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Nominal debt and the dynamics of currency crises*

Abstract

We present a model to study a currency crisis associated with a fiscal imbalance. The stock and maturity of nominal government liabilities emerge as key determinants of the magnitude and predictability of the crisis. Fiscal costs of peg defense reduce net fiscal revenue from money creation. While the imbalance makes a devaluation inevitable, an interest rate rule followed by the central bank determines the timing of the speculative attack. However, the date of the collapse may be indeterminate in the presence of a large stock of short-term public debt. Among notable features of our model, budget deficits need not be high before the crisis, post-devaluation inflation may exhibit little persistence, and money demand need not fall after the crisis.

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1 Introduction

This paper presents a model to study the dynamics of a currency crisis associated with a fiscal imbalance, defined as current or anticipated future decline in real primary surpluses.[1] We show that the stock and maturity of nominal government liabilities are key determinants of the magnitude and predictability of the crisis. On impact, a fiscal shock induces a decline in bond prices that generates an immediate wealth transfer from the private to the public sector.[2] If the stock of long-term nominal liabilities is large enough, this transfer can guarantee, by itself, that the intertemporal government budget constraint holds with temporary price and exchange rate stability. Thus long-term nominal liabilities help governments that, for any reason, want to delay a devaluation.[3] While public sector solvency implies an upper bound on the latest possible date of an exchange rate adjustment, the timing of the devaluation depends on a policy rule followed by the central bank. The timing, however, is not necessarily unique. A stock of short-term liabilities that is large relative to the stock of international reserves raises the possibility of indeterminacy and self-fulfilling speculative runs on public debt.

Our framework shows important equilibrium implications of fiscal costs of defending the peg against speculative attacks.[4] Consider the policy experiment underlying the analysis of Krugman (1979). A fiscal deficit drives money creation above the rate that is consistent with a perma-

[1] Policy analysts have discussed fiscal imbalances as causal factors in the collapses of the Russian ruble in 1998 and of the Brazilian real in 1999. According to a leading interpretation, the fall of Asian currencies in 1997 was associated with anticipated fiscal costs of bailing out private companies in distress. Corsetti, Pesenti and Roubini (1999, 1999) and Burnside, Eichenbaum and Rebelo (1998) present this thesis. The homepage "What caused the Asian Currency and Financial Crisis and its global contagion" by Nouriel Roubini (www.stern.nyu.edu/~nroubini/asia/AsiaHomepage.html) includes an extensive reference list.

[2] There may be other fiscal benefits from anticipated inflation. For example, see Persson, Persson, and Svensson (1998) for a case study emphasizing the role of nominal features in the tax and transfer system. Allowing for these additional benefits would reinforce the results of our analysis.

[3] Sims (1997) contains a discussion of currency crises that emphasizes nominal goverment liabilities. Daniel (1998) presents a model of the dynamics of crises including an analysis of fiscal gains from inflation and devaluation due to a decline of the real value of long-term government debt.

[4] Upon completing our paper, we have become aware that Lahiri and Vegh (1998) stress a closely related point. These authors analyze how the present value of seigniorage in a Krugman-style model depends on the interest elasticity of money demand. According to their analysis, the "feasibility of a BOP crisis" depends on a low enough interest elasticity of money demand that makes it possible for the government to raise enough seigniorage after the crisis. Our specification is consistent with their proposition concerning the critical value of this elasticity.

nently fixed exchange rate, yet the central bank pursues a policy aimed at delaying the ultimately inevitable devaluation. When a speculative attack eventually precipitates a crisis, the loss of seigniorage associated with handing over reserves to speculators is endogenous and increasing in the anticipated money expansion. Our model shows that this loss can be large enough to drive the overall net revenue from seigniorage to zero. Then, if the government wants to delay the devaluation, monetary financing of a fiscal imbalance is not even an option: in equilibrium, financing the imbalance via seigniorage and the goal of delaying the exchange rate adjustment are inconsistent with each other. In our model, if there are no long-term liabilities, and therefore no fiscal transfer associated with bond price movements, a fiscal shock forces the central bank to devalue immediately.

With enough long-term liabilities outstanding, the government can temporarily postpone the abandonment of the peg even if net seigniorage revenue is zero. The timing of the crisis is then determined by the behavior of the central bank. We propose a specification in which the central bank is willing to defend the peg as long as it can maintain the interest rate below a given threshold. The higher this threshold, the stronger is the central bank's commitment to delay a collapse — an upper bound on the interest rate indexes the tenacity with which the authorities defend the parity. We show that our 'interest rate rule' implies, as a special case, the assumption of a lower bound on reserves — a common specification in the literature.

For the timing of the crisis to be uniquely determined under our interest rate rule, the government must have full access to international financial markets.[5] However, the timing may be indeterminate if a credit constraint results from a coordination problem among public debt holders. We analyze a self-fulfilling crisis in which a run on public debt forces a devaluation by preventing the government from borrowing to defend the exchange rate.

Our framework reveals important links between first-generation models of currency crises after

[5] There may exist policy and constitutional rules limiting government borrowing – such as the bounds on public debt and deficits in the Euro area. In this case, the timing of devaluation is still uniquely determined, but will depend on these borrowing constraints.

Krugman (1979),[6] and the fiscal theory of the price level (FTPL).[7] The fiscal shock that we consider in this paper is common in the currency crises literature, but also corresponds to the fiscal policy that Leeper (1991) calls "active" and Woodford (1995, 1996) calls "non-Ricardian". Leeper refers as "passive" to the kind of monetary policy that both the first-generation literature and this paper consider. Absent coordination problems among agents, active fiscal policy, when coupled with passive monetary policy, guarantee a determinate, unique price and exchange rate level. We provide an intuitive graphical apparatus that illustrates how fiscal and monetary policies interact in equilibrium so as to generate a crisis. Consistent with recent episodes of currency instability, budget deficits need not be high before the crisis, post-devaluation inflation may exhibit little persistence, and seigniorage revenues may be moderate. While a speculative attack coincides with a contraction in money demand, money demand need not fall after the collapse of the exchange rate.

The paper proceeds as follows. The next section develops the model. In section 3, we discuss the role of seigniorage. In section 4 we characterize the equilibrium with short-term nominal liabilities. In section 5, we discuss the model with long-term nominal government debt, studying a delayed devaluation and the dynamics of a speculative attack. We offer concluding remarks in section 6.

2 A framework for study of a currency crisis associated with a fiscal imbalance

This section presents what we think of as the simplest fully-specified model in which to study a currency crisis associated with a fiscal imbalance. We will first focus on a specification with short term debt only – long-term debt will be introduced in Section 5. Consider a small, open,

[6] The first-generation models of currency crises of Krugman (1979) and Flood and Garber (1984) build on Salant and Henderson (1978). Examples of micro-founded versions of the Krugman-style analysis include: Obstfeld (1986b), Calvo (1987), Daniel (1998), Kumhof (1998), and Calvo and Vegh (1999). Recent discussions of the literature are: Calvo(1998), Buiter, Corsetti and Pesenti (1998), and Flood and Marion (1998).

[7] The literature on the FTPL includes, for example: Leeper (1991), Woodford (1994, 1995, 1996), Sims (1994, 1997, 1998), and Cochrane (1998, 1999).

endowment economy where a representative individual consumes a single perishable good. The law of one price holds. We denote with $\mathcal{E}$ the exchange rate, or the price of one unit of foreign currency in terms of domestic currency. Normalizing to unity the foreign currency price of the consumption good, the law of one price implies that the domestic currency price of the consumption good is equal to $\mathcal{E}$. Throughout the paper, we assume that the government initially fixes the price level at $\overline{\mathcal{E}}$, and we study the impact of a single unanticipated shock to fiscal transfers on the stability of the fixed exchange rate.

2.1 Utility, budget and resource constraints

The representative individual maximizes:

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{\mathcal{E}_t}\right) = \sum_{t=0}^{\infty} \beta^t \ln\left[C_t^{\gamma}\left(\frac{M_t}{\mathcal{E}_t}\right)^{1-\gamma}\right] \tag{1}$$

Note that we choose a functional form for the utility function such that the marginal utility from consumption does not depend on real balances – *i.e.* consumption and the current account do not depend on the path of nominal interest rates.

