Entry Deterrence in Durable-Goods Monopoly*

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Abstract

There are industries that tend to remain monopolized, with successive generations of a good being introduced by an incumbent monopolist. This paper investigates the tendency of persistent leadership in durable goods industry. In particular it explores the implications of the durability of a good on the pricing and innovation behavior of both the incumbent monopolist and a potential entrant. It is shown that the durability of the good either acts as an entry barrier itself or creates an opportunity for the incumbent firm to deter entry by limit pricing. Moreover, we demonstrate that entry deterrence by limit pricing may cause underinvestment in innovation.

Keywords: Entry deterrence, durable goods monopoly, Coasian dynamics, innovation.

JEL classification: D420, L110

1 Introduction

Some industries tend to remain monopolized, with successive generations of a product being introduced by the incumbent monopolist. In particular, the computer industry, which seems transforming modern society, provides a few outstanding examples, such as Microsoft in the software market and Intel in the computer CPU market.

Given the importance of this industry, as is expressed by the worldwide interest in the current antitrust case between the United States and Microsoft Corporation, in which Microsoft is accused of unlawfully maintaining its monopoly power by anticompetitive behavior in the software market,[1] this paper takes up some important questions that have attracted much attention ever since Schumpeter (1942). In particular, what factors determine entry into monopolized industries through innovation? Moreover, do industries that exhibit a single, persistent technological leadership implement the socially optimal rate of technological progress?

Previous studies have addressed these issues in the context of non-durable goods industries (see, for instance, the prominent debate between Gilbert and Newbery (1982) and Reinganum (1983) on preemptive patenting). However software and personal computers as well as many other products from high-tech industries, such electronics or machinery, are typically durable goods.

It is the purpose of this paper to reexamine these questions in the context of a durable-goods monopoly where entry can take place via a new generation of the durable good. We find that the durability of the good either acts as an entry barrier itself or creates an opportunity for the incumbent firm to deter entry by limit pricing. We also show that this practice of entry deterrence leads to underinvestment in innovation whenever the incumbent chooses not to innovate. These results may have implications for empirical studies on innovation and entry dynamics as well as antitrust policies.

It is often argued that Coasian pricing dynamics in a durable-goods monopoly warrants the competitiveness of that industry, even in the absence of competitors. A durable-goods monopolist will optimally reduce the price once high-valuation consumers have bought. Knowing this, even high-valuation consumers will postpone their purchases. Accordingly, prices converge to the competitive level as price adjustments become more frequent. However, the monopolist may attempt to fix the time-inconsistency problem by limiting the effective durability of the good through (i) contracts, i.e. renting the product rather than selling it (Coase (1972), Bulow

[1] The case reference is 97-5343: U.S.A. vs. Microsoft.

(1982)), (ii) physical obsolescence, i.e. producing shorter useful product lives (Coase (1972), Bulow, (1986)), or (iii) innovation, i.e. inventing and introducing a new generation of the good (Waldman (1993), Fudenberg and Tirole (1998), Lee and Lee (1998)).

But the picture may change when the existence of a potential entrant is taken into account. As demonstrated by Bucovetsky and Chilton (1986) and Bulow (1986), the incentives to rent or produce short-lived products may be reversed when the monopolist is faced with a future entrant who can produce the same good. The reason is that higher durability limits future demand and, hence, future profits, which may in turn prevent entry when entry is costly.

Our paper examines the interplay between Coasian pricing dynamics and the incentives for innovation in a durable-goods monopoly when potential competitors threaten to innovate as well. We construct a two-period model in which a new generation of a durable good can be invented and introduced in the second period by the incumbent monopolist or a potential entrant. The old generation of the durable good lasts two periods so that consumers who buy it in the first period can use it until the second period. The new generation of the durable good is characterized by a higher quality. Consumers can consume only one unit of the durable good in each period and have uniformly distributed quality tastes. They can prove the purchase of the old generation to be eligible for an upgrade discount on the new one.

First, we demonstrate that preemptive innovation cannot prevent entry in the absence of effective patent protection. The intuition is analogous to that behind Judd's (1985) result on spatial preemption. A multiproduct incumbent firm may optimally respond to entry into one market segment by withdrawing his product in that segment if postentry price competition reduces demand in another, monopolized segment of the market. In the context of quality improvement, the argument obviously holds for durable-goods industries as well as non-durable-goods industries, as long as the demand for the incumbent's original product is positively related to the price for the high-quality version of the good.

Second, we show that a durable-goods monopolist may credibly deter entry by means of limit pricing. Lowering the price of the old generation of the durable good in the first period increases first-period demand and hence the number of second-period consumers who are willing to pay only for the incremental utility derived from the new generation of the product over the old one. Interestingly, this may prevent the entrant from investing in innovation without necessarily making the innovation investment unattractive to the incumbent. The reason is that innovation by the potential entrant results in price competition with vertically differentiated products, while innovation by the incumbent yields a multiproduct monopoly. In particular, we show that the entrant would never implement a cross-upgrade policy due to competitive pressure, whereas the multiproduct monopolist may find it optimal to offer upgrade discounts in order to price discriminate between former and new customers. Since competition dissipates post-innovation profits, the entrant's incentive for innovation is smaller than that of the incumbent monopolist.

Third, we demonstrate that entry deterrence by limit pricing may lead to consumer leapfrogging. That is, consumers who have bought the old product do not upgrade to the new version, while others who have not obtained the old version decide to purchase the new one.

Finally, we show that innovation investments in a durable-goods monopoly under entry threat are not necessarily made in an efficient manner. When innovation occurs, inefficiency can take either of two forms: the incumbent may be the single innovator even though the entrant has lower innovation costs, and vice versa. Furthermore, we show that any entry-deterrence equilibrium without innovation implies underinvestment in innovation.

The findings appear largely consistent with empirical observations in the software industry mentioned above. There is a common consent that Microsoft holds a virtual monopoly. But, as Schmalensee notes in the *Boston Globe*, "a real monopolist - one who extracted the last dollar of profit from consumers - would charge hundreds of dollars more for the software that runs modern PCs."[2] We argue that Microsoft charges low prices to make entry via a new generation unattractive by flooding the market with the old one. Such a view is supported by Microsoft's mission "a PC on every desk and in every home, running Microsoft software", and the observation that it is often Microsoft that brings the new generation of products to the market, and not a competitor.

The idea behind limit pricing in our model differs from that put forth by Milgrom and Roberts (1982). In Milgrom and Roberts' model, limit pricing is based on asymmetric information between the entrant and the incumbent about the incumbent's cost of production, while our paper assumes complete information. Furthermore, in our paper limit pricing, when exercised, removes the possibility of entry unambiguously. This is consistent with the original idea of limit pricing due to Bain (1949). By contrast, Milgrom and Roberts' result is ambiguous on the probability of entry.

Complete-information limit pricing as an entry-deterrence practice has previously been at-

[2]See *The Boston Globe*, City Edition, July 10, 1999.

tributed to suppliers of network goods. Katz and Shapiro (1992) and more recently Fudenberg and Tirole (1999) show that an incumbent may charge low prices to build a large installed base of users of a network good in order to deter entry with an incompatible product. These papers, however, assume away any Coasian pricing dynamics and incentives for upgrade pricing which are associated with many durable-goods industries. Entry deterrence by limit pricing relies therefore solely on the presence of network externalities in the demand for compatible products. By contrast, our paper attributes entry deterrence by limit pricing solely to the durability of the goods in the absence of any network externalities. We regard the arguments as complementary in nature.

The paper is organized as follows. In the next section we present a two-period model of a durable-goods monopoly threatened by entry through innovation. Section 3 analyzes the subgames after the innovation decisions. Section 4 provides main analysis of the whole game. Section 5 discusses welfare implications, and Section 6 concludes.

2 The Model

Consider a two-period model of a durable-good market. In period 1, the market is monopolized by an incumbent, I. The incumbent produces a durable good, associated with quality level s_L, that lasts two periods after which it vanishes. Between period 1 and period 2, the incumbent can invest in innovation, which enables him to produce a new generation of the good, characterized by the higher quality level $s_H = (s_L + s_\Delta)$, $s_\Delta > 0$. Hence, conditional on innovation, the incumbent may sell both generations of the good in period 2, the low-quality one and the high-quality one. There is also a potential entrant, E. By investing in innovation, the entrant is able to produce and sell the new generation of the good with quality s_H in period 2. Variable costs of production are independent of quality and set equal to zero. Following Fudenberg and Tirole (1998), we assume that the quality improvement is not too large:[3]

$$s_L > s_\Delta \tag{A1}$$

It is further assumed that the firms cannot change the quality when the good is already produced.

On the demand side, there is a continuum of consumers with different utility from consumption of the durable good. Each consumer is associated with a type θ known only to himself.

[3] The assumption is necessary for the uniqueness of the equilibrium.

Consumer types are uniformly distributed over the range $[0, 1]$. Consumers may consume at most one unit of the durable good in each period. The consumer of type θ gets utility $s_i\theta$ from consumption of the good of quality s_i per period, $i = L, H$. There is no externality among the consumers such as a network effect. Consumers and firms have a common discount factor which is normalized to 1. There is no second-hand market.

