ , is clearly decreasing in the joint inside options  $j(A)$ .

It follows from this discussion that the increase in expected social surplus of moving from  $\bar{A}$  to  $\underline{A}$  is given by

$$(18) \quad W(\underline{A}, \bar{A}) = \frac{1}{r(\pi_h - \pi_l)} \left[ \int_{\pi_c^*(\underline{A})}^{\pi_c^*(\bar{A})} (r\pi - j(\bar{A})) (1 - e^{-r\tau(\pi, \bar{A})}) d\pi + \int_{\pi_l}^{\pi_c^*(\underline{A})} (r\pi - j(\bar{A})) (e^{-r\tau(\pi, \underline{A})} - e^{-r\tau(\pi, \bar{A})}) d\pi - \int_{\pi_l}^{\pi_c^*(\underline{A})} (j(\bar{A}) - j(\underline{A})) (1 - e^{-r\tau(\pi, \underline{A})}) d\pi \right].$$

The acceleration effect is given by the first two integrals. The last integral represents the resource cost of reducing the inside options. Note that, since  $\pi_c^*(\bar{A}) > \pi_c^*(\underline{A}) > \pi_l > j(\bar{A}) > j(\underline{A})$ , both the benefit (the acceleration effect) and the resource cost of reducing inside options are always weakly positive. Note also that, for  $\alpha = 0$ , the parties reach agreement instantaneously, independent of the ownership distribution. Thus,  $W(\underline{A}, \bar{A}) = 0$  for  $\alpha = 0$ .

It is worth to note the similarity of the expressions for  $\Delta W(\underline{A}, \overline{A})$  in the static (equation (6)) and the dynamic model (equation (18)). In the static model reducing  $j(A)$  increases the probability of agreement, while making permanent disagreement more costly. Here a reduction in  $j(A)$  accelerates agreement, while making temporary disagreement more costly.

In contrast to the static model we have, so far, not been able to find a complete analytic solution for the dynamic model<sup>11</sup>. To make further progress we now rely on simple simulations of equation (18). Consider figure 2 which plots the efficiency gain of moving from  $\overline{A}$  to  $\underline{A}$  for several different parameter configurations<sup>12</sup>. As expected,  $\Delta W(\underline{A}, \overline{A}) = 0$  for  $\alpha = 0$ . For small positive values of  $\alpha$  there is an expected efficiency gain of moving from  $\overline{A}$  to  $\underline{A}$ . In this region the acceleration effect dominates the resource cost effect. As  $\alpha$  becomes larger, however, the resource cost effect becomes larger relative to the acceleration effect. For large enough  $\alpha$  the resource cost effect dominates the acceleration effect. In this case a reduction in the joint inside options leads to an expected efficiency loss. Note again the similarity of figures 1 and 2 which, respectively, plot the expected efficiency gain of moving from  $\overline{A}$  to  $\underline{A}$  for the static and the dynamic model. In both cases moving from  $\overline{A}$  to  $\underline{A}$  is efficiency enhancing (reducing) if uncertainty is small (large) relative to the gains from trade. In figure 2 it can be seen that  $\max(W(j = 50), W(j = 20)) \geq W(j = 30)$ . This suggests that, as in the static model, only  $\underline{A}$  and  $\overline{A}$  can be efficient ownership distributions<sup>13</sup>. The players then agree on an ownership distribution that minimises their joint inside options if the degree of uncertainty is small relative to the gains from trade and they agree on an ownership distribution that maximises the joint inside options if the degree of uncertainty is large relative to the gains from trade.

---

<sup>11</sup>The problem we face is that we cannot find a closed form solution for  $\pi_c^*$  (see lemma 4).

<sup>12</sup>Obviously, simulations do not allow us to draw definite conclusion about the shape of the  $\Delta W(\underline{A}, \overline{A})$  function. We have made simulations for a large number of parameter values and the general shape of the graph is always as shown in figure 2. We hope that the insights from these simulations will allow us find an analytical solution for the shape of  $\Delta W(\underline{A}, \overline{A})$  in future work.

<sup>13</sup>We again repeated these simulations for a large number of parameter configurations which all gave the same result.

## 6 Discussion

The analysis above shows that the ownership distribution of physical assets influences the size of ex post inefficiencies. In this section we explore how the findings of our model relate to observed ownership patterns. We start by analysing a simple example. We then move on to discuss different ownership arrangements that reduce the agents' joint inside options. We conclude this section by briefly analysing the relationship between efficiency and the size of firms.