In addition to domestic money M, individuals hold a short-term (one-period) non-indexed domestic-currency-denominated bond issued by the domestic government, B, and a one-period foreign-currency-denominated foreign bond, B^*. The representative individual's budget constraint is:

$$B_{t+1} + \mathcal{E}_t B^*_{t+1} + M_t = (1 + i_t) B_t + \mathcal{E}_t (1 + r) B^*_t + M_{t-1} + \mathcal{E}_t (Y_t - C_t - \eta_t) \tag{2}$$

where Y and C denote endowment and consumption of the consumption good, respectively, r is the (constant) world real (and nominal) interest rate, i is the nominal interest rate on the domestic bond, and η are real lump-sum taxes. We assume that $r = (1 - \beta)/\beta$. The timing convention follows Obstfeld and Rogoff (1996). In particular, we denote by M_t the stock of nominal balances held at the end of period t, and by B_{t+1} the stock of bonds held at the end of period t. The nominal interest rate on a bond held between periods t and $t + 1$ is denoted i_{t+1}. Note that, absent shocks, the uncovered interest rate parity holds:

$$1+i_{t+1}=(1+r)\,\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \tag{3}$$

The budget identity of the consolidated public sector, which we simply refer to as the government, is:

$$(B_{t+1}-B_t)-\mathcal{E}_t\,(R_{t+1}-R_t)+(M_t-M_{t-1})=i_tB_t-r\mathcal{E}_tR_t-\mathcal{E}_t\eta_t \tag{4}$$

where R_{t+1} are net foreign assets of the government (foreign reserves) in the form of one-period foreign bonds held at the end of period t. Solving (4) forward,[8] we then obtain the intertemporal constraint of the government:

$$\frac{(1+i_t)\,B_t}{\mathcal{E}_t}-(1+r)\,R_t=\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s\left(\eta_{t+s}+\frac{M_{t+s}-M_{t-1+s}}{\mathcal{E}_{t+s}}\right) \tag{5}$$

where i_t, B_t, and R_t are predetermined at period $t-1$. At any time t, the real value of government's nominal debt net of foreign assets equals the present value of real primary surpluses plus the present value of seigniorage.[9]

The individual maximizes (1) subject to (2) and prohibitions on excessive borrowing,[10] choosing a lifetime consumption and portfolio plan $\{C_t, M_t, B_{t+1}, B^*_{t+1}\}$, while taking as given the time

[8] The solution makes use of the following condition:

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T\left(\frac{B_{t+1+T}+M_{t+T}}{\mathcal{E}_{t+T}}-R_{t+1+T}\right)=0$$

In an open economy model, it is theoretically possible for this expression to be non-zero. While we can be sure that optimizing individuals will not accumulate wealth indefinitely without spending it, non-optimizing governments might not satisfy this condition. However, as Bergin (1998) and Sims (1998) discuss, doing so would imply a reduction in welfare of the country's citizens and is therefore unappealing as a description of a possible equilibrium.

[9] The resource constraint is the current account identity:

$$R_{t+1}+B^*_{t+1}=(1+r)\,(R_t+B^*_t)+Y_t-C_t$$

External solvency implies that the economy's net foreign assets equal the present value of its trade deficits vis-à-vis the rest of the world.

[10] Specifically, we suppose that:

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T B^*_{t+1+T}\geq 0$$

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T \frac{B_{t+1+T}+M_{t+T}}{\mathcal{E}_{t+T}}\geq 0$$

paths of $\{Y_t,\ \eta_t, i_{t+1}, \mathcal{E}_t\}$ and r. There is a single first order condition with respect to B_{t+1} and B^*_{t+1}, since, with interest rate parity and purchasing power parity, domestic and foreign bonds are perfect substitutes:

$$C_{t+1} = (1+r)\,\beta C_t \tag{6}$$

The money demand equation is:

$$\frac{M_t}{\mathcal{E}_t} = \left(\frac{1-\gamma}{\gamma}\right)\left(1+\frac{1}{i_{t+1}}\right)C_t \tag{7}$$

and the transversality condition is:

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T\left(\frac{B_{t+1+T}+M_{t+T}}{\mathcal{E}_{t+T}}+B^*_{t+1+T}\right)=0$$

2.2 Fiscal imbalances and the government budget constraint

The economy is initially in a steady state equilibrium in which government policies are consistent with the exchange rate being permanently fixed at $\overline{\mathcal{E}}$. Given $\overline{\mathcal{E}}$, the money demand equation (7) and the uncovered interest rate parity (3) imply that $M_t = \overline{M}$ and $i_{t+1} = r$ at all times. In words, the exchange rate peg constrains monetary policy to keep the stock of nominal money constant and the nominal interest rate at the level of the world real (and nominal) interest rate.

Most importantly, the exchange rate peg constrains fiscal policy. With a permanently fixed $\overline{\mathcal{E}}$, the intertemporal government budget constraint (5) becomes:

$$\frac{(1+i_t)\,B_t}{\overline{\mathcal{E}}} - (1+r)\,R_t = \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^s \bar{\eta}_{t+s} \tag{8}$$

where $\{\bar{\eta}_s\}_{s=t}^{s=\infty}$ is a path of real primary surpluses such that (8) holds given B_t, R_t and $\overline{\mathcal{E}}$. When the public sector is a net debtor, this requirement could be satisfied, for instance, by a constant $\bar{\eta} > 0$, but is also consistent with a sequence of real primary deficits that are offset by appropriately large primary surpluses in the future.

We then consider an unanticipated fiscal shock: at time t, agents learn the news about the current and future path of government transfers, such that the present value of real primary surpluses declines by $\Delta > 0$. Without loss of generality, we assume that from time T_d onwards real primary surpluses are expected to fall by a constant $d > 0$:

$$\Delta \equiv \frac{(1+r)\,d}{r}\left(\frac{1}{1+r}\right)^{T_d - t} > 0$$

By letting $T_d \geq t$, we allow for the possibility that fiscal deficits start to deteriorate at a future date. To gain intuition, one could think that net transfers are expected to increase because the government will assume as its own liabilities of private companies.

The initial adverse shock underlying Δ can stem from a variety of sources. Once this shock occurs, the government may attempt to adjust the time path of its budgets in an effort to reverse the negative impact of the shock — an adjustment that is usually quite costly. What we assume is that the government cannot (or is not willing to) implement reforms such that (8) continues to hold. Thus, Δ is defined as the net decrease in the present value of real primary surpluses *after* the government has taken all fiscal measures it deemed feasible to reverse the impact of the original shock. *Monetary authorities and private agents take* Δ *as a datum.*

Let Ω_t denote the present value of seigniorage at time t. Clearly, Ω_t is zero in the initial constant-M equilibrium, but it may be different from zero in an equilibrium that follows the fiscal shock. Recalling that $\{\bar{\eta}_s\}_{s=t}^{s=\infty}$ denotes the time path of real primary surpluses before the policy change, we write the government's budget constraint *after* the news about the policy change at time t as follows:

$$\frac{(1+i_t)\,B_t}{\mathcal{E}_t} - (1+r)\,R_t = \left[\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s}\bar{\eta}_{t+s}\right] - \Delta + \Omega_t \tag{9}$$

3 Equilibrium implications of the fiscal costs of peg defense

In Krugman (1979) and the first-generation literature, the government's need to "monetize the deficit" after a fiscal shock causes a predictable collapse of the exchange rate peg. Yet, while current and future monetization of the deficit makes the crisis inevitable, the authorities are able to maintain the exchange rate temporarily fixed. The framework presented in the previous section makes a key assumption in this literature transparent. For $\mathcal{E}_t = \overline{\mathcal{E}}$ to be an equilibrium, the intertemporal budget constraint of the government (9) must hold accounting for all fiscal costs of peg defense. This section focuses on this condition and the implications of its violation for the equilibrium exchange rate.

3.1 Interest and seigniorage costs of peg defense

When all debt is short-term and uncovered interest parity holds, *anticipated* devaluation and inflation at and after the date of the peg collapse raises the interest bill of the government just enough to keep the real value of outstanding nominal public debt constant. Note that in our model this is true both ex ante and ex post, since, once the fiscal shock is revealed, there is no additional uncertainty. The key implication is that, for the intertemporal budget constraint (9) to hold with $\mathcal{E}_t = \overline{\mathcal{E}}$, it must be that $\Omega_t = \Delta$.

We thus need to examine seigniorage as a source of fiscal revenue when the government attempts to delay a devaluation. Here is the problem. While planning to raise positive seigniorage revenue *after* an anticipated collapse, the government also commits to spend resources defending the peg in the short run. It may well be that, once we account for the fiscal costs of fending off any speculative attacks, the overall present discounted value of seigniorage Ω_t falls short of Δ. We illustrate the importance of this issue by showing that in our quite standard setup Ω_t is — somewhat surprisingly — identically equal to zero.[11]

Define $\Omega_{t,T-1}$ and $\Omega_{t,T}$ as the present discounted value of seigniorage raised *before* and *after* the abandonment of the peg, respectively:

$$\Omega_t \equiv \Omega_{t,T-1} + \Omega_{t,T}$$

and consider first seigniorage after the collapse:

$$\Omega_{t,T} \equiv \left(\frac{1}{1+r}\right)^{T-t} \left[\sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^{s} \left(\frac{M_{T+s} - M_{T-1+s}}{\mathcal{E}_{T+s}}\right)\right] \tag{10}$$

[11] See Lahiri and Vegh (1998) for a detailed analysis of how the present value of seigniorage in a Krugman-style model depends on the interest elasticity of money.