The firms and consumers face the following multi-stage game. At the beginning of period 1, the incumbent sets a price for the original durable good. Consumers choose whether to purchase the good in period 1 or not. Hence, after period 1, the market divides into the following two segments: (i) the "upgrade market", which consists of the consumers who have purchased the good in period 1 and may want to upgrade in period 2 if that is an option, and (ii) the "new-purchase market", which consists of the consumers who have not purchased in period 1. Between the end of period 1 and the beginning of period 2, the incumbent and the potential entrant simultaneously choose whether to invest in innovation which encompasses the invention and introduction of a new generation of the product to the market. The innovation costs are $K_I \geq 0$ and $K_E \geq 0$ for the incumbent and the entrant, respectively. We allow the firms to observe the outcome of innovation game instantaneously. At the beginning of period 2, each firm decides whether to withdraw any product that it is able to produce from any market at zero cost[4], and sets a price for each product it wishes to offer in any market. In particular, each potential supplier of the new generation of the good can choose to price discriminate between consumers with respect to purchase history. That is, we allow the incumbent to give an upgrade discount to the consumers in the upgrade market, and the entrant to give a cross-upgrade discount to former customers of the incumbent. However, the pricing decision is subject to the incentive compatibility constraint that the upgrade price cannot exceed the new purchase price, since consumers in the upgrade market can pretend not to have purchased previously. If the incumbent wishes to offer the original durable good in period 2, he may set a new price for it. Finally, consumers choose in period 2 whether to buy any product that is offered.

We use the subgame-perfect equilibrium as the solution for the game.

[4]We follow Judd (1985) in allowing for an intermediate exit stage. Exit is assumed to be costless to apply Judd's argument on the non-credibility of spatial preemption and thereby obtain a unique solution for the second-period pricing subgame. Without this assumption, a certain parameter range would admit multiple equilibria, where one of them could be part of an entry-deterrence equilibrium similar to that in Gilbert and Newbery's (1983) model of preemptive patenting. But even in that case, the equilibrium that is unique under costless exit would remain an equilibrium.

3 Sales in Period 2

As is standard in the analysis of subgame-perfect equilibrium, we start with the examination of the second-period play. It comprises two sales decisions by each firm, i.e the decision in which market to offer any product that can be produced plus the decision of how to price the respective product, and the purchase decisions of the consumers. The second-period decisions depend on the sales history in period 1. For this, it is easy to verify the following monotonicity property. If the consumer of type θ_1 prefers to purchase in period 1, then all consumers with type $\theta \geq \theta_1$ prefer to purchase in period 1 (see Fudenberg and Tirole (1998 [Lemma 4])). Hence, we can represent the sales history by the type of the cutoff consumer θ_1. Furthermore, the second-period subgame is associated with four possible innovation histories, which are denoted as follows: N denotes the history in which no firm has innovated; I and E denote the histories in which only the incumbent or only the entrant has innovated, respectively; and B denotes the history in which both firms have innovated. We define four subgames $\Gamma^N, \Gamma^I, \Gamma^E, \Gamma^B$ for each innovation history, respectively. In this section, we will solve each of them separately. [5]

3.1 Γ^N: No Innovation

When no firm has innovated, the incumbent may choose to sell to consumers who have not purchased in the past, i.e. consumers of types $\theta < \theta_1$. Let p_L denote the second-period price for the original, low-quality good. The incentive constraint for the marginal consumer θ_2 is given by $\theta_2 s_L - p_L = 0$. Taking this constraint into account, the incumbent's problem is

$$\max_{p_L} p_L(\theta_1 - \frac{p_L}{s_L}) \tag{1}$$

subject to $p_L/s_L < \theta_1$, which is solved by

$$p_L = \frac{1}{2} s_L \theta_1. \tag{2}$$

3.2 Γ^I: Innovation by the Incumbent

This subgame has been analyzed by Lee and Lee (1998) for the case of two types of consumers and by Fudenberg and Tirole (1998) for a general distribution of consumer types. Our analysis

[5] We collect all computational results and present in a few tables at the end of the paper

for a uniform distribution of consumer types largely confirms their results. In addition, we obtain an explicit characterization of the equilibrium which is crucial for the analysis of the entire game.

Let p_U and p_H denote the price of the new, high-quality product offered to consumers in the upgrade market and the new-purchase market, respectively. For a given first-period cutoff consumer, θ_1, the incumbent can pursue the following sales strategies. First, the incumbent may choose to price discriminate between consumers in the upgrade market and consumers in the new-purchase market. Thereby the incumbent must take the incentive compatibility constraint $p_U \leq p_H$ into account, since upgrade consumers can pretend not to have purchased in period 1. Second, he can offer the new product to consumers in both markets at a uniform price $p_U = p_H$. Third, he can choose to withdraw the new product from the new-purchase market and sell it only to consumers in the upgrade market. In addition, he may choose to sell the old product at price p_L. However, since production of either quality is costless, it can easily be shown that the incumbent finds it optimal to sell only the new product, as long as no incentive compatibility constraint is binding. The next proposition indicates how the optimal strategy of the incumbent encompasses the alternative sales strategies depending on the parameter θ_1. All proofs are relegated to the appendix.

Proposition 1 *Consider subgame Γ^I. Define*

$$z_1 \equiv \frac{s_\Delta(s_L + s_\Delta - \sqrt{s_\Delta}\sqrt{(s_L + 2s_\Delta)})}{s_\Delta s_L + s_L^2 - s_\Delta^2}$$
$$z_2 \equiv \frac{s_\Delta}{s_L + s_\Delta}$$

where $0 < z_1 < z_2 < 1/2$.

1. *If $z_2 < \theta_1 \leq 1$, the incumbent sells the new product in the upgrade market at the price*

$$p_U = \begin{cases} \theta_1 s_\Delta & \text{if } \frac{1}{2} \leq \theta_1 \leq 1 \\ \frac{1}{2} s_\Delta & \text{if } z_2 < \theta_1 \leq \frac{1}{2} \end{cases} \tag{3}$$

and in the new-purchase market at the price

$$p_H = \frac{1}{2}(s_L + s_\Delta)\theta_1, \tag{4}$$

where $p_U < p_H$ *for* $z_2 < \theta_1 < 1$.

2. *If* $z_1 < \theta_1 \le z_2$*, the incumbent sells the new product in both markets, the upgrade market and the new-purchase market, at the uniform price*

$$p_U = p_H = \frac{1}{2} s_\Delta \frac{s_L + s_\Delta}{s_L + 2s_\Delta}(1 + \theta_1). \tag{5}$$

3. *If* $0 \le \theta_1 \le z_1$*, the incumbent sells the new product only in the upgrade market for the price*[6]

$$p_U = \frac{1}{2} s_\Delta \tag{6}$$

and the old product in the new-purchase market at the price given by (2).

Proposition 1 reveals that the incumbent will price discriminate between customers with different purchase history by offering an upgrade discount to those who have purchased in period 1, provided that the upgrade market is not too large (statement 1). Two effects matter for this result: First, consumers in the new-purchase market are willing to pay $(s_L + s_\Delta)\theta$, while those in the upgrade market are willing to pay only $s_\Delta\theta$ for the incremental utility. This implies a higher new-purchase price (the reservation-utility effect). Second, as the upgrade market gets large, the maximal valuation across consumers in the new-purchase market decreases. This drives the new-purchase price down relative to the upgrade price (the ratchet effect). For a large upgrade market, the ratchet effect dominates the reservation utility effect such that the incentive compatibility constraint $p_U = p_H$ is binding. The incumbent charges then a uniform price for the new product (statements 2 and 3). When the upgrade market gets very large, the optimal uniform price will exceed the maximal willingness to pay of any consumer in the new-purchase market. The incumbent can therefore gain from foregoing sales of the new product in the new-purchase market entirely (statement 3).

Finally, we shall analyze whether there are consumers who possess the old product and do not upgrade to the new version, while there are others who have not bought the old version and decide to purchase the new one. Such consumer leapfrogging implies that a consumer with a higher valuation may use a product of lower quality than a consumer with a lower valuation.

[6] For $\theta_1 = z_1$, the incumbent is indifferent between the policies described in statements 2 and 3. We assume that for $\theta_1 = z_1$ he chooses to sell the new product only in the upgrade market.

The analysis might therefore be of an independent interest in the context of technology adoption as discussed in the growth literature.

Corollary 1 *Leapfrogging occurs in* Γ^I *if* $z_1 < \theta_1 < 1/2$.

Corollary 1 indicates that leapfrogging may take place in Γ^I for a range of the first-period sales history. Note that this does not necessarily imply that leapfrogging occurs in the overall game.

3.3 Γ^E: Innovation by the Entrant

When the entrant is the only innovator, he can monopolize the upgrade market but faces price competition between vertically differentiated goods in the new-purchase market. While the incumbent's strategy set is simply a choice of $p_L \geq 0$, the entrant's strategy set is composed of the following sales policies. First, the entrant can price discriminate between the consumers in the new-purchase market and those in the upgrade market by giving a cross-upgrade discount $p_U < p_H$. Second, he can charge a uniform price in both markets $p_U = p_H$. Third, he can forego sales in the new-purchase market completely. As a preliminary step, we show in the next lemma that a cross-upgrade discount is never optimal for the entrant. Proposition 2 then summarizes the equilibrium behavior in Γ^E.

Lemma 1 *Price discrimination between consumers with respect to purchase history is never optimal for the entrant.*

Lemma 1 is due to the competition between the entrant and the incumbent in the new-purchase market which calls for a low new-purchase price p_H. It establishes that this competition effect together with the ratchet effect always dominates the reservation utility effect so that the incentive compatibility constraint $p_U \leq p_H$ is always binding.

Proposition 2 *Consider subgame* Γ^E. *Define*

$$x_1 \equiv \frac{(7\sqrt{2}-8)s_L + (8\sqrt{2}-8)s_\Delta}{8s_L + 8s_\Delta}$$
$$x_2 \equiv \frac{2s_L + 2s_\Delta}{5s_L + 6s_\Delta}$$
$$x_3 \equiv \frac{2s_L + 2s_\Delta}{3s_L + 4s_\Delta}$$

where $0 < x_1 < x_2 < 1/2 < x_3 < 1$.