### 6.1 A Simple Example

Suppose there are only two assets,  $a_1$  and  $a_2$ . Let  $A_B$  and  $A_S$  denote the assets which are owned, respectively, by the buyer and the seller given an ownership distribution  $A$ . We assume that the players are symmetric, in the sense that  $s(A_S) = b(A_B), \forall A_S = A_B$ . We refer to 'integration' as the ownership structure in which both assets are owned by one agent, and 'non-integration' as the one in which each agent owns one asset<sup>14</sup>. Furthermore, it is useful to introduce the following definitions:

**Definition 1** *The assets  $a_1$  and  $a_2$  are 'synergetic' if*

$$b(a_1, a_2) + b(\phi) \geq b(a_1) + b(a_2).$$

**Definition 2** *The assets  $a_1$  and  $a_2$  are 'non-synergetic' if*

$$b(a_1, a_2) + b(\phi) < b(a_1) + b(a_2).$$

**Definition 3** *The assets  $a_1$  and  $a_2$  are 'strictly complementary' if*

$$b(a_1, a_2) > b(a_1) = b(a_2) = b(\phi).$$

---

<sup>14</sup>Because of the symmetry of the example we do not need to distinguish between buyer and seller integration or between the two possible non-integration cases.

Thus, assets are synergetic if, in the case of temporary disagreement, the joint payoff is higher under integration than under non-integration. Correspondingly, assets are non-synergetic if, in the case of temporary disagreement, the joint payoff is higher under non-integration than under integration. In the case of a restaurant, for instance, the room and the furniture used to equip it are likely to be synergetic assets: if the chef owns the room and the waiter owns the furniture, then neither is able to generate much surplus without the cooperation of the other. If, instead, the chef owns both assets, then, in the case of non-cooperation, he would be able to generate some surplus by serving a smaller number of customers himself. In contrast, a restaurant and the plant used by the restaurant's supplier of foodstuffs are likely to be non-synergetic: in the case of disagreement more surplus can be generated if each is run independently of the other.

The analysis in the previous sections has shown that the parties optimally choose an ownership distribution which minimises their joint inside options when the degree of uncertainty is small relative to the gains from trade. When  $a_1$  and  $a_2$  are jointly owned by the buyer and the seller, then neither can use any assets during the delay period and their joint inside option is minimised to  $2b(\phi)$ . To see why joint ownership is optimal in this parameter range, consider moving from joint ownership to any other ownership distribution. Independent of whether the assets are synergetic or non-synergetic, such a change in the ownership distribution would lead to higher joint inside options. On the one hand, this implies a reduction in the cost of disagreement. On the other hand, however, it also implies a longer duration of the disagreement period. The latter effect dominates the former when the gains from trade are large relative to the degree of uncertainty. In this case moving from joint ownership to any other ownership distribution does, on average, lead to an efficiency loss.

The parties optimally agree on an ownership distribution that maximises their joint inside options when the degree of uncertainty is large relative to the gains from trade. Thus, integration is optimal when assets are synergetic, and non-integration is optimal when assets are non-synergetic. Note that this might provide an explanation for why firms own a large number of assets and do not distribute them among their work

force<sup>15</sup>: a firm can expect to spend a lot of time haggling with its work force, even after distributing its assets among the workers, when it faces a very uncertain environment. When the firm's assets are synergetic, it is optimal for the firm to reduce the cost of haggling by becoming the owner of all the assets.

Note that, in the case of non-synergetic assets, the above arguments imply a negative relationship between the average size of firms and the degree of uncertainty that they are exposed to<sup>16</sup>: it is optimal to have one large (jointly owned) firm when uncertainty is small, while two small, separately owned firms are optimal when uncertainty is high (relative to the gains from trade). It is well known that small firms tend to have more volatile share prices than larger firms. Taking the volatility of the share price as a proxy for the uncertainty in a firm's environment our argument reverses the causality that is usually used to explain this stylized fact: it is not because firms are small that they have a very volatile share price. Instead, it might be that, because a firm faces a very uncertain environment, it optimally chooses to be small so as to minimise bargaining inefficiencies. These arguments suggest a way of empirically testing our theory<sup>17</sup>.

## 6.2 Reducing Joint Inside Options

We have already seen that the joint ownership of physical assets reduces joint inside options. Giving veto rights over certain aspects of a firm's operation to one of its patrons is another closely related arrangement<sup>18</sup>. In this section we briefly discuss other observed ownership arrangements that reduce the parties' joint inside options.

Agents who are locked-in sometimes increase their interdependence further by ex-

---

<sup>15</sup>Holmström (1999) argues that this asset clustering is one of the most striking ownership patterns that we observe and that it is not easily explained by the property rights literature.

<sup>16</sup>In the case of synergetic assets there is no relationship between the size of firms and the degree of uncertainty.

<sup>17</sup>To our knowledge the existing empirical literature on the size of firms (see, for example, Kumar, Rajan, and Zingales (1999)) does not take into account the degree of uncertainty in a firm's environment as a determinant of its size.

<sup>18</sup>Another similar arrangement is the German law on codetermination which entitles workers to elect half of the members of the supervisory board in all large German firms (see Hansmann (1996), p.110-112). Codetermination further increases the interdependence between firms and their work force. Note, however, that codetermination is required by law and is only observed in Germany.

changing ‘ugly princess hostages’<sup>19</sup>. The term refers to a practice in which agents exchange ownership of assets that are very valuable to themselves, but which have little value for the other party. The Japanese car industry provides an example of such an arrangement. There it can be observed that physical assets which are specific to a particular car manufacturer are often not owned by that firm but by its supplier<sup>20</sup>.