In the above expression, $\mathcal{E}_{T+s}$ denotes the equilibrium exchange rate at the time of the collapse and afterwards. For simplicity, we assume that money supply after T grows at a constant rate $\mu \geq 0$. When $\mu > 0$, the process of money creation is:

$$M_s = \begin{cases} \overline{M} & \text{for} \quad s < T-1 \\ \overline{M}\,(1+\mu) & \text{at} \quad s = T \\ (1+\mu)M_{s-1} & \text{for} \quad s \geq T+1 \end{cases} \tag{11}$$

When $\mu = 0$, instead, money supply is constant from time T onwards and any adjustment in M must occur in period T. In either case, the money demand equation (7) implies that, after the collapse ($s \geq T+1$), the exchange rate grows at the same constant rate μ: $\mathcal{E}_s = (1+\mu)\mathcal{E}_{s-1}$. Using this property, in the first appendix we derive:

$$\Omega_{t,T} = \left(\frac{1}{1+r}\right)^{T-t} \frac{\overline{M}}{\overline{\mathcal{E}}} \left[\frac{(1+r)\left(\mathcal{E}_T/\overline{\mathcal{E}} - 1\right)}{(1+r)\,\mathcal{E}_T/\overline{\mathcal{E}} - 1}\right] \tag{12}$$

Let the government fix a target post-collapse seigniorage revenue Ω^*. By setting $\Omega^* = \Omega_{t,T}$ in the above equation, we obtain the size of the devaluation at time T, $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$, that is consistent with the target seigniorage revenue.[12] Two properties of $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$ are noteworthy. First, $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$ is increasing in the magnitude of the target revenue from seigniorage, Ω^*, and in the time span between the arrival of the news and the crisis $(T-t)$. We can therefore write $\mathcal{E}_T/\overline{\mathcal{E}}\left[\overset{+}{\Omega^*}, \overset{+}{T}\right]$. Second, for a given Ω^*, $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$ is independent of the post-collapse money growth rate μ: the devaluation rate at T is the same whether seigniorage is raised continuously ($\mu > 0$), implying a post-collapse steady state characterized by constant inflation and exchange rate depreciation, or money supply increases in period T only ($\mu = 0$).

To assess the fiscal costs of defending the fixed exchange rate, observe that, in a discrete-time

[12] The expression for $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$ is:

$$\frac{\mathcal{E}_T}{\overline{\mathcal{E}}} = \frac{(1+r)\,\frac{\overline{M}}{\overline{\mathcal{E}}} - (1+r)^{T-t}\,\Omega^*}{(1+r)\,\frac{\overline{M}}{\overline{\mathcal{E}}} - (1+r)^{T+1-t}\,\Omega^*}$$

The exchange rate at T *depreciates* (i.e. $\mathcal{E}_T/\overline{\mathcal{E}} > 1$) if and only if: $\left(\overline{M}/\overline{\mathcal{E}}\right) - \Omega^*\,(1+r)^{T-t} > 0$. If this condition failed, raising the target seigniorage revenue $\Omega_{t,T} = \Omega^*$ would only be possible via a sharp *appreciation* of the exchange rate at T — so sharp that the implied nominal interest rate at $T-1$, i_T, would have to be negative.

model with forward-looking agents, a speculative attack precedes the peg collapse by one period. At time $T-1$, money demand declines and the interest rate increases in anticipation of currency depreciation at T. In the first appendix we show [equation (32)] that the fall in money demand at $T-1$ can be written as:

$$\frac{\overline{M}-M_{T-1}}{\overline{\mathcal{E}}}=\left[\frac{\mathcal{E}_T/\overline{\mathcal{E}}-1}{(1+r)\,\mathcal{E}_T/\overline{\mathcal{E}}-1}\right]\frac{\overline{M}}{\overline{\mathcal{E}}} \tag{13}$$

The partial derivative of the above expression with respect to $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$ has a positive sign.

The contraction in money preceding the devaluation is negative seigniorage: by reducing foreign reserves, the stock of treasury liabilities at the central bank, or both, it increases the interest burden on the government. The fiscal cost of the peg defense, $\Omega_{t,T-1}$, is therefore equal to the present value of the revenue loss associated with the fall in money demand at time $T-1$:

$$\Omega_{t,T-1}=-\left(\frac{1}{1+r}\right)^{T-t-1}\frac{\overline{M}-M_{T-1}}{\overline{\mathcal{E}}}$$

Combining the above expression with (12) and (13) yields this section's main result: if $T>t$, then $\Omega_t=\Omega_{t,T-1}+\Omega_{t,T}=-\Omega^*+\Omega^*=0$. *If, for $\Delta>0$, the government attempts to delay a devaluation, the seigniorage raised after the collapse is exactly equal in present value to the fiscal costs of defending the peg before the collapse, so that the overall present value of seigniorage is zero.* Note that an attempt by the government to increase the collection of seigniorage at and after T will be self-defeating. The larger the target seigniorage revenue after the currency collapse, the larger the equilibrium depreciation of the exchange rate, and the more severe the speculative attack preceding the collapse.

The key lesson from our analysis is that financing the imbalance via seigniorage and the goal of delaying devaluation may be inconsistent with each other. A government that wants to buy time before allowing a necessary exchange rate adjustment limits its ability to raise fiscal revenue through money creation.

3.2 Devaluation without speculative attacks

While our result $\Omega_t = 0$ is independent of the maturity structure of government debt, its implication for price and exchange rate instability is not. In the model with short-term nominal debt and indexed tax and transfers presented in Section 2, the government budget constraint (9) clearly fails to hold with $\mathcal{E}_t = \overline{\mathcal{E}}$. Thus, upon the arrival of the adverse fiscal news, the government cannot defend the exchange rate parity, not even for one period. If an equilibrium in our economy exists with $\Delta > 0$, it must be the case that the exchange rate jumps up immediately. This will not be the case in the model with long-run liabilities, introduced in Section 5 below.

We analyze the exchange rate collapse highlighting two equilibrium relationships between the size of the devaluation in period t and the present discounted value of seigniorage Ω_t. The *first relationship* follows from the intertemporal government budget constraint, and includes the size of the fiscal imbalance as well as the amount of outstanding nominal government bonds. Subtracting (9) from (8) yields:[13]

$$\frac{\mathcal{E}_t}{\overline{\mathcal{E}}} = \frac{(1+i_t)\, B_t}{(1+i_t)\, B_t - (\Delta - \Omega_t)\, \overline{\mathcal{E}}} \tag{14}$$

The government intertemporal constraint induces a *negative* relationship between $\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$ and Ω_t. The logic of this result is reminiscent of the FTPL. The fiscal imbalance is inconsistent with equilibrium at $\overline{\mathcal{E}}$. The increase in the present value of transfers causes individuals to believe that their budget sets have expanded, and generates an upward pressure on the exchange rate. The resulting jump in the exchange rate in period t reduces the real value of the outstanding government nominal liabilities: an unanticipated devaluation lowers the *ex-post* real rate of return on public debt below its *ex-ante* value. Thus, the exchange rate adjustment induces a wealth transfer from the private to the public sector, offsetting the initial fiscal imbalance. Holding Δ and B_t constant, the larger the present value of seigniorage Ω_t, the smaller the wealth transfer via an immediate and unanticipated devaluation required for equilibrium. Hence the *negative* relationship between

[13] In our specification, consumption does not respond to the kind of fiscal shock we analyze.

$\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$ and Ω_t. By the same logic, given Δ and Ω_t, a larger stock of government liabilities B_t implies a smaller equilibrium jump in $\mathcal{E}_t$.

The *second relationship* between $\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$ and Ω_t is derived from money demand, uncovered interest parity and the definition of seigniorage. Using the equilibrium present value of seigniorage at $T = t$ [expression (30) in the appendix], we obtain:

$$\frac{\mathcal{E}_t}{\overline{\mathcal{E}}} = \frac{\overline{M}}{\overline{M} - \Omega_t \overline{\mathcal{E}}} \tag{15}$$

which establishes a *positive* relationship between Ω_t and $\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$ — consistent with the traditional monetary view of the exchange rate (and the price level). Observe that this relationship is independent of the growth rate of money μ.[14]

Combining equations (14) and (15), we see that the size of the initial equilibrium jump in the exchange rate is determined by Δ, and the nominal value of total government nominal liabilities, including both debt and money:

$$\frac{\mathcal{E}_t}{\overline{\mathcal{E}}} = \frac{(1 + i_t)\, B_t + \overline{M}}{(1 + i_t)\, B_t + \overline{M} - \overline{\mathcal{E}}\Delta} \tag{16}$$

The exchange rate is uniquely defined and, with $\Delta > 0$, always depreciates on impact.[15] The equilibrium devaluation at t is large if the outstanding interest-bearing and monetary liabilities of the government B_t and $\overline{M}$ are small, and/or the fiscal imbalance Δ is large.

Notably, the equilibrium present value of seigniorage is determined residually: Ω_t is increasing in Δ and $\overline{M}$, and decreasing in B_t. Following a decline in the present value of primary surpluses, the central bank expands money supply to generate the amount of seigniorage consistent with equilibrium. As B_t goes to zero, the present discounted value of seigniorage becomes as high as Δ.

[14] If $\mu = 0$, there is no continuous need for seigniorage generating inflation in the post-shock equilibrium. Then equations (7) and (3) imply that, in period t, money increases in proportion to the exchange rate (price level): $(M_t/\bar{M} = \mathcal{E}_t/\overline{\mathcal{E}})$. If $\mu > 0$, the post-shock equilibrium is characterized by a sustained rate of inflation, and the present value of seigniorage can be written as: $\Omega_t = (\overline{M}/\mathcal{E}_t)\mu(1 + r)/r$. Then we substitute expression (40) from the appendix for μ.