1. *If* $x_3 < \theta_1 \leq 1$, *the entrant sells the new product in both markets at the uniform price*

$$p_H = p_U = 2s_\Delta \frac{s_L + s_\Delta}{3s_L + 4s_\Delta} \tag{7}$$

such that the first-period cutoff type θ_1 *prefers to buy the new product. The incumbent sells the old product at the price*

$$p_L = s_\Delta \frac{s_L}{3s_L + 4s_\Delta}. \tag{8}$$

2. *If* $x_2 \leq \theta_1 \leq x_3$, *the entrant sells the new product in both markets at the uniform price*

$$p_H = p_U = s_\Delta \theta_1 \tag{9}$$

such that the first-period cutoff type θ_1 *is indifferent between buying the new product or not. The incumbent sells the old product at the price*[7]

$$p_L = \frac{1}{2} s_\Delta \frac{s_L}{s_L + s_\Delta} \theta_1. \tag{10}$$

3. *If* $x_1 < \theta_1 < x_2$, *the entrant sells the new product in both markets at the uniform price*

$$p_H = p_U = 2s_\Delta \frac{s_L + s_\Delta}{7s_L + 8s_\Delta}(1 + \theta_1) \tag{11}$$

such that the first-period cutoff type θ_1 *prefers not to buy the new product. The incumbent sells the old product at the price*

$$p_L = s_\Delta \frac{s_L}{7s_L + 8s_\Delta}(1 + \theta_1). \tag{12}$$

4. *If* $0 \leq \theta_1 \leq x_1$, *the entrant sells the new product only to consumers in the upgrade market*

[7]For $\theta_1 = x_3$, the entrant is indifferent between the policies described in statements 1 and 2 of this Proposition. We assume that for $\theta_1 = x_3$ firms coordinate on the strategies specified in statement 2, since it yields a higher profit for the incumbent.

at the price

$$p_U = \frac{1}{2} s_\Delta. \tag{13}$$

The incumbent sells the old product at the price given by (2).[8]

5. *The entrant's profit in Γ_E is continuous and weakly increasing in θ_1.*

The proposition exhibits an interesting discontinuity at $\theta_1 = x_1$. As θ_1 falls from above x_1 below that value, the entrant stops selling the new product to new-purchase consumers, and the price of the new durable good jumps upwards to $p_U = (1/2)\, s_\Delta$ (statements 3 and 4). The reason is that for a sufficiently large upgrade market, i.e. $\theta_1 \leq x_1$, it is profitable for the entrant to avoid competition in the new-purchase market. Note that the optimal upgrade price does not depend on the low end of the upgrade market, when the upgrade market is already of substantial size (statement 4). For a smaller upgrade market, i.e. $\theta_1 > x_1$, the entrant is subject to substantial competitive pressure from the incumbent who continues selling the old durable good. The competitive pressure prevents the entrant from price-discriminating between upgrade consumers and new-purchase consumers (statements 1-3 and Lemma 1).

Finally, similar to the previous subsection we analyze the consumers' equilibrium purchase decision and check whether leapfrogging is possible in subgame Γ^E.

Corollary 2 *If $x_1 < \theta_1 < x_2$, leapfrogging occurs in Γ^E.*

The corollary shows that consumer leapfrogging may occur in Γ^E for a range of the first-period sales history. The range is similar to the case of Γ^I, but it is narrower here.

3.4 Γ^B: Innovation by Both Firms

Consider the subgame where both firms have innovated. In this case, the incumbent can sell both goods, the old one and the new one, while the entrant can sell only the new version. We will demonstrate, however, that the incumbent prefers to offer only the old product.

Proposition 3 *In subgame Γ^B, it is optimal for the incumbent to withdraw the new product entirely and sell only the old product for all $\theta_1 > 0$.*

[8] For $\theta_1 = x_1$, the entrant is indifferent between the policies described in statements 3 and 4. We assume that for $\theta_1 = x_1$ firms coordinate on the strategies specified in statement 4, since it yields a higher profit for the incumbent.

Proposition 3 describes a striking result. When both firms introduce the new version of the durable good, the optimal response of the incumbent is to withdraw the new product from both markets, the upgrade market and the new-purchase market. The result can be explained as follows. If the incumbent remains in both markets, Bertrand price competition drives the new-purchase price and the upgrade price down to zero. As a consequence, the price for the old product is zero as well. Hence, each firm makes zero profits. It is obvious that the entrant cannot gain by exiting either market, since this would yield zero profits as well. However, the incumbent may want to avoid Bertrand price competition in the new-purchase market. Since the old product is directly competing against the new one, the incumbent has an incentive to withdraw the new product from the new-purchase market in order to generate positive profits with the old product.[9]

Moreover, the incumbent can do even better by withdrawing the new product from the upgrade-market as well and offering only the old product, as with history E. To understand this point, remember that for history E the entrant charges a uniform price for the new product in both markets as the incentive compatibility constraint $p_U \leq p_H$ is binding (Lemma 1). This price is higher than the entrant's optimal (unconstrained) new-purchase price. It is clear that the incumbent benefits from a higher price charged by its rival. Hence, when both markets are linked by the incentive compatibility constraint, as with history E, the incumbent is actually better off relative to when markets are separated, as with history B and free upgrading. Corollary 3 follows immediately from Proposition 3.

Corollary 3 *In Γ^B the incumbent and the entrant set prices as in Γ^E. Leapfrogging occurs under the same circumstances as in Γ^E.*

It is interesting to note that this result implies that innovation has no preemptive power in deterring an entry, in contrast to the previous debate between Gilbert and Newbery (1982) and Reinganum (1983). The difference follows from the multiple product feature of the present model and the absence of any effective patent protection.

[9] A similar result has been obtained by Judd (1985) for a multiproduct incumbent with horizontally differentiated goods who is threatened by an entrant.

4 Sales in Period 1

The analysis of the second-period subgame in the previous section allows us to solve for the subgame-perfect equilibrium of the entire game. There are two stages at which the incumbent makes decisions prior to the second-period sales: the pricing decision in the first period and the innovation investment decision immediately before the second period.

The payoff matrix at the time of the innovation decision, given the costs K_I and K_E for the incumbent and the entrant, respectively, is given by Table 1, in which the incumbent is the row player and the entrant is the column player. $\pi_j^h(\theta_1)$ denotes the second-period optimal profit accruing to firm j as a function of the first-period sales level θ_1, where the subscript $j = I, E$ represents the incumbent and the entrant, and the superscript $h = N, I, E, B$, represents the innovation history. Note that $\pi_j^B(\theta_1) = \pi_j^E(\theta_1)$ for all θ_1 by Corollary 3.

	No Innovation	Innovation
No Innovation	$\pi_I^N(\theta_1), 0$	$\pi_I^E(\theta_1), \pi_E^E(\theta_1) - K_E$
Innovation	$\pi_I^I(\theta_1) - K_I, 0$	$\pi_I^E(\theta_1) - K_I, \pi_E^E(\theta_1) - K_E$

Table 1: Payoff Matrix for Second Period

Rolling back we can write the total profit of the incumbent as a function of θ_1:

$$\Pi(\theta_1) = p_1(1 - \theta_1) + \pi_I^h(\theta_1) - K_I \mathcal{I}\{h = I, B\} \tag{14}$$

where p_1 is the first-period price that generates the marginal consumer of type θ_1, and $\mathcal{I}\{\cdot\}$ is an indicator function. The incumbent's optimal strategy at the beginning of the whole game is to choose a first-period cutoff type θ_1 that maximizes $\Pi(\theta_1)$.

Define $\Lambda_{K_E} = \{\theta_1 \left| \pi_E^E(\theta_1) \leq K_E \right.\}$, namely the set of sales histories which yield a non-positive profit to the entrant when entry via innovation takes place.[10] The incumbent can prevent entry by setting the first-period price in such a way that all consumers of type $\theta \geq \theta_1 \in \Lambda_{K_E}$ purchase in the first period. We call Λ_{K_E} the no-entry set.

Using Bain's terminology, we distinguish between three forms of first-period behavior: *blockaded entry*, where the incumbent chooses a first-period price as if there were no entry

[10] We assume that the entrant stays out, i.e. chooses not to innovate, if the profit from entry is zero.

threat, but no entry occurs; *deterred entry*, where entry cannot be blockaded, but is prevented through limit pricing; and *accommodated entry*, where the incumbent prefers a first-period price that does not prevent entry, and entry occurs. The subgame-perfect equilibrium of the entire game has different properties, depending on whether entry takes place or not. We analyze first the no-entry equilibrium in which entry is either blockaded or deterred, then the entry equilibrium in which entry is accommodated, and finally, we examine how the incumbent chooses in the first stage between the two equilibrium paths if both are available.

4.1 No-Entry Equilibrium

In the no-entry equilibrium the incumbent maximizes the total profit subject to the constraint that the first-period sales prevent entry. Hence, the second-period subgame is either Γ^N or Γ^I. We can write the incumbent's optimization problem as

$$\max_{\{\theta_1, h=N,I\}} \Pi(\theta_1) = p_1(1-\theta_1) + \pi_I^h(\theta_1) - K_I \mathcal{I}\{h = I\}, \tag{15}$$

subject to $\theta_1 \in \Lambda_{K_E}$.

The incumbent can prevent entry only when he has the means to impose a non-positive profit on the entrant in the second period. It is therefore crucial to know when the no-entry set is non-empty, i.e. $\Lambda_{K_E} \neq \emptyset$. We will first establish useful properties of the no-entry set.