Agents can also reduce joint inside options by abolishing assets that can be used unilaterally during a disagreement period. For instance, Holmström and Roberts (1998) report the case of an airline alliance between KLM and Northwest Airlines<sup>21</sup>. In this case the airlines deliberately increased their interdependence by eliminating their duplicate support operations. Interestingly, they did so after running into a costly dispute which led to the dismantling of the companies’ cross-ownership structure.

Firms that are engaged in long term vertical relationships often make themselves remarkably dependent on each other by using exclusive sourcing arrangements. The Ford case that was described in the introduction is one example. Holmström and Roberts (1998) provide another example. They report the case of Nucor, which has been the most successful steel manufacturer in the US in the last 20 years. Instead of following the industry standard by integrating backward, the firm made itself very dependent on only one independent supplier. Given the size of Nucor, the supplier is also very dependent on this relationship<sup>22</sup>. We can interpret such an arrangement as an example of non-integration in the case of synergetic assets. By not integrating backwards, Nucor and its supplier increase their interdependence which, in turn, reduces the expected duration of potential disputes. This argument is in line with Holmström and Roberts (1998) who hypothesise that, in the case of Nucor, “one reason why the partnership has been working so well may be the high degree of mutual dependence”<sup>23</sup>.

Finally, it can be observed that firms at times agree on the separate ownership of strictly complementary assets. For instance, Dnes (1993) observes that, in some franchise agreements, the franchisee and the franchisor agree on the separate ownership

---

<sup>19</sup>See, for example, Williamson (1985).

<sup>20</sup>Holmström and Roberts (1998), p.80-81.

<sup>21</sup>Holmström and Roberts (1998), p.84.

<sup>22</sup>The supplier is estimated to make about 50% of its scrap business with Nucor.

<sup>23</sup>Holmström and Roberts (1998), p.83.

of the land on which the premise is built and the assets used to equip it. The separate ownership of strictly complementary assets is similar to joint ownership and minimises the parties' joint inside options.

The analysis in the previous section has shown that lowering joint inside options reduces ex post inefficiencies when the gains from trade are large relative to the degree of uncertainty. To some extent our model is therefore consistent with the type of ownership arrangements described in this section. Note, however, that in our model it is optimal to either maximise or minimise joint inside options. Ownership arrangements that reduce joint inside options, but do not minimise them, can therefore not be fully explained with the given framework<sup>24</sup>. Note, also, that our model is not the only one in the property rights literature in which joint ownership can be optimal. In particular, Rajan and Zingales (1998), de Meza and Lockwood (1998), and Chiu (1998) show that joint ownership can be optimal in models that focus on ex ante investment inefficiencies.

### 6.3 Decreasing Returns from Managerial Inputs

It is often argued that there are decreasing returns from managerial inputs and that this puts an upper bound on the size of firms. For instance, Coase (1937) argues that: “[...] as a firm gets larger, there may be decreasing returns to the entrepreneur function, that is, the costs of organizing additional transactions within the firm may rise”<sup>25</sup>. The analysis in this paper suggest another reason for why large firms may be inefficient. To see this, consider a firm that increases its size by buying additional assets from a third party. It seems natural to assume that, at least in a weak sense, the cost of disagreeing with any patron is decreasing in the number of assets owned by the firm. This may, for instance, be due to the firm's ability to produce some of the inputs it usually obtains from a supplier. As a consequence, the size of a firm is positively related to the expected duration of disagreement, and it is negatively related to the cost of disagreement. If the gains from trade are large relative to the degree of uncertainty, then the former effect dominates the latter. In this case firms that increase in size become less efficient

---

<sup>24</sup>We conjecture that interior solutions can be optimal in a version of our model in which the gains from trade can be negative, and in which there is uncertainty about the agents' *outside* options.

<sup>25</sup>Coase (1937), p.340.

*because* they are in a strong bargaining position vis-a-vis their patrons and are expected to spend a long time haggling with them.

## 7 Conclusion

There are many situations in which agents, firstly, depend on each other to generate a surplus and, secondly, anticipate that bargaining over the sharing of the surplus may be costly. In this paper we have shown that, in such a situation, agents may have an incentive to take actions prior to the bargaining stage to further increase their interdependence. This can be achieved by lowering the payoffs the parties realise during the bargaining process. Increased interdependence accelerates decision making but also makes disagreement more costly. In the analysis we have shown that the former effect dominates the latter if uncertainty is small relative to the agents' expected gains from trade. In this case it is efficient for the agents to increase their interdependence by minimising their joint pre-agreement payoffs. The opposite is true if uncertainty is large relative to the gains from trade. In this case it is efficient for the agents to minimise their interdependence by maximising their joint pre-agreement payoffs.