[15] Yet the fiscal imbalance cannot be too large, so as to exceed the combined stock of government liabilities, i.e. it cannot be that $\overline{\mathcal{E}}\Delta > (1 + i_t)\, B_t + \overline{M}$. If this were the case, our $\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$ equation would yield an economically meaningless solution of a negative exchange rate.

The first relationship (14) becomes irrelevant, and the equilibrium exchange rate is determined exclusively by (15) setting $\Omega_t = \Delta$. With a positive B, we have: $\Delta > \Omega_t > 0$.

We display the equilibrium solution and its properties in Figure 1, where we graph equation (14) as the government budget constraint GBC curve and equation (15) as the money market MM curve. The GBC curve is vertical at $\Omega_t = \Delta$ if B_t is zero.

Figure 1 here.

4 The dynamics of speculative attacks

We have seen that, in the economy with short-term nominal public debt, the equilibrium involves an *unanticipated* increase in the price level (the exchange rate) that reduces the real value of public debt. The logic of our argument suggests that the government may be able to delay an exchange rate depreciation if *anticipated* devaluation and inflation produce fiscal benefits independent of seigniorage. Indeed, this section shows that, provided the outstanding stock of long-term nominal public debt is sufficiently large, a devaluation can be postponed even if net seigniorage is zero. The intuition for this result is straightforward. An imbalance in the government budgets that cannot be matched by seigniorage revenues requires an offsetting wealth transfer from the private to the public sector. With non-indexed bonds of long maturities, a wealth transfer can occur via an unanticipated jump in bond prices in period t.

In what follows, we augment our model to include long-term non-indexed public debt (for simplicity, we consider perpetuities), and characterize the equilibrium devaluation rate using, as before, the money market equilibrium and the government budget constraint.[16] We then derive the timing of the delayed devaluation when the central bank follows a policy rule such that the interest rate is kept below a certain threshold, and discuss the possibility of indeterminacy.

[16] While we consider perpetuities for simplicity, our argument clearly applies to the case of a large stock of nominal public debt of sufficiently long maturity.

4.1 A model with both short- and long-term debt

Since the model with long-term debt is almost identical to what we discussed in section 2, we display only the key characteristics in the main text, and complete the model in our second appendix. In addition to M, B^*, and B, the individual may now hold L, a perpetuity issued by the domestic government. The perpetuity pays one unit of domestic currency forever, so that the price of the perpetuity in units of domestic currency at time t, denoted by θ_t, is:

$$\theta_t = \left(\frac{1}{1+i_{t+1}}\right) + \left(\frac{1}{1+i_{t+1}}\right)\left(\frac{1}{1+i_{t+2}}\right) + ... \tag{17}$$

We note a useful relationship:

$$\theta_t \left(1 + i_{t+1}\right) = 1 + \theta_{t+1} \tag{18}$$

The flow budget constraint of the government is now:

$$\left(B_{t+1} - B_t\right) + \theta_t \left(L_{t+1} - L_t\right) - \mathcal{E}_t \left(R_{t+1} - R_t\right) + \left(M_t - M_{t-1}\right) = i_t B_t + L_t - r\mathcal{E}_t R_t - \mathcal{E}_t \eta_t \tag{19}$$

In analogy to the case of short-term debt, we solve (19) forward to obtain the intertemporal budget constraint of the government:

$$\frac{\left(1+\theta_t\right) L_t}{\mathcal{E}_t} + \frac{\left(1+i_t\right) B_t}{\mathcal{E}_t} - \left(1+r\right) R_t = \sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s \left(\eta_{t+s} + \frac{M_{t+s} - M_{t-1+s}}{\mathcal{E}_{t+s}}\right) \tag{20}$$

where i_t, L_t, B_t, and R_t are predetermined in period $t-1$, while both $\mathcal{E}_t$ and θ_t can jump upon the arrival of the news about the fiscal imbalance.

As before, we assume that fiscal policy is initially consistent with the exchange rate being permanently fixed at $\overline{\mathcal{E}}$: at any time t, the exogenously set time path of real primary surpluses is such that the government's intertemporal budget constraint holds given L_t, B_t, and $\overline{\mathcal{E}}$:

$$\frac{\left[1 + (1/r)\right] L_t}{\overline{\mathcal{E}}} + \frac{\left(1+i_t\right) B_t}{\overline{\mathcal{E}}} - \left(1+r\right) R_t = \sum_{s=0}^{\infty} \left(\frac{1}{1+r}\right)^s \overline{\eta}_{t+s} \tag{21}$$

Note that, in the above expression, we substitute $(1/r)$ for the price of the perpetuity. The policy change is, as before, that in period t individuals learn of a decline in the present value of real primary surpluses by $\Delta > 0$.

4.2 Fiscal and monetary determinants of a delayed devaluation

Given a decline in the present value of real primary surpluses by $\Delta > 0$, fiscal policy is no longer consistent with the exchange rate being permanently fixed at $\overline{\mathcal{E}}$. A comparison of equations (21) and (20) shows that, for an equilibrium to exist, the news about Δ at time t must be immediately followed by an increase in $\mathcal{E}_t$ and/or by a decline in θ_t. The fiscal policy shock makes the peg unsustainable: either the peg collapses immediately, or a devaluation occurs in the future, implying an immediate decline in the price of the perpetuity.

Now, suppose that the exchange rate remains at $\overline{\mathcal{E}}$ in period t and depreciates at some $T > t$. We can then write the government budget constraint *after* the policy change as follows:

$$\frac{(1+\theta_t)\,L_t}{\overline{\mathcal{E}}} + \frac{(1+i_t)\,B_t}{\overline{\mathcal{E}}} - (1+r)\,R_t = \left[\sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s}\bar{\eta}_{t+s}\right] - (\Delta - \Omega_t) \tag{22}$$

Subtracting (22) from (21), and recalling that our result $\Omega_t = 0$ if $T > t$ does not depend on the maturity structure of government debt, we obtain an expression for the equilibrium perpetuity price:

$$\theta_t = \frac{1}{r} - \Delta\frac{\overline{\mathcal{E}}}{L_t} \tag{23}$$

Importantly, since θ_t has a lower bound of zero, a fixed exchange rate and an immediate decline in the perpetuity price are an equilibrium only if the outstanding stock of perpetuities L_t is sufficiently large.[17] Otherwise, the government intertemporal constraint requires an immediate collapse of the peg in equilibrium — an outcome we analyze in the third appendix.

Consider next the money market equilibrium. Using identity (18) and the interest rate parity, we derive a positive relationship between θ_t and θ_{T-1}:

$$\theta_t = \frac{1}{r} + \left(\frac{1}{1+r}\right)^{T-t-1}\left(\theta_{T-1} - \frac{1}{r}\right) \tag{24}$$

[17] Specifically, for $\theta_t > 0$, it must be that $L_t > r\overline{\mathcal{E}}\Delta$.

and conclude that:

$$\theta_{T-1} = \frac{1+\theta_T}{(1+r)\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)} \tag{25}$$

where $\theta_T = (1/i_{T+1})$ and $(1+i_{T+1}) = (1+r)(1+\mu)$. Combining (25) and (24), we obtain a monotonically decreasing relationship between θ_t and $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$.

Figure 2 here.

It is instructive to consider a graphical solution of the model. Figure 2 displays equation (23) as a horizontal line *GBC*, government budget constraint, as well as equations (24) and (25) as a downward sloping curve *MM*, money market.[18] The intersection of *GBC* and *MM* curves depicts the equilibrium values of θ_t and $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$, conditional on Δ, T, L_t, and μ. With our graphical apparatus, it is easy to see the intuitive properties of the equilibrium. Other things equal, a larger Δ requires a larger jump in the bond price and a larger devaluation rate. A larger L_t is associated with a smaller jump in the bond price and a smaller devaluation rate. Delaying the collapse further into the future (i.e. an increase in T) raises the devaluation rate. Notably, with long-term debt the equilibrium devaluation rate is no longer independent of the post-collapse money growth rate μ, since the time profile of inflation affects the market value of public debt. Other things equal, a larger μ corresponds to a smaller equilibrium devaluation rate.[19]

Setting $\mu = 0$, we have a simple algebraic expression for the equilibrium devaluation rate. The intersection of *GBC* and *MM* curves corresponds to (details are in the second appendix):

$$\frac{\mathcal{E}_T}{\overline{\mathcal{E}}}\left[\overset{+}{\Delta},\overset{+}{T}\right] = \frac{L_t}{L_t - \overline{\mathcal{E}}\,(1+r)^{T-t-1}\,\Delta r} \tag{26}$$

[18] >From the government budget constraint, Δ and L_t uniquely determine the equilibrium perpetuity price θ_t: an increase in Δ shifts the *GBC* locus down, since the bond price must fall further if the fiscal imbalance is larger; an increase in L_t instead shifts the locus up, since more perpetuities imply that a smaller decline in the bond price is needed to re-establish equilibrium. The *MM* curve slopes down since, fixing the time of the collapse and the post-collapse growth rate of money, a larger devaluation rate implies a higher interest rate in period $T-1$, and therefore a smaller bond price in period t. For a given devaluation size, a larger T shifts the curve up: a devaluation and an interest rate increase further into the future correspond to a higher perpetuity price at time t. Given $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$ and T, a higher money growth rate shifts the curve down, since θ_{T-1} is decreasing in μ.