Lemma 2 *1. If $K_E < \frac{1}{4}s_\Delta$, then $\Lambda_{K_E} = \emptyset$.*

2. If $K_E \geq \frac{1}{4}s_\Delta$,[11] then $\Lambda_{K_E} = [0, \lambda_{K_E}] \neq \emptyset$, where $\lambda_{K_E} \geq x_1 > 0$.

The first part of the lemma implies that the no-entry set is empty if the entrant's innovation cost is low. The second part reveals that, if the entrant's entry cost is high enough, the no-entry set is non-empty, and the upper bound of the set is greater than or equal to x_1. This property has an important implication: in solving for the no-entry equilibrium we do not have to consider the range of θ_1 smaller than x_1. The following proposition characterizes the no-entry equilibrium.

Proposition 4 *1. In the no-entry equilibrium without innovation,*

[11] The weak inequality follows from our tie-breaking rule that the entrant stays out when the profit is 0.

(a) *if* $3/5 < \lambda_{K_E} \leq 1$*, the incumbent chooses a first-period price* p_1 *as if there were no entry threat (blockaded entry),*

(b) *if* $x_1 \leq \lambda_{K_E} \leq 3/5$*, the incumbent sets* p_1 *such that* $\theta_1 = \lambda_{K_E}$ *(deterred entry).*

2. *In the no-entry equilibrium with innovation,*

(a) *if* $(3s_L + s_\Delta) / (5s_L + s_\Delta) < \lambda_{K_E} \leq 1$*, the incumbent chooses a first-period price* p_1 *as if there were no entry threat (blockaded entry),*

(b) *if* $x_1 \leq \lambda_{K_E} \leq (3s_L + s_\Delta) / (5s_L + s_\Delta)$*, the incumbent sets* p_1 *such that* $\theta_1 = \lambda_{K_E}$ *(deterred entry).*

The proposition shows that the no-entry equilibrium comprises a blockaded-entry equilibrium and an entry-deterrence equilibrium. In the latter, the incumbent deters entry by producing at the boundary of the no-entry set. This reveals that the concept of the limit pricing due to Bain (1949) is valid in the durable-goods industry. As is well known, an argument which essentially amounts to the requirement of subgame perfection makes the limit pricing strategy ineffective in non-durable goods industries that are not characterized by network externalities.[12] By contrast, our model of a durable-goods monopoly without network externalities indicates that the incumbent may choose to deter entry by selling more in the first period than in the absence of any entry threat. The reason is that the second-period demand function is determined by the first-period sales volume in the case of durable goods, whereas it is independent of the first-period sales in the case of non-durable goods.

4.2 Entry Equilibrium

The incumbent may choose a strategy that allows entry. Proposition 3 implies that in such a case the incumbent never innovates. The second-period subgame is then always Γ^E in which the incumbent sells only the old product (Proposition 2). Given entry, the incumbent's optimization problem in the first period is

$$\max_{\{\theta_1\}} \Pi(\theta_1) = p_1(1 - \theta_1) + \pi_I^E(\theta_1). \tag{16}$$

[12] For the analysis of limit pricing by suppliers of network goods, see Fudenberg and Tirole (1999).

The next proposition states the optimal first-period price, given that the entry equilibrium is played.

Proposition 5 *In the entry equilibrium, the incumbent chooses a first-period price p_1 such that $\theta_1 = x_1$.*

According to Proposition 5 entry accommodation involves a large sales volume in the first period. In fact it is the maximum quantity the incumbent would be willing to sell for entry deterrence (Lemma 2). This intriguing result can be explained by the externality of the incumbent's first-period sales, i.e. their negative impact upon the entrant's second-period profits.[13]

Observe that the first-period sales in general affect the incumbent's second-period sales as well as the entrant's second-period sales. Proposition 2 implies that a first-period sales level of $\theta_1 = x_1$ limits the extent of entry by making the new-purchase market unattractive for the entrant. By contrast, any θ_1 above x_1 admits competition in the new-purchase market which lowers the second-period price for the old product and hence the second-period profit for the incumbent. Once the entrant leaves the new-purchase market, further first period sales affect only the incumbent's profit since the entrant's profit comes entirely from the upgrade market. Hence, any θ_1 below x_1 affects the incumbent's profit through a lower first-period price as well as a lower second period price. Therefore the discontinuity at $\theta_1 = x_1$ due to the change in the market structure plays a crucial role for Proposition 5.

4.3 Choice between Equilibrium Paths

Proposition 5 has an immediate consequence for the choice between the entry equilibrium and the no-entry equilibrium. The following proposition shows that the incumbent always prevents entry, whenever he has the means to do so.

Proposition 6 *The incumbent plays the entry equilibrium if and only if $\Lambda_{K_E} = \emptyset$.*

As discussed above, the impact of the first-period sales on the second-period market structure induces the incumbent to set a low first-period price when he plans to accommodate entry. However, for $\Lambda_{K_E} \neq \emptyset$ the implied first-period sales volume is larger than the optimal sales

[13]Carlton and Gertner (1989) exploit the same intuition to demonstrate that a durable-goods oligopolist has an incentive to sell rather than rent for strategic reasons.

volume under entry deterrence. Hence entry is prevented almost by default even if the incumbent plans to concede entry. The result indicates that the incumbent in a durable-good industry enjoys a substantial advantage in securing his monopoly power.

4.4 Consumer Leapfrogging

In this subsection we analyze the consumers' purchase decision and ask whether and when leapfrogging occurs.

Proposition 7 *Leapfrogging occurs if the entry-deterrence equilibrium with innovation is played for $x_1 < \lambda_{K_E} < 1/2$.*

In Fudenberg and Tirole's (1998) model of a durable-goods monopoly without entry threat, leapfrogging can only occur when production is costly. By contrast, our model with costless production predicts the occurrence of leapfrogging under a certain condition. The intuition is that the practice of limit pricing induces some consumers to purchase in period 1 whose valuations are not high enough to warrant an upgrade in the second period. On the other hand, the even larger first-period sales volume chosen in the entry equilibrium does not imply consumer leapfrogging, because the valuation of the consumers who have not purchased in period 1 is so low that the entrant finds it optimal to serve only the consumers in the upgrade market. These two observations suggest that the occurrence of leapfrogging can also be attributed to the competitive pressure under entry threat.

5 Welfare Analysis of Innovation Investment

The threat of entry has the following straightforward effects on social welfare. First, the practice of limit pricing allows more consumers to consume the durable good compared to the situation without entry threat. Second, an even higher sales volume is also obtained when entry is accommodated. However, even the entry equilibrium entails a loss of efficiency against the first best in which both product are provided at prices equal to the marginal cost of 0.

In this section we focus on the non-trivial question, albeit of partial nature, of whether the durable-goods monopolist and the potential entrant have proper incentives to invest in innovation.[14] The next proposition shows that when innovation occurs in equilibrium, inefficiency in innovation can be caused by either firm: the incumbent may innovate even though innovation by the entrant is more efficient, i.e. $K_E < K_I$, while the entrant may innovate even if though innovation by the incumbent is more efficient, i.e. $K_I < K_E$.

Proposition 8 *Suppose that innovation occurs in the equilibrium.*

1. *When $K_E \geq \frac{1}{4}s_\Delta$ so that the no-entry equilibrium is played, the incumbent may innovate even if $K_E < K_I$.*
2. *When $K_E < \frac{1}{4}s_\Delta$ so that the entry equilibrium is played, the entrant may innovate even if $K_I < K_E$.*

The proposition follows from the fact that the possibility of entry deterrence depends only on the entrant's innovation cost and his profit from entry and not on the incumbent's innovation cost. When the incumbent successfully deters entry, he may invest in innovation although the entrant has a cost advantage. On the other hand, the inefficiency in innovation can occur in the opposite way as well. To see this, note that if the no-entry set is empty, the incumbent is forced to accommodate entry via the new generation of the durable good. By Proposition 3, however, the incumbent never innovates in the entry equilibrium, irrespectively of his innovation costs.

The previous proposition has revealed two forms of inefficiency when innovation occurs. The next proposition discovers an inefficiency when no firm innovates.

Proposition 9 *Underinvestment in innovation occurs in any entry-deterrence equilibrium without innovation: the entrant's innovation cost is lower than the social gain from innovation so that the entrant's innovation is welfare-enhancing.*

The intuition behind the proposition can be explained using two elements. First when the entrant has a low innovation cost, the social gain from the consumption of the new durable good easily dominates the innovation cost. Second even when the entrant has a moderate innovation cost, entry deterrence by the incumbent induces an inefficiency. This inefficiency increases with the rise in the entrant's innovation cost since the first period sales volume necessary for entry deterrence is decreasing in K_E. Note that for very high innovation cost K_E entry need

[14]The question lies at the center of the recent trial on Microsoft although it is admittedly of a more limited scope. Our approach highlights the most controversial issue in the trial.

not be deterred, but is blockaded. The proof of Proposition 9 establishes that the sum the social benefit of the new durable good and the social cost of entry deterrence outweighs the entrant's innovation cost, as long as entry is deterred and not blockaded.

The proposition indicates that the practice of entry deterrence may imply less innovation than optimal. This finding provides a scope for possible government intervention in encouraging innovation by a potential entrant. Furthermore, the proposition has an interesting implication for the recent trial of Microsoft, who consistently argued that it faces the correct innovation incentive because of the time-inconsistency problem in durable-goods industries: once the old generation of the durable is sold, the firm has to innovate to generate further revenue. A careful examination of the argument reveals that this is an unwarranted extrapolation of the Coasian argument to the problem of entry via innovation. Indeed the analysis in this section suggests that their claim is not true in general.