In the paper we have applied these arguments to the theory of the firm and discussed the efficiency implications of various observed ownership patterns. We believe that the basic effects in our model can also help us understand other institutions and legal arrangements. An obvious example is the marriage contract which can be interpreted as a contract that increases the interdependence of two partners who anticipate to be locked-in in future<sup>26</sup>. In this interpretation the marriage contract is a means of accelerating domestic decision making which, however, comes at the cost of lower disagreement payoffs<sup>27</sup>. While we believe that the basic effects in our model can help us understand

---

<sup>26</sup>Here lock-in may not only be due to exogenous factors, such a mutual attraction, but also to relationship-specific investments.

<sup>27</sup>In so far as our model can be extended to multilateral bargaining situations, arguments similar to those presented above might also be used to explain the institution used by the Roman Catholic church to elect a new pope. A new pope is elected by an assembly of cardinals who are locked up in a part of Vatican Palace until they reach agreement. This institution, called a 'conclave', originates in the 13th century when the cardinals failed to elect a new pope for two years. A local magistrate then decided to improve the cardinals' incentives by locking them up in the episcopal palace, removing its roof, and allowing them nothing but bread and water until they elected the next pope (for more

a number of institutions, it is also evident that our formal model is quite restrictive. In particular, among other assumptions, we only allow for two players, one-sided asymmetric information, and assume a particular extensive form bargaining game. Relaxing any of these assumption may be interesting future work.

---

details see [www.britannica.com](http://www.britannica.com)). The observation that this institution has not been abandoned (but only somewhat adapted) suggests it might be efficient for the church as a whole (including the decision making cardinals) to accelerate the decision making process by lowering the payoff the cardinals realise during their negotiations.

## 8 Bibliography

### References

- [1] **Admati, A. and Perry, M.** (1987), “Strategic Delay in Bargaining,” *Review of Economic Studies*, LIV: 345-64.
- [2] **Aghion, P. and Tirole, J.** (1997), “Formal and Real Authority in Organizations,” *Journal of Political Economy*, 105(1): 1-29.
- [3] **Arrow, K.** (1975), “Vertical Integration and Communication,” *Bell Journal of Economics*, 6(1): 173-83.
- [4] **Binmore, K., Rubinstein, A. and Wolinsky, A.** (1986), “The Nash Bargaining Solution in Economic Modelling,” *Rand Journal of Economics*, 17: 176-88.
- [5] **Chiu, S.** (1998), “Noncooperative Bargaining, Hostages, and Optimal Asset Ownership,” *American Economic Review*, 88(4): 882-901.
- [6] **Coase, R.** (1937), “The Nature of the Firm,” *Economica*, 4: 386-405.
- [7] **Crampton, P.** (1992), “Strategic Delay in Bargaining with Two-Sided Uncertainty,” *Review of Economic Studies*, 59: 205-25.
- [8] **De Meza, D. and Lockwood, B.** (1998), “Does Asset Ownership Always Motivate Managers? Outside Options and the Property Rights Theory of the Firm,” *Quarterly Journal of Economics*, 113(2): 361-86.
- [9] **Dessein, W.** (1999), “Authority and Communication in Organizations,” mimeo.
- [10] **Dnes, W.** (1993), “A Case-Study Analysis of Franchise Contracts,” *Journal of Legal Studies*, 22, 367-93.
- [11] **Grossman, S. and Hart, O.** (1986), “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration,” *Journal of Political Economy*, 94: 691-719.

- [12] **Hansmann, H.** (1996), *The Ownership of Enterprise*, Harvard University Press.
- [13] **Hart, O.** (1995), *Firms, Contracts and Financial Structure*, Oxford University Press.
- [14] **Hart, O. and Moore, J.** (1990), "Property Rights and the Nature of the Firm," *Journal of Political Economy*, 98: 1119-58.
- [15] **Hart, O. and Moore, J.** (1998), "Cooperatives versus Outside Ownership," STICERD Discussion Paper No. TE/98/346.
- [16] **Hart, O. and Moore, J.** (1999), "Foundations of Incomplete Contracts," *Review of Economic Studies*, 66(1): 115-138.
- [17] **Holmström, B.** (1999), "The Firm as a Subeconomy," mimeo.
- [18] **Holmström, B. and Roberts, J.** (1998), "The Boundaries of the Firm Revisited," *Journal of Economic Perspectives*, 12(4): 73-94.
- [19] **Klein, B., Crawford, R. and Alchian, A.** (1978), "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process," *Journal of Law and Economics*, 21: 297-326.
- [20] **Kumar, K., Rajan, R. and Zingales L.** (1999), "What Determines Firm Size?," mimeo.
- [21] **Muthoo, A.** (1999), *Bargaining Theory with Applications*, Cambridge University Press.
- [22] **Rajan, R. and Zingales, L.** (1998), "Power in a Theory of the Firm," *Quarterly Journal of Economics*, 386-432.
- [23] **Reiche, S.** (1999), "Foundation of Incomplete Contracts: Asymmetric Information and Renegotiation," mimeo.
- [24] **Riordan, M.** (1990), "What is Vertical Integration?," in M. Aoki, B. Gustafsson, and O. Williamson (eds.), *The Firm as a Nexus of Treaties*, London, Sage: 94-111.