[19] Since θ_t is determined by L_t and Δ, and θ_{T-1} is higher with smaller μ, the devaluation must be higher with a smaller μ so as to generate a decline in θ_{T-1} compatible with θ_t.

where we assume that Δ is no larger than the maximum fiscal gain from the anticipated future peg collapse:

$$\Delta < \frac{L_t}{\overline{\mathcal{E}}(1+r)^{T-t}} \left(\frac{1+r}{r} \right) \tag{27}$$

For this condition to hold, given Δ and L_t, a currency collapse must occur before the passage of time changes the sign of the above inequality. It is straightforward to show that a currency crisis must occur before $\widehat{T}$ such that:

$$\widehat{T} \cong t + \frac{\log \left[\frac{L_t}{\overline{\mathcal{E}}} \left(\frac{1+r}{r} \right) \right] - \log \Delta}{r} \tag{28}$$

Given L_t, Δ, and μ, there exists a finite upper bound, $\widehat{T}$, to the date of the peg collapse. A currency crisis occurring at a date close to this upper bound corresponds to an extreme scenario in which, while the exchange rate is pegged, the government borrows up to the point in which the maximum fiscal gain from inflation is just enough to guarantee solvency. Evaluating expression (26) as $T \to \widehat{T}$, we find that such a solvency-driven crisis is associated with extreme rates of devaluation and inflation.[20]

Note the properties of the equilibrium devaluation rate $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$. Holding (27), equation (26) establishes that the devaluation rate at T is increasing in the size of the fiscal imbalance Δ, as well as in the time span between the adverse fiscal news and the collapse $(T-t)$. It is instead decreasing in the nominal value of long-term bonds L_t. Again, these properties are reminiscent of the FTPL.[21]

4.3 Monetary policy and the timing of currency crises

While the behavior of the fiscal authority makes a devaluation inevitable, it fails to pin down the date of the collapse — the crisis may occur in any period between t and $\widehat{T}$. We now show

[20] If the government attempted to delay the devaluation past $\widehat{T}$, its budget contraint would fail to hold with $\mathcal{E}_t = \overline{\mathcal{E}}$, and the peg would collapse in period t.

[21] After Flood and Garber (1984), the exchange rate conditional on abandoning the peg, given by equation (26), is often referred to as the "shadow exchange rate". The shadow rate could be calculated for all dates $s \geq t$ and would share the properties of $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$ discussed here.

how completing the model with a policy rule of the monetary authority uniquely determines the timing.

Suppose that the central bank is willing to defend the current exchange rate parity so long as the nominal interest rate remains below some threshold $\bar{i}$ — the exchange rate target is matched by an asymmetric target zone for the nominal interest rate. Such a policy rule can be motivated in terms of economic costs implied by very high interest rates. We do not, however, model these costs explicitly in our framework.[22] We stress that, in addition to being a realistic way to describe the conduct of the central bank, our policy rule is closely related to the rule in Krugman (1979). Note that, using the money demand equation (7), an upper bound on the interest rate translates into a lower bound on money supply. We can therefore restate our interest rate rule as saying that the monetary authorities are willing to defend the exchange rate so long as money supply does not fall below some lower bound. As is well known, the Krugman model posits an exogenous rate of domestic credit expansion coupled with a lower bound on reserves. If one abstracts from the possibility of borrowing to defend the exchange rate – as Krugman does – the combination of these two assumptions *is* a lower bound on money supply, implying an upper bound on the interest rate.[23]

We now define the condition for the timing of a speculative attack triggering an exchange rate crisis. The government will abandon the peg in the first period $T \geq t$ such that keeping the exchange rate fixed would cause the interest rate to rise to or above $\bar{i}$ at T. The period $T-1$ is then the last period in which the interest rate is below the policy-specified threshold:

$$T = \max\ \tau \quad \text{such that} \quad i_\tau < \bar{i} \tag{29}$$

With $\bar{i}$ fixed exogenously and known to all agents, the perfect foresight equilibrium is unique. To see this, note first that the interest rate parity (3) implies that i_τ is monotonically increasing in

[22] For simplicity, we assume that the upper bound on the interest rate is time invariant. Extending our analysis to the case with time variation in the policy rule would not affect our conclusions.

[23] See Cavallari and Corsetti (in press).

$\left(\mathcal{E}_\tau/\overline{\mathcal{E}}\right)$. Second, recall from (26) that the equilibrium devaluation rate $\left(\mathcal{E}_\tau/\overline{\mathcal{E}}\right)$ is also monotonically increasing in calendar time, τ. Thus, i_τ is monotonically increasing in time τ. Individuals have no incentive to attack the peg "too early", at some $\tau^* < T-1$. Since an "early" attack is associated with a relatively small contraction in money demand, the government does not abandon the peg in the following period, the opportunity cost of real balances at τ^* is given by r, and there is no incentive for portfolio reallocation. Similarly, agents will not wait "too long", and launch a speculative attack at some τ^{**} for which condition (29) fails to hold. As i_τ increases over time, we know that a speculative attack at τ^{**}will drive the interest rate to or above $\bar{\imath}$, forcing the government to abandon the peg immediately. But, in a perfect foresight equilibrium, a collapse at τ^{**} implies that the interest rate already rises above $\bar{\imath}$ at time $\tau^{**}-1$, in anticipation of the crisis. A speculative contraction of money demand will therefore occur before τ^{**}. Since to ensure solvency the collapse must occur before $\widehat{T}$, by backward induction rational agents calculate the unique timing of a speculative attack, as the last period in which such an attack is resisted by the authorities.

The unique timing of a collapse is increasing in the upper bound on the interest rate $\bar{\imath}$. Other things equal, a central bank willing to let the interest rate grow higher can defend longer the current parity. It is noteworthy that a larger post-collapse money growth rate μ implies a further delay of the devaluation. This result follows from our earlier finding that the equilibrium devaluation rate is decreasing in μ.

We summarize our analysis in Figure 3, where we assume that the rate of growth of money in the new regime after the peg collapse is zero ($\mu = 0$). The top half shows the time plot of the exchange rate $\mathcal{E}_s$ and the equilibrium exchange rate conditional on the abandonment of a peg, denoted by $\widetilde{\mathcal{E}}_s$ — this is the so-called shadow exchange rate. The bottom half shows the time plot of the interest rate threshold of the central bank $\bar{\imath}$, the current interest rate and the equilibrium interest rate conditional on a devaluation one period ahead, denoted by $\widetilde{\imath}$ — the shadow interest rate. Note that, when the adverse fiscal news arrives at t, the shadow exchange rate immediately depreciates,[24] then keeps depreciating until time T, when it coincides with the actual exchange rate. The shadow interest rate also jumps up at t, but not enough to pass the central bank threshold $\bar{i}$. The shadow interest rate is still below $\bar{i}$ at $T-1$, but is above $\bar{i}$ at T. The realized interest rate coincides with its shadow value only at time $T-1$.

Figure 3 here

4.4 Liquidity and indeterminacy

An important feature of our analysis so far is that, before $\widehat{T}$, the authorities can fight speculative attacks counting on a large stock of borrowed reserves — as large as is consistent with the government solvency constraint.[25] Thus the size of gross reserves at the central bank is at no time a key indicator of the government's ability to withstand a speculative attack.

This is no longer true when credit constraints limit the ability of a solvent government to borrow to defend the exchange rate peg. For instance, there could exist policy rules and/or laws imposing upper bounds on debt and deficits. Alternatively, a government attempting to delay a devaluation may face a borrowing constraint as a result of a coordination problem among public debt holders. In what follows, we briefly discuss the latter case.

Consider a set of assumptions that are typical in models of crises allowing for coordination problems. Suppose that, in the economy described above, there is no single large investor (each investor holds a small amount of government debt), and short sales are not allowed (no single investor can sell an arbitrarily large amount of public debt). In each period, individuals decide whether to roll over their short-term loans to the government, or to exchange them for foreign currency. If sales of short-term public debt exceed the amount of international reserves at the

[24] Interestingly, the shadow exchange rate appreciates on impact if the perpetuity price $\theta_t < (1/r) - \left(\Delta\overline{\mathcal{E}}/L_t\right)$. When a delayed collapse is possible, $(1/r) - \left(\Delta\overline{\mathcal{E}}/L_t\right) > 0$, so by choosing a large μ the central bank can generate a capital loss to holders of perpetuities that is more than enough to finance the imbalance.