6 Conclusion

Our result that only one of the firms innovates appears compatible with a few outstanding cases in the computer industry. In the software market for operating systems Microsoft holds a virtual monopoly while in the computer CPU market Intel holds a comparable position. Our result indicates that the incumbent in a durable-good industry enjoys a certain degree of entry-deterrence power.

Although the power to deter entry is not equivalent to the lack of incentive to innovate, the power allows the incumbent to cause underinvestment in innovation or make an inefficient innovation decision. It is rather surprising that the inefficiency in innovation may go in the opposite direction as well, namely that the entrant may innovate even though the incumbent has a cost advantage in innovation.

Finally, we would like to emphasize that the issue of dynamic competition considered here could be crucial for issues of economic growth since durable goods are often used as factors of production. Hence, results which draw on a careful analysis of entry deterrence in durable-goods monopoly may provide important implications for policies on growth.

Appendix

Proof. (Proposition 1)

The incumbent's optimal second-period policy is given by

$$\max_{\{p_U, p_H\}} \left[(1 - \frac{p_U}{s_\Delta}) p_U + (\theta_1 - \frac{p_H}{s_L + s_\Delta}) p_H \right] \tag{17}$$

subject to

$$\frac{p_U}{s_\Delta} \geq \theta_1 \tag{18}$$
$$\frac{p_H}{s_L + s_\Delta} \leq \theta_1 \tag{19}$$
$$p_U \leq p_H \tag{20}$$

Constraint (18) [(19)] implies that the marginal consumer who is willing to pay p_U [p_H] for the new product belongs to the upgrade [new-purchase] market. Constraint (20) stems from the fact that upgrade consumers can pretend not to have purchased in period 1.

Suppose first that (20) is not binding for some θ_1. The solution is then

$$p_U = \begin{cases} \theta_1 s_\Delta & \text{if } \theta_1 > \frac{1}{2} \\ \frac{1}{2} s_\Delta & \text{if } \theta_1 \leq \frac{1}{2} \end{cases} \tag{21}$$
$$p_H = \frac{1}{2}(s_L + s_\Delta)\theta_1. \tag{22}$$

Let us now add the incentive compatibility constraint (20). We obtain that, for $\theta_1 > 1/2$, $p_U \leq p_H$ if and only if $s_\Delta \leq s_L$, which is satisfied by assumption (A1), and for $\theta_1 \leq 1/2$, $p_U \leq p_H$ if and only if $\theta_1 \geq s_\Delta / [s_L + s_\Delta] \equiv z_2$. We conclude that the incumbent will price discriminate if $\theta_1 > z_2$ (statement 1).

Consider next the range of θ_1 in which (20) is binding, i.e. $\theta_1 \leq z_2$. The incumbent may then charge a uniform price for the new product in both markets or offer it only in one market. In serving both markets, there are in turn three options: (i) either the pricing ensures that the first-period cutoff type θ_1 strictly prefers to upgrade, or (ii) is indifferent between upgrading and not, or (iii) strictly prefers not to upgrade. We will first consider the different options separately and compare the resulting profits to determine the optimal sales policy.

Under option (i), optimal uniform pricing is the solution of

$$\max_{\{p_U, p_H\}} (1 - \frac{p_H}{s_L + s_\Delta}) p_H \tag{23}$$

subject to

$$\begin{aligned} \theta_1 &> \frac{p_U}{s_\Delta} && (24) \\ \frac{p_H}{s_L + s_\Delta} &\leq \theta_1 && (25) \\ p_U &= p_H && (26) \end{aligned}$$

which yields

$$p_H = p_U = \frac{1}{2}(s_L + s_\Delta). \tag{27}$$

Checking the constraints reveals that the relevant range of θ_1 for option (i) coincides with the range in which the incumbent finds it optimal to price discriminate with respect to purchase history. Hence, option (i) is not chosen.

Under option (ii), the incumbent solves

$$\max_{\{p_U, p_H\}} \left[(1 - \frac{p_U}{s_\Delta}) p_U + (\theta_1 - \frac{p_H}{s_L + s_\Delta}) p_H \right] \tag{28}$$

subject to

$$\begin{aligned} \frac{p_U}{s_\Delta} &\geq \theta_1 && (29) \\ \frac{p_H}{s_L + s_\Delta} &\leq \theta_1 && (30) \\ p_U &= p_H && (31) \end{aligned}$$

The maximization problem under option (iii) differs from the previous one only in constraint (29) which must hold with as strict inequality.

Maximizing (28) subject to (31), we obtain

$$p_H = p_U = \frac{1}{2} s_\Delta \frac{s_L + s_\Delta}{s_L + 2s_\Delta}(1+\theta_1). \tag{32}$$

Taking the other constraints into account, gives the relevant ranges for options (ii) and (iii) as $\theta_1 \leq z_2$, and $s_\Delta / (2s_L + 3s_\Delta) \leq \theta_1 \leq z_2$, respectively. An inspection of the implied profits reveals that option (iii) yields strictly greater profits in the relevant range than option (ii).

To complete the proof of statement 2, we determine the profits obtainable from foregoing sales of the new product in one of the markets. In particular, the incumbent can choose to offer the new product only to consumers in the upgrade market and continue to sell the old product to the consumers in the new-purchase market. The optimal upgrade price and the optimal old-product price are then given by (21) and (2), respectively. By comparing the profits obtainable with this policy and options (ii) and (iii), it is easy to verify the following result. There is a unique value

$$\theta_1 = \frac{s_\Delta(s_L + s_\Delta - \sqrt{s_\Delta}\sqrt{(s_L + 2s_\Delta)})}{s_\Delta s_L + s_L^2 - s_\Delta^2} \equiv z_1$$

with $s_\Delta / (2s_L + 3s_\Delta) < z_1 < z_2$, such that the incumbent prefers to sell the new product in both markets at a uniform price if $z_1 < \theta_1 \leq z_2$ (statement 2), and prefers to offer the new product only in the upgrade market along with the old product in the new-purchase market if $0 \leq \theta_1 \leq z_1$ (statement 3). ■

Proof. (Corollary 1)

We will analyze each of the ranges of θ_1, which are specified in statements 1-3 of Proposition 1 for Γ^I, and check whether leapfrogging occurs. First, for $1/2 \leq \theta_1 \leq 1$, the incumbent serves the whole upgrade market, which precludes leapfrogging. Second, for $z_2 < \theta_1 < 1/2$, the incumbent's optimal prices in the second period are given by (3) and (4) (statement 1 of Proposition 1). Then $p_U/s_\Delta > \theta_1$ if $\theta_1 < 1/2$, and $p_H/[s_L + s_\Delta] < \theta_1$ if $\theta_1 > 0$. That is, the marginal consumer who upgrades in the second period is of a type that is strictly higher than θ_1, and the new product is bought by consumers of type below θ_1, i.e. leapfrogging occurs. Third, for $z_1 < \theta_1 \leq z_2$, the incumbent sells the new product at the optimal uniform price given by (5) (statement 2 of Proposition 1). Then $p_H/s_\Delta > \theta_1$ if $\theta_1 < [s_L + s_\Delta] / [s_L + 3s_\Delta]$, which holds for all $\theta_1 < z_2$. And, $p_H/[s_L + s_\Delta] < \theta_1$ if $\theta_1 > s_\Delta/[2s_L + 3s_\Delta]$, which holds for $\theta_1 > z_1$, i.e.

leapfrogging occurs. Finally, for $0 \leq z_1 \leq \theta_1$, the new product is sold in the upgrade market only, which precludes leapfrogging. ■

Proof. (Lemma 1)

We first determine the unique candidate for a pure-strategy Nash equilibrium in prices, given that the incentive compatibility constraint $p_U \leq p_H$ is not binding. Checking this constraint reveals then that it is binding for all θ_1.

Given the entrant's new-purchase price for the new product, p_H, the incumbent solves

$$\max_{\{p_L\}} \left(\frac{p_H - p_L}{s_\Delta} - \frac{p_L}{s_L} \right) p_L \tag{33}$$

subject to

$$\frac{p_H - p_L}{s_\Delta} \leq \theta_1 \tag{34}$$

which yields the incumbent's reaction function

$$R_L = \frac{1}{2} \frac{s_L}{s_L + s_\Delta} p_H \tag{35}$$

for any $p_H \geq 0$.

Given the incumbent's price for the old product, p_L, the entrant solves

$$\max_{\{p_U, p_H\}} \left[\left(1 - \frac{p_U}{s_\Delta}\right) p_U + \left(\theta_1 - \frac{p_H - p_L}{s_\Delta}\right) p_H \right] \tag{36}$$

subject to

$$\frac{p_U}{s_\Delta} \geq \theta_1 \tag{37}$$

$$\frac{p_H - p_L}{s_\Delta} \leq \theta_1 \tag{38}$$

$$p_U \leq p_H \tag{39}$$

Suppose first that the incentive compatibility constraint $p_U \leq p_H$ is non-binding for some θ_1. The entrant's optimal upgrade price p_U is then the same as given by (21) for Γ^I, while the new-purchase price is chosen as a best response to the incumbent's second-period price

$$R_H = \frac{1}{2}(s_\Delta \theta_1 + p_L). \tag{40}$$

Solving the firm's reaction functions (40) and (35) simultaneously yields

$$p_H = 2s_\Delta \frac{s_L + s_\Delta}{3s_L + 4s_\Delta}\theta_1 \tag{41}$$

$$p_L = s_\Delta \frac{s_L}{3s_L + 4s_\Delta}\theta_1 \tag{42}$$

as the unique candidate for the price equilibrium in the new-purchase market. But as we show in the following the incentive constraint $p_U \leq p_H$ is binding for all θ_1. For $\theta_1 > 1/2$, $p_U \leq p_H$ if and only if $s_L + 2s_\Delta \leq 0$, which is never satisfied. For $\theta_1 \leq 1/2$, we obtain that $p_U \leq p_H$ if and only if $\theta_1 \geq [3s_L + 4s_\Delta] / [4s_L + 4s_\Delta]$. But $1/2 < [3s_L + 4s_\Delta] / [4s_L + 4s_\Delta]$, hence there is a contradiction. ■

Proof. (Proposition 2)

To proof Proposition 2, we need to find the pure-strategy Nash equilibrium in subgame Γ^E. The incumbent's reaction function has been derived in the proof of Lemma 1 and is repeated here:

$$R_L = \frac{1}{2}\frac{s_L}{s_L + s_\Delta}p_H \tag{43}$$

for any $p_H \geq 0$.