- [25] **Rubinstein, A.** (1982), “Perfect Equilibrium in a Bargaining Model”, *Econometrica*, 50: 97-109.
- [26] **Segal, I.** (1999), “Complexity and Renegotiation: A Foundation for Incomplete Contracts,” *Review of Economic Studies*, 66(1): 39-56.
- [27] **Williamson, O.** (1975), *Markets and Hierarchies*, The Free Press.
- [28] **Williamson, O.** (1985), *The Institutions of Capitalism*, The Free Press.

## 9 Appendix

### Proof of lemma 2:

From (4) it follows that

$$\begin{aligned}\pi_c^*(\underline{A}) &= \pi_c^*(\overline{A}) = \pi_l \text{ for } \alpha \in [0, \frac{1}{3}(\mu - j(\overline{A}))] \\ \pi_c^*(\overline{A}) &> \pi_c^*(\underline{A}) = \pi_l \text{ for } \alpha \in (\frac{1}{3}(\mu - j(\overline{A})), \frac{1}{3}(\mu - j(\underline{A})))] \\ \pi_c^*(\overline{A}) &> \pi_c^*(\underline{A}) > \pi_l \text{ for } \alpha \in (\frac{1}{3}(\mu - j(\underline{A})), \mu - j(\underline{A}))].\end{aligned}$$

Consider first  $\alpha \in [0, \frac{1}{3}(\mu - j(\overline{A}))]$ . It then follows from (6) that  $W(\underline{A}) - W(\overline{A}) = 0$ .

Consider next  $\alpha \in (\frac{1}{3}(\mu - j(\overline{A})), \frac{1}{3}(\mu - j(\underline{A}))]$ . From (6) we get

$$W(\underline{A}) - W(\overline{A}) = \frac{1}{\pi_h - \pi_l} \int_{\pi_l}^{\pi_c^*(\overline{A})} \pi - j(\overline{A}) d\pi.$$

Note that in this parameter range  $\pi_c^*(\overline{A}) > \pi_l$  and that, by assumption (1),  $\pi_l \geq j(\overline{A})$ . It then follows immediately that  $W(\underline{A}) - W(\overline{A}) > 0$ . Substituting (4) into (6) and solving gives

$$\Delta W(\underline{A}, \overline{A}) = \frac{1}{16\alpha} [3\alpha - (\mu - j(\overline{A}))][3(\mu - j(\overline{A})) - \alpha].$$

Differentiating gives

$$\frac{\partial \Delta W(\underline{A}, \overline{A})}{\partial \alpha} = \frac{1}{16\alpha^2} ((\mu - j(\overline{A}))^2 - \alpha^2).$$

Because of assumption (1) it then follows that  $\frac{\partial \Delta W(\underline{A}, \overline{A})}{\partial \alpha} > 0$ .

Finally, consider  $\alpha \in (\frac{1}{3}(\mu - j(\underline{A})), \mu - j(\underline{A}))]$ . Substituting (4) into (6) and solving gives

$$\Delta W(\underline{A}, \overline{A}) = \frac{j(\overline{A}) - j(\underline{A})}{16\alpha} (6\mu - 3j(\overline{A}) - 3j(\underline{A}) - 10\alpha)$$

Hence,  $\Delta W(\underline{A}, \overline{A}) > 0$  if  $\alpha \in (\frac{1}{3}(\mu - j(\underline{A})), \frac{3}{5}(\mu - \frac{1}{2}(j(\overline{A}) + j(\underline{A}))))$  and  $\Delta W(\underline{A}, \overline{A}) \leq 0$  if  $\alpha \in [\frac{3}{5}(\mu - \frac{1}{2}(j(\overline{A}) + j(\underline{A}))), \mu - j(\underline{A})]$ . Also, taking the derivative gives

$$\frac{\partial \Delta W(\underline{A}, \overline{A})}{\partial \alpha} = \frac{-3(j(\overline{A}) - j(\underline{A}))}{8\alpha^2} (\mu - \frac{1}{2}(j(\overline{A}) + j(\underline{A}))) < 0. \quad \blacksquare$$

**Proof of proposition 1:**

Let  $j(q) = qj(\bar{A}) + (1 - q)j(\underline{A})$ . Furthermore, let  $\tilde{q}$  be implicitly defined by  $\frac{1}{2}(\pi_h + j(\tilde{q})) = \pi_l$ . Then (5) can be rewritten as

$$(19) \quad W(q) = \frac{1}{\pi_h - \pi_l} \left( \int_{\pi_c^*(j(q))}^{\pi_h} \pi d\pi + \int_{\pi_l}^{\pi_c^*(j(q))} j(q) d\pi \right) \text{ for } q \in [0, 1].$$