[25] Our analysis assumes that domestic and foreign bonds yield the same return. If the supply of reserves were upward sloping, as in Buiter (1987), $\widehat{T}$ would depend on the net reserve position of the government.

central bank, the currency immediately devalues. At the same time, agents selling their holdings of debt obtain some small fraction of reserves at the pre-devaluation parity $\overline{\mathcal{E}}$. This assumption is an alternative to the 'sequential service constraint' in models of bank runs such as Diamond and Dybvig (1983): when deciding not to roll over their one-period domestic currency loans to the government B, agents can always obtain some foreign reserves from the central bank at the pre-devaluation exchange rate.[26]

Under these conditions, in the presence of a fiscal imbalance, a run on public debt can force a devaluation as soon as the stock of short-run nominal liabilities of the government is at least as large as the stock of reserves, evaluated at the ongoing parity, i.e. $B_\tau \geq R_\tau \overline{\mathcal{E}}$. To see why, suppose that, at the beginning of each period $\tau \geq t$, an individual agent does not anticipate a run on public debt during the period. Then, she has no incentive not to roll over her loans to the government. When all agents focus on this expectations regime, no one will sell debt and the government will be able to keep the peg: the expectation of peg survival during the period is self-fulfilling. Conversely, suppose that an individual agent anticipates that all other investors will stop rolling over government debt. Then the agent's best response is to run as well, in anticipation of obtaining with positive probability some reserves from the central bank at the ongoing parity $\overline{\mathcal{E}}$. Given a run on its debt, without sufficient reserves to back up its short-term liabilities, the government cannot but abandon immediately the fixed exchange rate.

The equilibrium rate of devaluation can be calculated analogously to (45) in the third appendix – with the difference that now Δ will be augmented by the fiscal cost of the reserves sold to the public at $\overline{\mathcal{E}}$ during the crisis. Note that the loss of reserves at the central bank is not necessarily associated with a speculative contraction of money demand. At the time of devaluation, money adjusts consistent with the post-collapse regime.

By means of a simple amendment to our model, we have shown that a liquidity crisis may suddenly frustrate the government's attempt to delay a devaluation. An unpredictable collapse of

[26] See the discussion in Allen and Gale (2000, p. 277).

the exchange rate is the consequence of a switch in the coordination of private agents' expectations across possible equilibria. The run on short-term public debt may or may not exhaust the stock of reserves at the central bank. What is key is that the run *prevents the government from borrowing additional reserves for peg defense.* While the fiscal imbalance makes a collapse ultimately inevitable at $\widehat{T}$, the exact timing is subject to indeterminacy, and is therefore unpredictable.[27]

5 Conclusions

This paper presents a model to study a currency crisis associated with a fiscal imbalance. In the spirit of the fiscal theory of the price level (FTPL), we emphasize nominal public liabilities and view fiscal and monetary policies symmetrically as determinants of the price level and the exchange rate. We study the equilibrium implications of the fiscal costs of defending the peg. Finally, we highlight the potential for coordination problems and indeterminacy in the timing of the crisis.

The FTPL bears similarities to and has lessons for the first-generation models of currency crises. A decline in the real value of nominal public liabilities is shown to be a key adjustment mechanism to a fiscal imbalance. The equilibrium devaluation rate is determined by the magnitude of the fiscal imbalance, the size and maturity of nominal public liabilities, and the time delay between the arrival of the adverse fiscal news and the peg collapse. The timing of the delayed devaluation is pinned down by the monetary authority, provided that the government has full access to international financial markets.

In our specification, fiscal costs of defending an exchange rate peg exactly offset post-devaluation seigniorage revenue. Although this precise result may not hold in other model specifications, the costs of peg defense will still constrain feasible seigniorage policies, increasing endogenously the revenue need of the government. Our analysis points to these costs and their equilibrium implications for policy and exchange rate stability as important topics for future research.

[27] Refining the analysis of coordination problems among investors in speculative attacks remains an important area of research. See Morris and Shin (1998) and Corsetti, Morris, and Shin (1999).

A Appendix

This appendix derives equation (12) for $\Omega_{t,T}$, the present value of seigniorage raised at and after T, discounted to period t. To start with, we use the monetary rule (11) to substitute into the definition of the present value of seigniorage (10):

$$\Omega_{t,T} \equiv \left(\frac{1}{1+r}\right)^{T-t}\left[\left(\frac{M_T - M_{T-1}}{\mathcal{E}_T}\right) + \left(\frac{M_T}{\mathcal{E}_T}\right)\left(\frac{\mu}{1+\mu}\right)\left(\frac{1}{r}\right)\right] \tag{30}$$

We then derive an expression for M_{T-1} in terms of $\overline{\mathcal{E}}, \mathcal{E}_T$,and $\overline{M}$. Note that at $T-1$ the interest rate parity (3) implies that: $1 + i_T = (1+r)\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$. Since the exchange rate remains fixed in period $T-1$ and the jump in $\mathcal{E}_T$ is anticipated, M_{T-1} must decline as individuals adjust down their desired real balances.

Since the money demand equation (7) and the interest rate parity (3) imply that:

$$\left(\frac{r}{1+r}\right)\frac{\overline{M}}{\overline{\mathcal{E}}} = \left(\frac{i_T}{1+i_T}\right)\frac{M_{T-1}}{\overline{\mathcal{E}}}, \tag{31}$$

we can write the following expression for the fall in money demand:

$$\frac{\overline{M} - M_{T-1}}{\overline{\mathcal{E}}} = \left[1 - \left(\frac{r}{1+r}\right)\left(\frac{1+i_T}{i_T}\right)\right]\frac{\overline{M}}{\overline{\mathcal{E}}} = \frac{1}{i_T}\left(\frac{\mathcal{E}_T - \overline{\mathcal{E}}}{\overline{\mathcal{E}}}\right)\frac{\overline{M}}{\overline{\mathcal{E}}} \tag{32}$$

from which we derive the equation (13) in the main text. Also, multiplying both sides of the above equation by $\overline{\mathcal{E}}/\mathcal{E}_T$, we obtain:

$$\frac{M_{T-1}}{\mathcal{E}_T} = \frac{\overline{M}r}{(1+r)\,\mathcal{E}_T - \overline{\mathcal{E}}} \tag{33}$$

The next step is to substitute (33) into (30). We consider separately the case where $\mu = 0$ and the case where $\mu > 0$. In the case where $\mu = 0$, equation (30) becomes:

$$\Omega_{t,T} = \left(\frac{1}{1+r}\right)^{T-t}\left(\frac{M_T - M_{T-1}}{\mathcal{E}_T}\right) \tag{34}$$

The money demand equation (7) and the interest rate parity (3) imply that:

$$\frac{M_T}{\mathcal{E}_T} = \frac{\overline{M}}{\overline{\mathcal{E}}} \tag{35}$$

We substitute (33) and (35) into (34) to obtain:

$$\Omega_{t,T} = \left(\frac{1}{1+r}\right)^{T-t}\left[\frac{\overline{M}}{\overline{\mathcal{E}}} - \frac{\overline{M}r}{(1+r)\,\mathcal{E}_T - \overline{\mathcal{E}}}\right] \tag{36}$$

which is the solution for the present value of seigniorage raised at and after T and equation (12) in the main text.

For the case where $\mu > 0$, given that $M_T = \overline{M}\,(1+\mu)$, we can write equation (30) as:

$$\Omega_{t,T} = \left(\frac{1}{1+r}\right)^{T-t}\left[\frac{\overline{M}\,(1+\mu)}{\mathcal{E}_T} - \frac{M_{T-1}}{\mathcal{E}_T} + \left(\frac{\overline{M}\mu}{\mathcal{E}_T r}\right)\right] \tag{37}$$

In order to proceed further, we derive a relationship between μ and $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$. We note that the interest rate parity condition (3) implies that:

$$1 + i_{T+1} = (1+r)\,(1+\mu) \tag{38}$$

while the money demand equation (7) can be used to write:

$$\left(\frac{r}{1+r}\right)\frac{\overline{M}}{\overline{\mathcal{E}}} = \left(\frac{i_{T+1}}{1+i_{T+1}}\right)\frac{\overline{M}\,(1+\mu)}{\mathcal{E}_T} \tag{39}$$

The above two relationships, (38) and (39), can be combined to derive:

$$1 + \mu = \frac{\mathcal{E}_T r}{\overline{\mathcal{E}}\,(1+r)} + \frac{1}{1+r} \tag{40}$$

We then make use of (40) to substitute for μ in (37):

$$\Omega_{t,T} = \left(\frac{1}{1+r}\right)^{T-t}\left[\frac{\overline{M}r}{\overline{\mathcal{E}}\,(1+r)} + \frac{\overline{M}}{\mathcal{E}_T\,(1+r)} - \frac{M_{T-1}}{\mathcal{E}_T} + \frac{\overline{M}}{\overline{\mathcal{E}}\,(1+r)} + \frac{\overline{M}}{\mathcal{E}_T r\,(1+r)} - \frac{\overline{M}}{\mathcal{E}_T r}\right] \tag{41}$$

and finally substitute expression (33) for $(M_{T-1}/\mathcal{E}_T)$, simplify, and arrive at:

$$\Omega_{t,T} = \left(\frac{1}{1+r}\right)^{T-t}\left[\frac{\overline{M}}{\overline{\mathcal{E}}} - \frac{\overline{M}r}{(1+r)\,\mathcal{E}_T - \overline{\mathcal{E}}}\right] \tag{42}$$

which is identical to equation (36) that we obtained above for the case where $\mu = 0$.