By Lemma 1, the entrant may charge a uniform price for the new product in both markets or offer it only in one market. As in Γ^I, there are in turn three options in serving both markets: (i) either the pricing ensures that the first-period cutoff type θ_1 strictly prefers to upgrade, or (ii) is indifferent between upgrading and not, or (iii) strictly prefers not to upgrade. We will analyze each of these options separately and then compare the implied profit levels.

Under option (i), the entrant's problem for a given price p_L is then

$$\max_{\{p_U, p_H\}} \left(1 - \frac{p_H - p_L}{s_\Delta}\right) p_H$$

subject to

$$\theta_1 > \frac{p_U}{s_\Delta} \tag{44}$$
$$p_U = p_H \tag{45}$$

which yields the entrant's reaction function

$$R_H = R_U = \frac{1}{2}(s_\Delta + p_L) \tag{46}$$

for any $p_L \geq 0$. Solving the reaction functions (43) and (46) simultaneously yields

$$p_H = p_U = 2s_\Delta \frac{s_L + s_\Delta}{3s_L + 4s_\Delta} \tag{47}$$
$$p_L = s_\Delta \frac{s_L}{3s_L + 4s_\Delta} \tag{48}$$

as the unique candidate for the price equilibrium. Checking constraint (44) gives the relevant range for (47) and (48) to be part of the equilibrium of subgame Γ^E:

$$\theta_1 > \frac{2s_L + 2s_\Delta}{3s_L + 4s_\Delta} \equiv x_3.$$

Under option (ii), the entrant's problem for a given price p_L is to solve

$$\max_{\{p_U, p_H\}} \left[(1 - \frac{p_U}{s_\Delta})p_U + (\theta_1 - \frac{p_H - p_L}{s_\Delta})p_H\right] \tag{49}$$

subject to

$$\frac{p_U}{s_\Delta} \geq \theta_1 \tag{50}$$
$$\frac{p_H - p_L}{s_\Delta} \leq \theta_1 \tag{51}$$
$$p_U = p_H \tag{52}$$

The maximization problem under option (iii) differs from the previous one only in the strict inequality sign of constraint (50).

The first-order condition yields the reaction function

$$R_H = R_U = \frac{1}{4}[s_\Delta(1+\theta_1) + p_L]. \tag{53}$$

for any $p_L \geq 0$. Note that (53) specifies the best response to (35) only for $\theta_1 \leq x_3$, where x_3 is defined above. Otherwise, options (ii) and (iii) are dominated by option (i). Solving (53) and (35) simultaneously and taking the constraints into account, yields the unique price equilibrium

$$p_H = p_U \quad = \quad \begin{cases} s_\Delta \theta_1 & \text{if } x_2 \leq \theta_1 \leq x_3 \text{ (option (ii))} \\ 2s_\Delta \frac{s_L + s_\Delta}{7s_L + 8s_\Delta}(1+\theta_1) & \text{if } x_4 \leq \theta_1 < x_2 \text{ (option (iii))} \end{cases} \tag{54}$$

$$p_L \quad = \quad \begin{cases} \frac{1}{2} s_\Delta \frac{s_L}{s_L + s_\Delta}\theta_1 & \text{if } x_2 \leq \theta_1 \leq x_3 \text{ (option (ii))} \\ s_\Delta \frac{s_L}{7s_L + 8s_\Delta}(1+\theta_1) & \text{if } x_4 \leq \theta_1 \leq x_2 \text{ (option (iii))} \end{cases} \tag{55}$$

where

$$x_4 \equiv \frac{s_L + 2s_\Delta}{6s_L + 6s_\Delta} < \frac{2s_L + 2s_\Delta}{5s_L + 6s_\Delta} \equiv x_2 < \frac{1}{2}.$$

To complete the proof, we need to examine the policy of ignoring sales in the new-purchase market entirely. For this policy, we obtain the same optimal upgrade price as is given by (3) in Proposition 1, which is part of the incumbent's optimal price discrimination strategy. Finally, we need to derive the profits obtainable under all different sales policies and compare them. It is easy to verify that the resulting profit from selling only in the upgrade market is higher than the profit from selling in both markets at the uniform price given by (54) if and only if

$$0 \leq \theta_1 < \frac{(7\sqrt{2}-8)s_L + (8\sqrt{2}-8)s_\Delta}{8s_L + 8s_\Delta} \equiv x_1$$

where $x_4 < x_1 < x_2 < 1/2 < x_3$. Statements 2, 3 and 4 follow immediately.

The proof of statement 5 is straightforward and omitted. ■

Proof. (Corollary 2)

We will analyze each of the ranges of θ_1 that are specified in Proposition 2 for Γ^E and check whether leapfrogging occurs. First, for $x_3 < \theta_1 \leq 1$, the entrant charges a uniform price given by (7) such that the cutoff type θ_1 prefers to buy. In addition, the entrant sells the new product in the new-purchase market, i.e. no leapfrogging occurs. Second, for $x_2 \leq \theta_1 \leq x_3$, the argument is similar as for $x_3 < \theta_1 \leq 1$. Third, for $x_1 < \theta_1 < x_2$, the equilibrium prices are given by (11) and (12) (statement 3 of Proposition 2). It is easy to show that $p_H/s_\Delta > \theta_1$ if $\theta_1 < x_2$, and $[p_H - p_L]/s_\Delta < \theta_1$ if $\theta_1 > [s_L + 2s_\Delta]/[6s_L + 6s_\Delta] < x_1$, i.e. leapfrogging occurs if $\theta_1 < x_2$. Finally, for $0 \leq \theta_1 \leq x_1$, the new product is not sold to consumers in the new-purchase market, which prevents leapfrogging. ■

Proof. (Proposition 3)

Consider the incumbent's strategy of selling both products. The incumbent has three alternative sales strategies for the new product. First, selling the new product in both, the upgrade market and the new-purchase market. Second, selling the new product only in the new-purchase market. And third, selling the new product only in the upgrade market. Among these sales strategies, the first two yield zero profit to the incumbent, since Bertrand competition reduces the price of the new product as well as the price of the old product to 0. The third strategy of offering the new product only in the upgrade market reduces the upgrade price to 0. This strategy effectively produces the market structure of vertical product differentiation in the new-purchase market, with the entrant as is the high-quality firm and the incumbent as the low-quality firm.[15]

However, the incumbent can gain by withdrawing the new product entirely. It is easy to verify that the incumbent's profit obtained in the case of the history E, in which only the entrant sells the new product and the incumbent continues to sell the old version, strictly exceeds the profit obtainable with history B and free upgrading for $\theta_1 < 1$, and is the same for $\theta_1 = 1$. ■

Proof. (Lemma 2) From Proposition 2 we know that $\pi_E^E(\theta_1)$ is monotone increasing in θ_1 and bounded from below by $\frac{1}{4}s_\Delta$. It follows that $\pi_E^E(\theta_1) \leq K_E$ only if $K_E \geq \frac{1}{4}s_\Delta$ and the first part follows.

To prove the second part, notice that $\pi_E^E(\theta_1) \leq \pi_E^E(\lambda_{K_E}) \leq K_E$ for all $\theta_1 \leq \lambda_{K_E}$ by the monotonicity of $\pi_E^E(\theta_1)$ and the definition of λ_{K_E}. For $\theta_1 \leq x_1$, $\pi_E^E(\theta_1)$ is constant at $\frac{1}{4}s_\Delta$ so for $K_E \geq \frac{1}{4}s_\Delta$ there exists $\lambda_{K_E} \geq x_1$ such that $\pi_E^E(\lambda_{K_E}) = K_E$. ■

Proof. (Proposition 4)

In the no-entry equilibrium without innovation, the incumbent's problem is to maximize

$$\Pi(\theta_1) = p_1(1 - \theta_1) + \pi_I^N(\theta_1) \tag{56}$$

[15] See, for example, Choi and Shin (1992).

subject to $\theta_1 \in \Lambda_{K_E}$.

When the no-entry constraint is not binding, the incumbent's problem reduces to the standard maximization problem of a durable-goods monopolist, which is solved, for instance, by Bulow (1982). That is, $\theta_1 = 3/5$ and the respective first-period price is $p_1 = (9/10)\, s_L$.

For $\lambda_{K_E} \leq 3/5$, the no-entry constraint is binding, i.e. the incumbent is constrained to supply at least λ_{K_E} to prevent entry. To find the respective optimal first-period price, we need to derive the first-period demand. That is, we need to determine, for any price p_1, the θ_1-type consumer who is indifferent between buying the durable good in period 1 for p_1 or not. The concavity of the total profit function implies that the optimal quantity is exactly λ_{K_E}. Collecting these points yields statement 1 of the proposition.