Differentiating (19) with respect to  $q$  gives

$$W'(q) = \begin{cases} 0 & \text{if } q < \tilde{q} \\ \frac{1}{8\alpha}(j(\bar{A}) - j(\underline{A}))(\pi_h + 3j(q) - 4\pi_l) & \text{if } q > \tilde{q} \end{cases}$$

and

$$\begin{aligned} W'_-(\tilde{q}) &= 0 \\ W'_+(\tilde{q}) &= \frac{1}{8\alpha}(j(\bar{A}) - j(\underline{A}))(\pi_h + 3j(\tilde{q}) - 4\pi_l). \end{aligned}$$

Differentiating twice gives

$$W''(q) = \begin{cases} 0 & \text{if } q < \tilde{q} \\ \frac{3}{8\alpha}(j(\bar{A}) - j(\underline{A}))^2 & \text{if } q > \tilde{q} \end{cases}$$

and

$$\begin{aligned} W''_-(\tilde{q}) &= 0 \\ W''_+(\tilde{q}) &= \frac{3}{8\alpha}(j(\bar{A}) - j(\underline{A}))^2. \end{aligned}$$

Hence,  $W(q)$  has a kink at  $q = \tilde{q}$ . It is flat for  $q < \tilde{q}$  which implies that  $W(\tilde{q}) = W(0)$  for  $q \leq \tilde{q}$ . For  $q > \tilde{q}$  the function  $W(q)$  is convex. Clearly, local maxima of a convex function are given by the corner solutions, i.e.  $q = \tilde{q}$  or  $q = 1$ . Hence,  $\max[W(q = 1), W(q = 0)] \geq W(\hat{q})$  for any  $\hat{q} \in [0, 1]$ . ■

**Proof of lemma 4:**

Substituting (8), (9) and (14) into (15) and differentiating with respect to  $\pi_c$  gives

$$\begin{aligned} R'(\pi_c) &= \frac{1}{(\pi_l - \pi_h)} [\delta p'_c(\pi_c)(\pi_h - \pi_c) - \delta p_c(\pi_c) \\ &\quad + \delta^2 p^B(\pi_c) + (1 - \delta^2) \frac{s}{r} - \frac{\delta^4}{1 + \delta} \int_{\pi_l}^{\pi_c} \left( \frac{r\pi - j}{r\pi_c - j} \right)^{\delta+1} d\pi] \end{aligned}$$

and

$$(20) \quad R''(\pi_c) = \frac{-1}{(2 + \delta)(\pi_h - \pi_l)} [2(2 - \delta)\delta + \delta^4 \left( \frac{r\pi_l - j}{r\pi_c - j} \right)^{\delta+2}].$$

Note that  $R''(\pi_c) < 0$  for any  $\pi_c \in [\pi_l, \pi_h]$ . For  $\delta \rightarrow 1$  the first order condition reduces to

$$(21) \quad R'(\pi_c) = \frac{1}{2(\pi_h - \pi_l)} \left( \pi_h - \pi_c - \int_{\pi_l}^{\pi_c} \left( \frac{r\pi - j}{r\pi_c - j} \right)^2 d\pi \right) = 0.$$

Note that

$$R'(\pi_l) = \frac{3r(r\pi_l - j)(\pi_h - \pi_l)}{2r^3(\pi_h - \pi_l)} > 0$$

and

$$R'(\pi_h) = \frac{-[(r\pi_h - j)^3 - (r\pi_l - j)^3]}{2r^3(\pi_h - \pi_l)} < 0.$$

Thus, for  $\delta \rightarrow 1$  it must be that  $\pi_c^* \in (\pi_l, \pi_h)$ .

Since  $R''(\pi_c^*) < 0$  we can apply the implicit function theorem to (21). Totally differentiating (21) gives

$$\left( \int_{\pi_l}^{\pi_c} \frac{r(r\pi - j)(\pi_c^* - \pi)}{(r\pi_c^* - j)^3} d\pi \right) dj - \frac{1}{3} \left( 2 + \left( \frac{r\pi_l - j}{r\pi_c^* - j} \right)^3 \right) d\pi_c^* = 0$$

Hence,

$$\frac{\partial \pi_c^*}{\partial j} = \frac{r(\pi_c^* - \pi_l)^2 (r\pi_c^* + 2r\pi_l - 3j)}{4(r\pi_c^* - j)^3 + 2(r\pi_l - j)^3} > 0. \quad \blacksquare$$

### Proof of proposition 2:

To verify that these beliefs and strategies are an equilibrium we have to show that each strategy is a best response given the other strategy and that the beliefs are consistent with Bayes' rule.

