Job Upgrading, Job Creation and Job Destruction

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Abstract

This paper proposes and develops a dynamic matching model à la Mortensen and Pissarides (1994, 1999a, 1999b) where firms respond to idiosyncratic and aggregate shocks by upgrading, creating, and destroying jobs. By allowing firms to invest in the productivity of existing jobs, the paper sheds light on: i) the impact of labor market policy on the economy's rates of job upgrading, job creation and job destruction; ii) the impact of labor market policy on the response of the job upgrading rate to aggregate shocks; and iii) the impact of training policy on labor market equilibrium outcomes.

- Key Words: Matching, Job Flows, Job Upgrading

- JEL classification: J6

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1. Introduction

When and how do firms invest in the productivity of their workforce in the presence of aggregate and idiosyncratic shocks? While the problem of the firm's investment in capital goods in the presence of aggregate and idiosyncratic shocks has received much (well-deserved) attention, the problem of the firm's investment in the productivity of its workforce in the presence of aggregate and idiosyncratic shocks has received considerably less attention. Clearly, both problems not only seem tremendously important to an understanding of aggregate productivity growth, and, concomitantly, aggregate welfare, but are also likely to be closely related, despite traditionally being treated in the literature as separate (Cooper, Haltiwanger, and Power 1999).

The firm's decision to invest in the productivity of its workforce surely is affected by the aggregate and idiosyncratic shocks, either to demand or to supply conditions, that the firm faces, and by trade-frictions (in labor markets) and labor market policy and institutions. However, there are surprisingly few studies in the literature that capture this simple real world observation. For instance, a closer look at the literature on business cycles reveals that most authors specify the evolution of productivity as exogenous.[1] Studies of certain facets of this problem, such as training, often emphasize the role of market failures and institutional arrangements (see, e.g., Acemoglu and Pischke 1996), paying no attention to the position of the firm in the cycle and to the stream of idiosyncratic shocks hitting the firms that fuel the need for reallocation of labor across productive units. At the same time, recent advances in dynamic matching models and in stochastic dynamic general equilibrium models of economic fluctuations allow for new models of the problem of the firm's investment in the productivity of its workforce (Mortensen and Pissarides 1994, 1999a, 1999b).

While the interest in such models can be motivated solely by the desire to understand the importance of trade frictions and labor market policy and institutions for aggregate productivity growth from a theoretical perspective, recent advances in data collection at the establishment level also provide increased empirical interest in these models (Davis, Haltiwanger, and Schuh 1996).

It is with this motivation in mind that this paper proposes and solves a dynamic matching model à la Mortensen

[1] King and Rebelo in their chapter in the Handbook of Macroeconomics (1999) survey recent advances in stochastic dynamic general equilibrium models of economic fluctuations. It is clear from this survey that these studies of economic fluctuations tend to ignore altogether the firm's decisions with respect to the productivity of its workforce over the cycle.

and Pissarides (1994, 1999a, 1999b) (MP hereafter) where firms react to idiosyncratic and aggregate shocks by upgrading, creating, and destroying jobs. In the original MP model, firms cannot change the productivity of existing jobs (i.e., technology is irreversible in MP's terminology) and hence can respond to aggregate and idiosyncratic shocks only by destroying or creating jobs. Here, in contrast, firms can invest in the productivity of existing jobs through training and reorganization activities, *in addition to* destroying and creating jobs.

I will argue that the model is a flexible and tractable framework that naturally lends itself to the analysis of the importance of trade-frictions and labor market policy and institutions to the considered investment decisions by the firm. In addition, under the natural interpretation of job upgrading as discretionary firm-specific training, the model becomes a useful tool to analyze the impact of training policy on equilibrium labor market outcomes, including the natural unemployment rate.

The paper is organized as follows. Section 2 presents a minimalist version of the model that clarifies the nature of the problem and its solution. Section 3 spells out a more general version of the problem where wages are endogenous and labor market policy is introduced. Section 4 contains some comparative statics exercises and computational experiments based on the general version of the model. Section 4 concludes with an empirical investigation that shows that the model's predictions with respect the relation between labor market policy and labor market equilibrium outcomes, including training or job upgrading incidence, are consistent with the observed cross-country differences in the data. Section 5 concludes and suggests further research. The Appendix contains particularly long derivations.