To obtain statement 2, observe that the incumbent maximizes

$$\Pi(\theta_1) = p_1(1 - \theta_1) + \pi_I^I(\theta_1) - K_I \tag{57}$$

subject to $\theta_1 \in \Lambda_{K_E}$.

Suppose first that the no-entry constraint is not binding. To find the optimal first-period choice of the incumbent, we will proceed in the following way: (i) We compute the first-period demand function in terms of the first-period cutoff type θ_1, given there is no entry in the second period. Using Proposition 1, we obtain four ranges of θ_1 with different first-period demand and profit functions. (ii) Second, we determine the optimum in each of the four ranges separately. (iii) Finally, we compare the associated profits across different ranges, and select the one which yields the highest total profit.

(i) Given that $0 \leq \theta_1 \leq z_1$, the θ_1-type is given by $2s_L\theta_1 - p_1 = s_L\theta_1 - p_L \Leftrightarrow p_1 = (3/2)\, s_L\theta_1$.

Given that $z_1 < \theta_1 \leq z_2$, the θ_1-type is given by $2s_L\theta_1 - p_1 = (s_L + s_\Delta)\, \theta_1 - p_H \Leftrightarrow p_1 = \theta_1 (s_L - s_\Delta) + \frac{1}{2} s_\Delta (s_L + s_\Delta) / (s_L + 2s_\Delta)(1 + \theta_1)$.

Given that $z_2 \leq \theta_1 \leq 1/2$, the θ_1-type is given by $2s_L\theta_1 - p_1 = (s_L + s_\Delta)\, \theta_1 - p_H \Leftrightarrow p_1 = \theta_1 (3s_L - s_\Delta)/2$.

Given that $1/2 < \theta_1 \leq 1$, the θ_1-type is given by $s_L\theta_1 - p_1 + (s_L + s_\Delta)\, \theta_1 - p_U = (s_L + s_\Delta)\, \theta_1 - p_H \Leftrightarrow p_1 = \theta_1 (3s_L - s_\Delta)/2$.

(ii) The next step is to determine the optimum of $\Pi(\theta_1)$ in each of the four ranges. It is easy to verify that, for $0 \leq \theta_1 \leq z_1$, and $z_1 < \theta_1 < z_2$ and $z_2 \leq \theta_1 \leq 1/2$, $\Pi(\theta_1)$ attains its optimum at the upper boundary of the respective range of θ_1, respectively. For $1/2 \leq \theta_1 \leq 1$, the optimum of $\Pi(\theta_1)$ lies at $\theta_1 = (3s_L + s_\Delta) / (5s_L + s_\Delta)$.

(iii) By the continuity of $\Pi(\theta_1)$, it follows immediately from (ii) that $\theta_1 = (3s_L + s_\Delta) / (5s_L + s_\Delta)$ is the optimal first-period choice given no entry threat. The respective optimal first-period price is $p_1 = (9s_L^2 - s_\Delta^2) / (10s_L + 2s_\Delta)$.

To complete the proof, observe that the incumbent is constrained by the no-entry set when $\lambda_{K_E} \leq (3s_L + s_\Delta) / (5s_L + s_\Delta)$. The concavity of the total profit function implies that the optimal first period sales is obtained at the boundary, λ_{K_E}. To get the optimal first-period price we substitute λ_{K_E} for θ_1 in the first-period demand obtained in step (i). ■

Proof. (Proposition 5)

To find the optimal first-period choice of the incumbent in the entry equilibrium, we proceed in a similar way as in the proof of Proposition 4: (i) We derive the first-period demand function in terms of the first-period cutoff type θ_1, given entry in the second period, where θ_1 is the consumer type that is indifferent between buying and not buying the durable good in period 1 for price p_1. Using Proposition 2, we obtain different first-period demand and hence profit functions for four ranges of θ_1. (ii) Second, we determine the optimum in each of the four ranges separately. (iii) Finally, we compare the associated profits across different ranges, and select the one which yields the highest total profit.

(i) Given that $0 \leq \theta_1 \leq x_1$, the θ_1-type is given $2s_L\theta_1 - p_1 = s_L\theta_1 - p_L \Leftrightarrow p_1 = (3/2)\, s_L\theta_1$.

Given $x_1 < \theta_1 < x_2$, the θ_1-type is given by $2s_L\theta_1 - p_1 = (s_L + s_\Delta)\,\theta_1 - p_H \Leftrightarrow p_1 = (s_L - s_\Delta)\,\theta_1 + 2s_\Delta\,(s_L + s_\Delta) / (7s_L + 8s_\Delta)\,(1 + \theta_1)$.

Given $x_2 \leq \theta_1 \leq x_3$, the θ_1-type is given by $2s_L\theta_1 - p_1 = (s_L + s_\Delta)\,\theta_1 - p_H \Leftrightarrow p_1 = s_L\theta_1$ or $s_L\theta_1 - p_1 + (s_L + s_\Delta)\,\theta_1 - p_H = (s_L + s_\Delta)\,\theta_1 - p_H \Leftrightarrow p_1 = s_L\theta_1$.

Given $x_3 < \theta_1 \leq 1$, the θ_1-type is given by $s_L\theta_1 - p_1 + (s_L + s_\Delta)\,\theta_1 - p_H = (s_L + s_\Delta)\,\theta_1 - p_H \Leftrightarrow p_1 = s_L\theta_1$.

(ii) The next step is to determine the optimum of $\Pi(\theta_1)$ in each of the four ranges. It is easy to verify that, for $0 \leq \theta_1 \leq x_1$, $\Pi(\theta_1)$ attains its optimum at $\theta_1 = x_1$. For $x_1 < \theta_1 < x_2$, the optimum of $\Pi(\theta_1)$ lies at x_1 for high values of s_Δ/s_L, and at $\theta_1 = x_2$ for low values of s_Δ/s_L, and between x_1 and x_2 for medium values of s_Δ/s_L. For $x_2 \leq \theta_1 \leq x_3$ and $x_3 < \theta_1 \leq 1$, $\Pi(\theta_1)$ attains its optimum at $\theta_1 = x_3$.

(iii) Comparing the associated profits across ranges yields $\theta_1 = x_1$ as the optimal first-period choice. The respective optimal first-period price is hence $p_1 = (3/2)\, s_L x_1$. ■

Proof. (Proposition 6)

If Λ_{K_E} is empty, then the incumbent has no choice but to concede and play the entry equilibrium. To prove the reverse, suppose that Λ_{K_E} is non-empty and the incumbent plans to play the entry equilibrium. Proposition 5 implies that the incumbent's optimal decision for the first period is to chooses $\theta_1 = x_1$. However Lemma 2 (Statement 2) implies that $\pi_E^E(x_1) < 0$ so that the entrant cannot earn a positive profit from entry. Therefore entry does not take place, which completes the proof. ■

Proof. (Proposition 7)

There is no possibility of leapfrogging in Γ^N since there is only one generation of the durable good. By Corollary 1, leapfrogging occurs in Γ^I for $z_1 < \theta_1 < 1/2$. Since x_1 is greater than z_1, leapfrogging occurs in the no-entry equilibrium with innovation for $x_1 < \theta_1 < 1/2$. Finally, leapfrogging does not occur in the entry equilibrium: by Corollary 2 leapfrogging in Γ^E would require that $x_1 < \theta_1 < x_2$ is satisfied, while the optimal first period sales quantity in the entry equilibrium is x_1. ■

Proof. (Proposition 8)

Suppose that $K_E \geq \frac{1}{4}s_\Delta$ such that Λ_{K_E} is non-empty. Proposition 6 implies that the incumbent will not play the entry equilibrium. Note that $\pi_I^I(\theta_1)$ does not depend on K_E and the incumbent chooses to innovate only if $\pi_I^I(\theta_1) - \pi_I^N(\theta_1) \geq K_I$. In this equilibrium, the incumbent chooses a $\theta_1$greater than or equal to x_1 such that $\pi_E^E(\theta_1) = K_E$ (statement 2 of Proposition 4). Hence, to prove the proposition, it suffices to show that $\pi_I^I(\theta_1) - \pi_I^N(\theta_1) > \pi_E^E(\theta_1)$ is possible for some values of s_Δ and s_L.

Consider the case in which s_Δ is sufficiently close to s_L. Then $z_1 < x_1 < x_2 < z_2 < 1/2$. We obtain that $\pi_I^I(\theta_1) - \pi_I^N(\theta_1) > \pi_E^E(\theta_1)$ for $x_1 \leq \theta_1 < z_2$.

To show the second statement, assume that $K_E < \frac{1}{4}s_\Delta$ and $K_I < K_E$. In this case the incumbent plays the entry equilibrium since the no-entry set is empty. Therefore the entrant innovates even if the incumbent has a lower innovation cost. ■

Proof. (Proposition 9) First compute the total gain from trade under the entry deterrence equilibrium without innovation:

$$W^N = 2\int_{\lambda_{K_E}}^{1} s_L\theta\, d\theta + \int_{\frac{1}{2}\lambda_{K_E}}^{\lambda_{K_E}} s_L\theta\, d\theta \tag{58}$$
$$= s_L(1 - \frac{5}{8}\lambda_{K_E}^2)$$

Next consider the total gain from trade under entry equilibrium where the entrant innovates. Proposition 2 and Proposition 5 imply the following total gain:

$$W^E = 2\int_{x_1}^{1} s_L\theta\, d\theta + \int_{\frac{1}{2}}^{1} s_\Delta\theta\, d\theta + \int_{\frac{1}{2}x_1}^{x_1} s_L\theta\, d\theta - K_E \tag{59}$$
$$= s_L(1 - \frac{5}{8}x_1^2) + \frac{3}{8}s_\Delta - K_E$$

If W^E dominates W^N for a given K_E, then the entry equilibrium is more efficient than the entry deterrence equilibrium without innovation. The computation above indicates that W^E dominates W^N if $\frac{5}{8}s_L(\lambda_{K_E}^2 - x_1^2) + \frac{3}{8}s_\Delta - K_E \geq 0$. Hence it remains to show that W^E is greater than W^N whenever given K_E is consistent with λ_{K_E}.