B Appendix

This appendix completes the model with long-term debt. As before, the period utility function of the representative individual is given by (1). In addition to M, B^*, and B, the individual may now hold L, a perpetuity issued by the domestic government. The perpetuity pays one unit of domestic currency forever and its price in units of domestic currency at time t is denoted by θ_t. The individual's flow budget constraint is now:

$$\begin{aligned}\theta_t L_{t+1} + B_{t+1} + \mathcal{E}_t B^*_{t+1} + M_t &= (1+\theta_t) L_t + (1+i_t) B_t + \mathcal{E}_t (1+r) B^*_{t+1} + \\ & \quad M_{t-1} + \mathcal{E}_t (Y_t - C_t - \eta_t)\end{aligned} \tag{B.43}$$

The government budget constraint is discussed in the main text. The individual maximizes (1) subject to (43) and prohibitions on excessive borrowing,[28] choosing a lifetime consumption and portfolio plan $\{C_t, M_t, L_{t+1}, B_{t+1}, B^*_{t+1}\}$, while taking as given the time paths of $\{Y_t,\ \eta_t, i_{t+1}, \mathcal{E}_t, \theta_t\}$ and r. The first order conditions for the individual's problem, the money demand equation, and the optimal consumption choice are identical to the case with short-term debt. In particular, the first order condition with respect to L_{t+1} is identical to the condition for B_{t+1}, and we assume that the initial quantities of bonds are supply determined. The transversality condition is:

$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T \left(\frac{\theta_{t+T} L_{t+1+T} + B_{t+1+T} + M_{t+T}}{\mathcal{E}_{t+T}} + B^*_{t+1+T}\right) = 0$$

We now focus on the money market equilibrium, and solve for θ_{T-1} as a function of the equilibrium devaluation rate. When $\mu = 0$, the interest rate parity (3) and identity (18) imply

[28] Specifically, we have that:

$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T B^*_{t+1+T} \geq 0$$

$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T \frac{\theta_{t+T} L_{t+1+T} + B_{t+1+T} + M_{t+T}}{\mathcal{E}_{t+T}} \geq 0$$

that:

$$1+i_T = (1+r)\left(\mathcal{E}_T/\overline{\mathcal{E}}\right), \qquad 1+i_{T+1} = 1+r$$

$$\theta_T = (1/r), \qquad \theta_{T-1}(1+i_T) = 1+\theta_T$$

We deduce that $\theta_{T-1} = \left(\overline{\mathcal{E}}/r\mathcal{E}_T\right)$.

When $\mu > 0$, the interest rate parity and identity (18) imply that:

$$1+i_T = (1+r)\left(\mathcal{E}_T/\overline{\mathcal{E}}\right), \qquad 1+i_{T+1} = (1+r)(1+\mu)$$

$$\theta_T = (1/i_{T+1}), \qquad \theta_{T-1}(1+i_T) = 1+\theta_T.$$

from which we derive:

$$\theta_{T-1} = \frac{\left(\frac{\overline{\mathcal{E}}}{\mathcal{E}_T}\right)(1+\mu)}{(1+r)(1+\mu)-1}$$

Money demand, equation (7), and $M_T = \overline{M}(1+\mu)$ imply that:

$$\left(\frac{r}{1+r}\right)\frac{\overline{M}}{\overline{\mathcal{E}}} = \left(\frac{1}{(1+r)(1+\mu)\theta_T}\right)\frac{\overline{M}(1+\mu)}{\mathcal{E}_T}$$

which simplifies to $\theta_T = \left(\overline{\mathcal{E}}/r\mathcal{E}_T\right)$. After substituting $\theta_T = (1/i_{T+1})$, we arrive at:

$$1+\mu = \frac{\mathcal{E}_T r}{\overline{\mathcal{E}}(1+r)} + \frac{1}{1+r}$$

Substituting for $(1+\mu)$ in the expression for θ_{T-1} above, we obtain:

$$\theta_{T-1} = \frac{\mathcal{E}_T\overline{\mathcal{E}}r + \overline{\mathcal{E}}^2}{(1+r)\,r\mathcal{E}_T^2}$$

Thus, we have an expression for θ_{T-1} as a function of the equilibrium devaluation rate:

$$\theta_{T-1} = \begin{cases} \frac{1}{r\left(\frac{\mathcal{E}_T}{\overline{\mathcal{E}}}\right)}, & \text{if } \mu = 0 \\ \frac{1}{(1+r)\left(\frac{\mathcal{E}_T}{\overline{\mathcal{E}}}\right)} + \frac{1}{(1+r)r\left(\frac{\mathcal{E}_T}{\overline{\mathcal{E}}}\right)^2} & \text{if } \mu > 0 \end{cases}$$

Combining the above expression with (24) and (23) gives the solution for $\left(\mathcal{E}_T/\overline{\mathcal{E}}\right)$. When the central bank chooses $\mu = 0$, the solution has a simple representation given in the main text. When $\mu > 0$, the solution is a quadratic equation, with one positive and one negative root.

C Appendix

When there are not enough nominal long-term bonds outstanding when the adverse fiscal news arrives, namely when $L_t \leq r\overline{\mathcal{E}}\Delta$, equilibrium requires an immediate devaluation. Although this case is similar to the one analyzed in section 4, there is a difference that makes a brief discussion worthwhile. In analogy to (14), the government budget constraint induces a negative relationship between the equilibrium devaluation rate, $\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$, and the equilibrium present value of seigniorage, Ω_t:

$$\frac{\mathcal{E}_t}{\overline{\mathcal{E}}} = \frac{(1+i_t)\,B_t + (1+\theta_t)\,L_t}{(1+i_t)\,B_t + [1+(1/r)]\,L_t - (\Delta - \Omega_t)\,\overline{\mathcal{E}}} \tag{44}$$

where $\theta_t = (1/i_{t+1})$, and $(1+i_{t+1}) = (1+r)(1+\mu)$. The money market equilibrium implies a positive relationship between $\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$ and Ω_t given by (15). We display the equilibrium solution and its properties in Figure 4, where we graph equation (44) as the government budget constraint *GBC* curve and equation (15) as the money market *MM* curve. Combining the two equations, we confirm that the size of the jump in the exchange rate is determined by Δ, and the size of the outstanding non-monetary and monetary public liabilities:

$$\frac{\mathcal{E}_t}{\overline{\mathcal{E}}} = \frac{(1+i_t)\,B_t + (1+\theta_t)\,L_t + \overline{M}}{(1+i_t)\,B_t + [1+(1/r)]\,L_t + \overline{M} - \overline{\mathcal{E}}\Delta} \tag{45}$$

The novel feature of the immediate devaluation with long-term debt is that the post-devaluation money growth rate matters for the government intertemporal constraint, because it affects the real value of public debt. Given Δ, B_t, and L_t, a larger μ implies a larger jump in the perpetuity price θ_t and a smaller jump in the exchange rate $\left(\mathcal{E}_t/\overline{\mathcal{E}}\right)$.

Figure 4 here

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FIGURE 1: Equilibrium with short-term bonds