We start by checking whether the buyer's strategy is a best response given the seller's strategy and beliefs. Suppose the seller offers  $p = p^S(\pi')$ . If the buyer is of type  $\pi'$ , then acceptance ensures her a payoff of  $\pi' - p^S(\pi')$  and rejection gives a payoff of  $\delta(\pi' - p^B(\pi')) + (1 - \delta)\frac{b}{r}$ . Hence, by (8) a buyer of  $\pi'$  is indifferent between accepting  $p^S(\pi')$  and rejecting it. It was shown above that any buyer of type  $\pi > \pi'$  strictly prefers to accept  $p^S(\pi')$ . Also, any buyer of type  $\pi < \pi'$  strictly prefers to reject  $p^S(\pi')$  and instead offer  $p^B(\pi')$  after a delay of  $t(\pi, p^S(\pi'))$ .

Consider next whether the seller's strategy is a best response given his beliefs and the buyer's strategy. Suppose the buyer offers  $p \geq p^B(\pi)$  and  $\lambda(\pi) = 1$  and the seller rejects  $p$  and counteroffers offers  $p'$ . If  $p' \leq p^S(\pi)$ , then the buyer accepts immediately, and the seller makes a payoff of  $\delta p' + (1 - \delta)\frac{s}{r}$  which, because of (8), is weakly less than

$p^B(\pi)$ . If, instead,  $p' > p^S(\pi)$ , then the seller expects the buyer to reject  $p'$  and offer  $p^B(\pi)$  again after a delay of  $t(\pi, p')$ . Hence, if  $\lambda(\pi) = 1$ , the seller cannot do better than to accept any  $p \geq p^B(\pi)$ . Suppose now that  $\lambda(\pi) = 1$  and the buyer offers  $p < p^B(\pi)$ . Accepting this offer gives  $p < p^B(\pi)$ , while offering  $p^S(\pi)$  gives an expected payoff of  $\delta p^S(\pi) + (1 - \delta)\frac{s}{r}$ . Because of (8) rejecting  $p < p^B(\pi)$  and offering  $p^S(\pi)$  therefore leads to a higher expected payoff than accepting  $p$ . Clearly, rejecting  $p$  and offering  $p' = p^S(\pi)$  gives a higher expected payoff than rejecting  $p$  and offering  $p' < p^S(\pi)$ . Also, rejecting  $p$  and offering  $p' > p^S(\pi)$  is expected to lead to a counteroffer by the buyer of  $p^B(\pi)$  after a delay of  $t(\pi, p')$ . Hence, if  $\lambda(\pi) = 1$  and the buyer offers  $p < p^B(\pi)$ , the seller's best response is to reject  $p$  and counteroffer  $p^S(\pi)$ . Finally, since  $\pi_c^*$  solves the maximisation problem (15), the first offer  $p_c(\pi_c^*)$  is optimal given the other strategies and beliefs.

At last, we need to check that the seller's beliefs are consistent with Bayes' rule. Given the strategies the seller's offer of  $p$  is accepted immediately by any buyer of type  $\pi \in [\pi', \pi_h]$  where  $p = p^S(\pi')$ . Any buyer of type  $\pi \in [\pi_l, \pi')$  rejects  $p$  and offers  $p^B(\pi)$  at  $\tau(\pi, p)$ . Hence, if an offer is made at  $\tau(\pi, p)$ ,  $\lambda(\pi) = 1$  is consistent with Bayes' rule.