Recall that λ_{K_E} is the first period sales when the entrant makes 0 profit from the entry. Proposition 2 implies that K_E and λ_{K_E} are related as follows for 3 ranges:

$$K_E = \begin{cases} 4s_\Delta\frac{(s_L+s_\Delta)^2}{(3s_L+4s_\Delta)^2} & \text{if } x_3 < \lambda_{K_E} \leq \frac{3}{5} \\ s_\Delta\theta_1 - \frac{1}{2}s_\Delta\frac{s_L+2s_\Delta}{s_L+s_\Delta}\theta_1^2 & \text{if } x_2 \leq \lambda_{K_E} \leq x_3 \\ 8s_\Delta\frac{(s_L+s_\Delta)^2}{(7s_L+8s_\Delta)^2}\left(1+\theta_1\right)^2 & \text{if } x_1 \leq \lambda_{K_E} \leq x_2 \end{cases} \tag{60}$$

Substituting K_E into $\frac{5}{8}s_L(\lambda_{K_E}^2 - x_1^2) + \frac{3}{8}s_\Delta - K_E$ and evaluating it for different range of λ_{K_E}, we can easily confirm that W^E dominates W^N for all ranges. The proof is complete. ■

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	θ_1	p_L	p_H	p_U	θ_L	θ_H	θ_U
Γ^N	-	$\frac{1}{2}s_L\theta_1$	-	-	$\frac{1}{2}\theta_1$	-	
Γ^I	$(\frac{1}{2},1]$	-	$\frac{1}{2}(s_L+s_\Delta)\theta_1$	$s_\Delta\theta_1$	-	$\frac{1}{2}\theta_1$	θ_1
	$(z_2,\frac{1}{2}]$	-	$\frac{1}{2}(s_L+s_\Delta)\theta_1$	$\frac{1}{2}s_\Delta$	-	$\frac{1}{2}\theta_1$	$\frac{1}{2}$
	$(z_1,z_2]$	-	$\frac{s_\Delta(s_L+s_\Delta)}{2(s_L+2s_\Delta)}(1+\theta_1)$	$\frac{s_\Delta(s_L+s_\Delta)}{2(s_L+2s_\Delta)}(1+\theta_1)$	-	$\frac{s_\Delta}{2(s_L+2s_\Delta)}(1+\theta_1)$	$\frac{s_L+s_\Delta}{2(s_L+2s_\Delta)}(1+\theta_1)$
	$[0,z_1]$	$\frac{1}{2}s_L\theta_1$	-	$\frac{1}{2}s_\Delta$	$\frac{1}{2}\theta_1$	-	$\frac{1}{2}$
Γ^E	$(x_3,1]$	$\frac{s_\Delta s_L}{3s_L+4s_\Delta}$	$\frac{2s_\Delta(s_L+s_\Delta)}{3s_L+4s_\Delta}$	$\frac{2s_\Delta(s_L+s_\Delta)}{3s_L+4s_\Delta}$	$\frac{s_L}{3s_L+4s_\Delta}$	$\frac{2s_\Delta}{3s_L+4s_\Delta}$	$\frac{2(s_L+s_\Delta)}{3s_L+4s_\Delta}$
	$(x_2,x_3]$	$\frac{s_\Delta s_L}{2(s_L+s_\Delta)}\theta_1$	$s_\Delta\theta_1$	$s_\Delta\theta_1$	$\frac{s_\Delta}{2(s_L+s_\Delta)}\theta_1$	$\frac{s_\Delta}{(s_L+s_\Delta)}\theta_1$	θ_1
	$(x_1,x_2]$	$\frac{s_\Delta s_L}{7s_L+8s_\Delta}(1+\theta_1)$	$\frac{2s_\Delta(s_L+s_\Delta)}{2(7s_L+8s_\Delta)}(1+\theta_1)$	$\frac{2s_\Delta(s_L+s_\Delta)}{2(7s_L+8s_\Delta)}(1+\theta_1)$	$\frac{s_\Delta}{7s_L+8s_\Delta}(1+\theta_1)$	$\frac{2s_\Delta}{2(7s_L+8s_\Delta)}(1+\theta_1)$	$\frac{2(s_L+s_\Delta)}{2(7s_L+8s_\Delta)}(1+\theta_1)$
	$[0,x_1]$	$\frac{1}{2}s_L\theta_1$	-	$\frac{1}{2}s_\Delta$	$\frac{1}{2}\theta_1$	-	$\frac{1}{2}$

Table 2: Equilibrium Strategy for Second Period Subgames

	θ_1	π_I	π_E
Γ^N	-	$\frac{1}{4}s_L\theta_1^2$	-
Γ^I	$(\frac{1}{2},1]$	$(1-\theta_1)\,\theta_1 s_\Delta+\frac{1}{4}(s_\Delta+s_L)\,\theta_1^2$	-
	$(z_2,\frac{1}{2}]$	$\frac{1}{4}s_\Delta+\frac{1}{4}(s_L+s_\Delta)\,\theta_1^2$	-
	$(z_1,z_2]$	$\frac{1}{4}s_\Delta\frac{s_L+s_\Delta}{s_L+2s_\Delta}(1+\theta_1)^2$	-
	$[0,z_1]$	$\frac{1}{4}s_\Delta+\frac{1}{4}s_L\theta_1^2$	-
Γ^E	$(x_3,1]$	$\frac{s_L s_\Delta(s_L+s_\Delta)}{(3s_L+4s_\Delta)^2}$	$4s_\Delta\frac{(s_L+s_\Delta)^2}{(3s_L+4s_\Delta)^2}$
	$(x_2,x_3]$	$\frac{s_L s_\Delta}{4(s_L+s_\Delta)}\theta_1^2$	$s_\Delta\theta_1-\frac{1}{2}s_\Delta\frac{s_L+2s_\Delta}{s_L+s_\Delta}\theta_1^2$
	$(x_1,x_2]$	$\frac{s_L s_\Delta(s_L+s_\Delta)}{(7s_L+8s_\Delta)^2}(1+\theta_1)^2$	$8s_\Delta\frac{(s_L+s_\Delta)^2}{(7s_L+8s_\Delta)^2}(1+\theta_1)^2$
	$[0,x_1]$	$\frac{1}{4}s_L\theta_1^2$	$\frac{1}{4}s_\Delta$

Table 3: Equilibrium profit for Second Period Subgames

	λ_{K_E}	p_1	θ_1	Π
Γ^N	$(\frac{3}{5}, 1]$	$\frac{9}{10}$	$\frac{3}{5}$	$\frac{9}{20}s_L$
	$(x_1, \frac{3}{5}]$	$\frac{3}{2}s_L\lambda_{K_E}$	λ_{K_E}	$\frac{3}{2}s_L\lambda_{K_E} - \frac{5}{4}s_L\lambda_{K_E}^2$
Γ^I	$(\frac{3s_L+s_\Delta}{5s_L+s_\Delta}, 1]$	$\frac{9s_L^2-s_\Delta^2}{2(5s_L+s_\Delta)}$	$\frac{3s_L+s_\Delta}{5s_L+s_\Delta}$	$\frac{(3s_L+s_\Delta)^2}{4(5s_L+s_\Delta)}$
	$(\frac{1}{2}, \frac{3s_L+s_\Delta}{5s_L+s_\Delta}]$	$\frac{1}{2}\left(3s_L - s_\Delta\right)\lambda_{K_E}$	λ_{K_E}	$\frac{1}{2}\left(3s_L + s_\Delta\right)\lambda_{K_E} - \frac{1}{4}\left(5s_L + s_\Delta\right)\lambda_{K_E}^2 - K_I$
	$(z_2, \frac{1}{2}]$	$\frac{1}{2}\left(3s_L + s_\Delta\right)\lambda_{K_E} - \frac{1}{2}s_\Delta$	λ_{K_E}	$-\frac{1}{4}s_\Delta + \frac{1}{2}(3s_L + 2s_\Delta)\lambda_{K_E} - \frac{1}{4}(5s_L + s_\Delta)\lambda_{K_E}^2 - K_I$
	$(x_1, z_2]$	$s_L\lambda_{K_E}$	λ_{K_E}	$\frac{s_\Delta(s_L+s_\Delta)}{4(s_L+2s_\Delta)} + \frac{2s_L^2+5s_Ls_\Delta+s_\Delta^2}{2(s_L+2s_\Delta)}\lambda_{K_E} - \frac{4s_L^2+7s_Ls_\Delta-s_\Delta^2}{4(s_L+2s_\Delta)}\lambda_{K_E}^2 - K_I$
Γ^E		$\frac{3}{2}s_L\frac{(7\sqrt{2}-8)s_L+(8\sqrt{2}-8)s_\Delta}{8s_L+8s_\Delta}$	x_1	$\frac{\left(448\sqrt{2}-597\right)\left(2647s_L+2472s_\Delta-220s_\Delta\sqrt{2}\right)\left(17s_L+4s_\Delta\sqrt{2}+24s_\Delta\right)s_L}{5759\,872(s_L+s_\Delta)^2}$

Table 4: Equilibrium Outcome for Γ