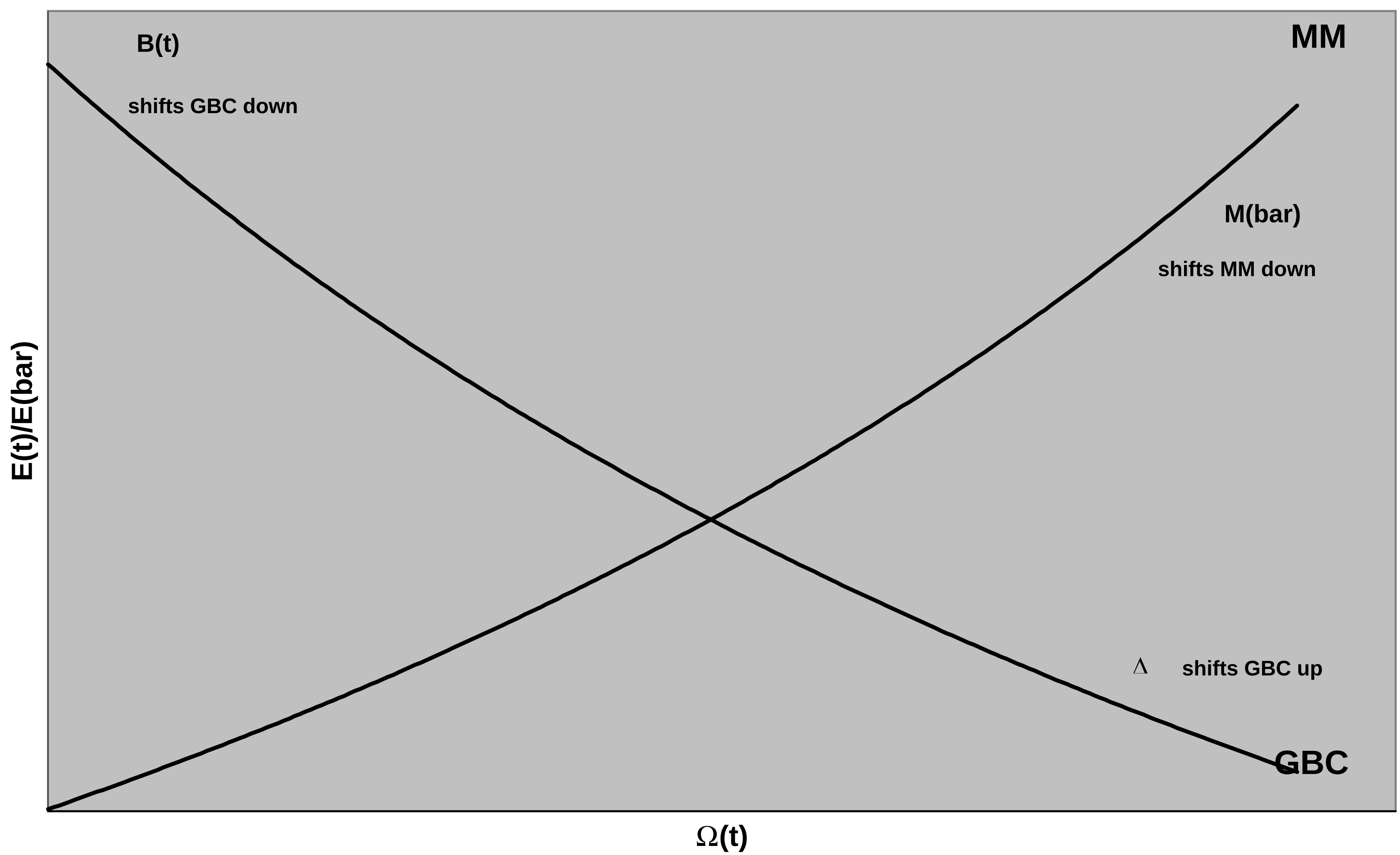
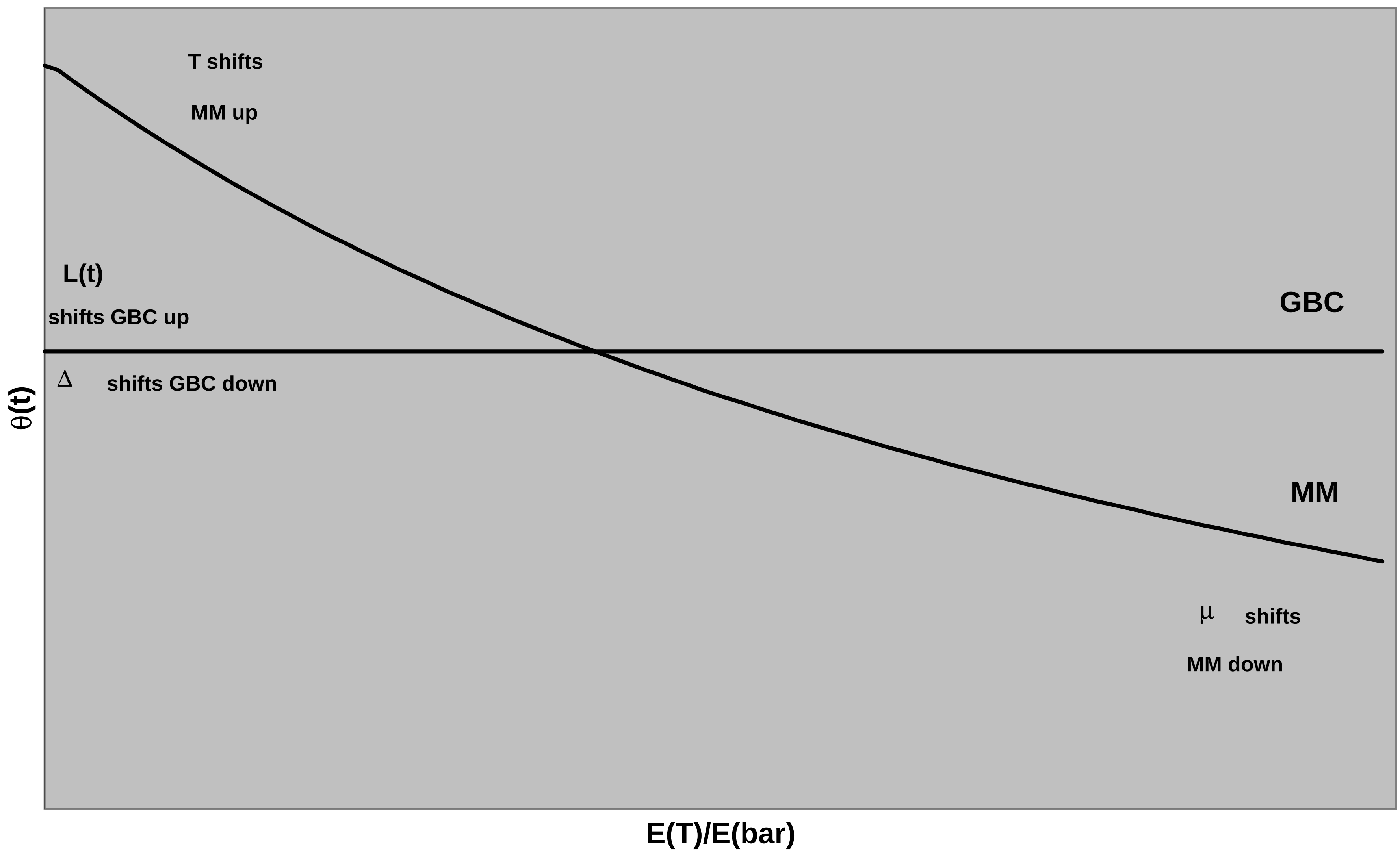


FIGURE 2: Delayed collapse with long bonds



Fg.3

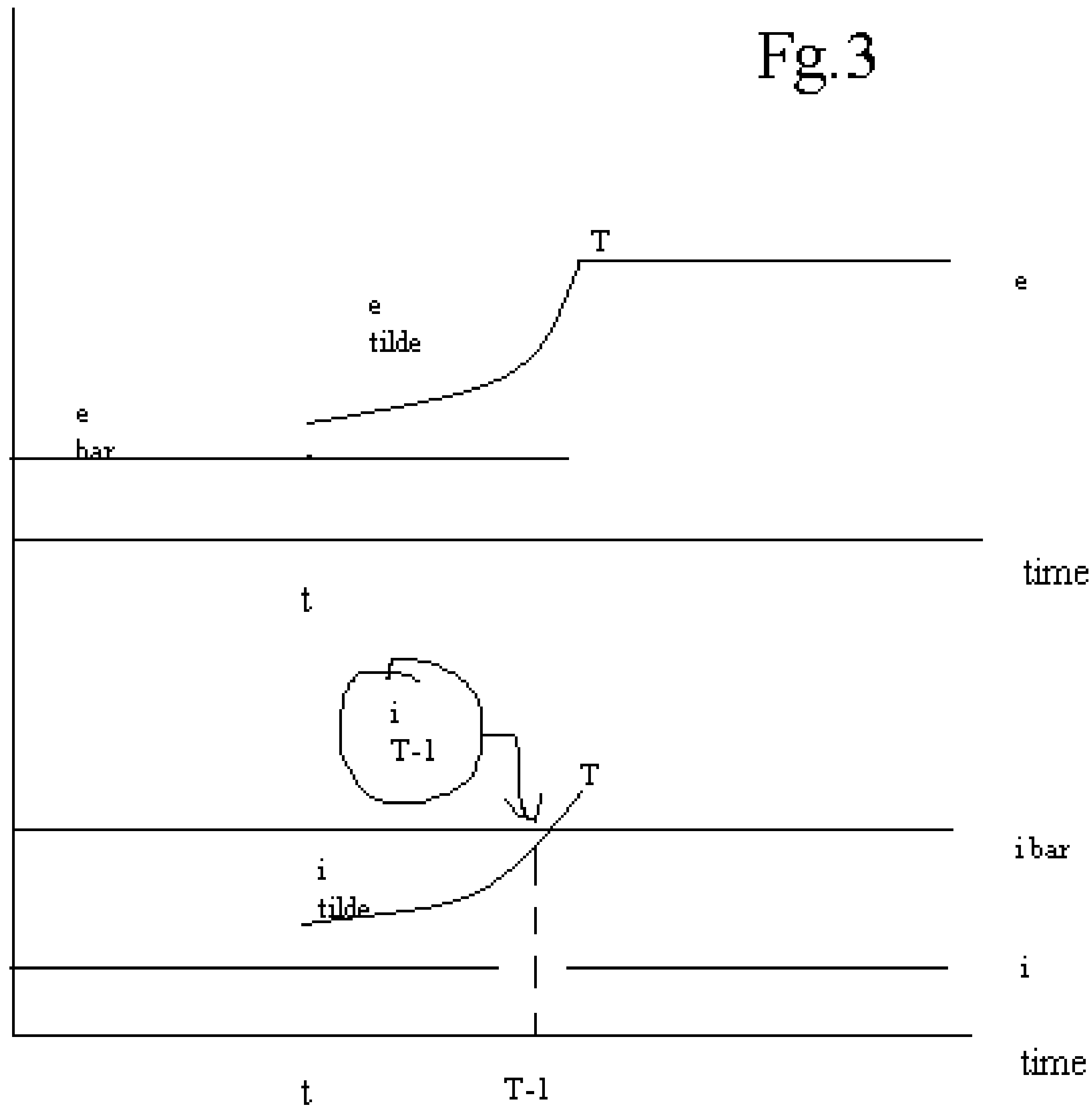


FIGURE 4: Immediate collapse with long bonds