■

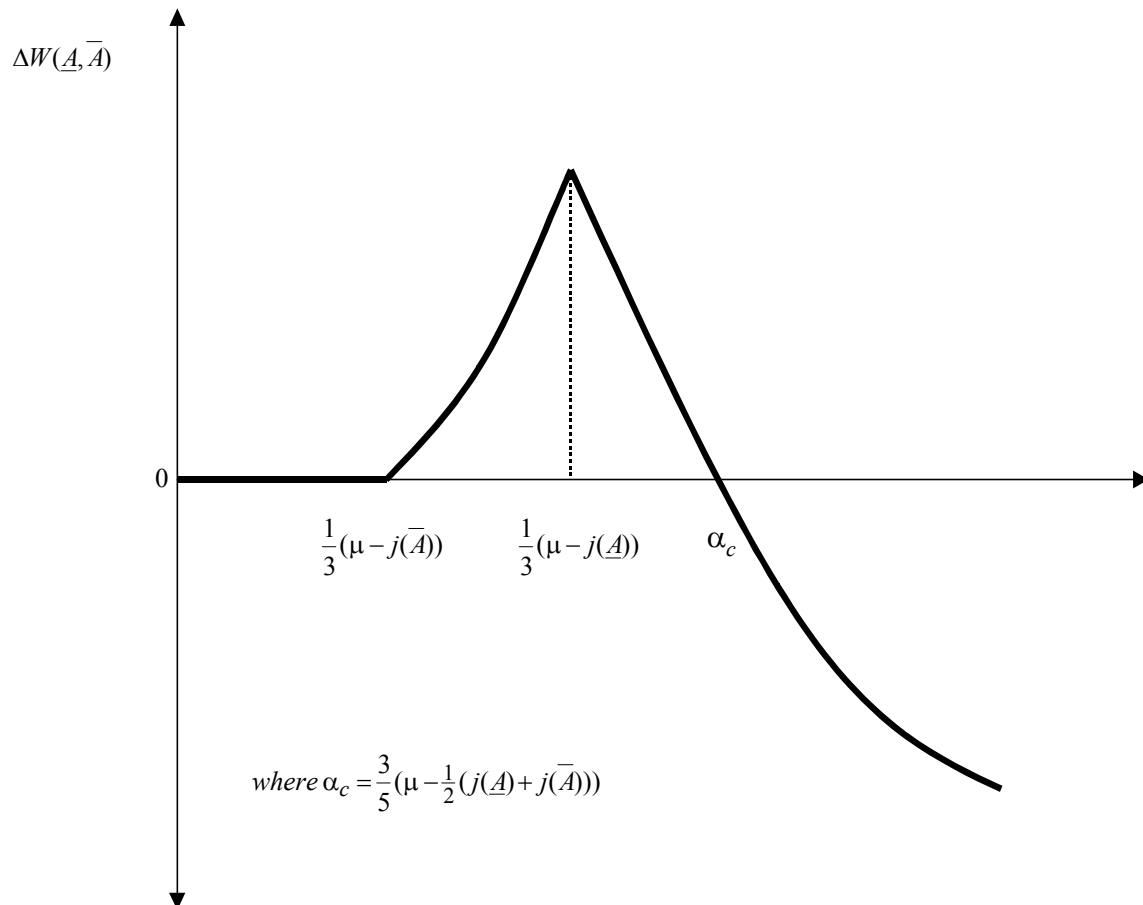


Figure 1: Efficiency gain of moving from  $\bar{A}$  to  $\underline{A}$  in the static model.

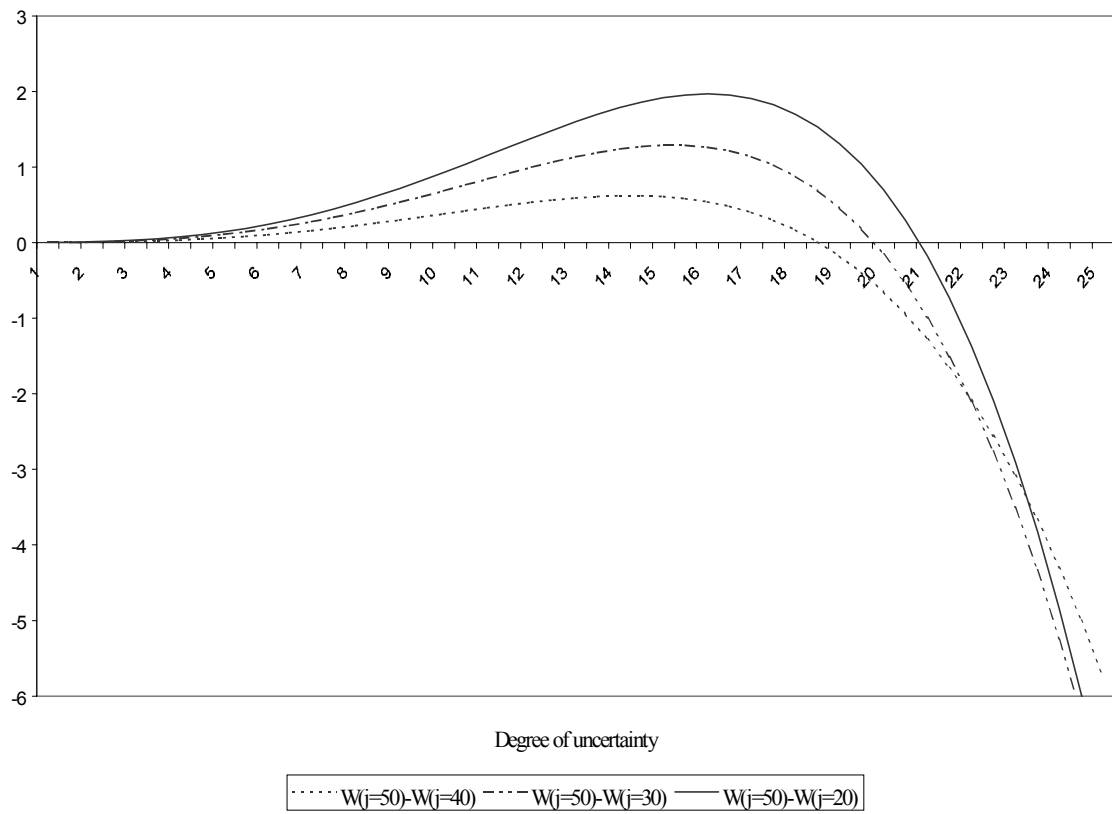


Figure 2: Simulated efficiency gain of moving from  $\bar{A}$  to  $\underline{A}$  in the dynamic model for  $\mu = 100$ ,  $r = 1$ ,  $j(\bar{A}) = 50$  and  $j(\underline{A}) = 40, 30, 20$ .