2. A Minimalist Model

2.1. Overview

The economy is populated by firms and workers, who search for one another in order to form productive matches. Search is time and resource consuming and the search technology is described by an aggregate matching function. When a firm meets a worker, a match is created that owing to search frictions generates a surplus ex-post, which accrues entirely to the firm.[2] This match, which can be thought of as a job, produces a quantity of the final good

[2] Since the aim of this section is to present a minimalist (version of the) model, it is assumed here that employed workers receive a wage equal to the flow benefit of being unemployed. This assumption considerably simplifies the structure of the problem and is a special case of the extended model of section 3, where the workers' bargaining-power in the Nash-Bargaining game is zero (or alternatively, to a model of efficiency wages where firms' monitoring capabilities are close to perfect).

that depends on its productivity. The productivity of each job can be decomposed into an aggregate component and an idiosyncratic component. Every period, the productivity of the job may change because the idiosyncratic productivity component of the match may change. Firms and workers are risk-neutral, forward-looking rational agents. The firm is able to invest in the productivity of the worker. The firm is also able to terminate the match at some finite cost. We will explore how these investment and firing decisions depend on the parameters of the model, and, concomitantly, on labor market policy. Figure 1 illustrates the sequence of events and flows studied in the model.

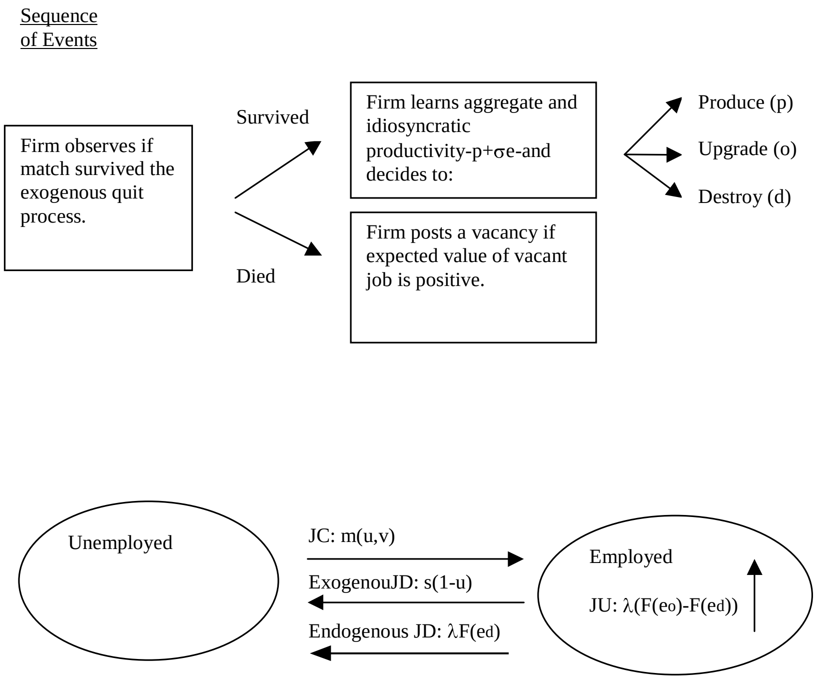


Figure 1: Sequence of Events and Flows of the Model. Notation described in text.

2.2. Notation and Concepts

The model treats time as discrete. The economy is populated by a continuum of firms and a continuum of workers with mass normalized to one. Units of production (or positions) consist in a match between one worker and one firm. Hence, a firm offers only one job. A job can be either filled or empty. If the job is empty, then the firm may post a vacancy (if the expected value of creating a vacancy is nonnegative), which will eventually be filled with an unemployed worker, through a matching process. All unemployed workers search for posted vacant jobs and receive a flow benefit b per period while unemployed which may be interpreted as unemployment insurance pay-outs or the value of home-produced goods, but is called simply unemployment benefit. Vacancies are costly to create, with the per-period cost of posting a vacancy denoted by c. Once a vacant job meets with an unemployed worker, the firm pays recruiting costs C, and production takes place at the beginning of the following period. New jobs are the most productive jobs in the economy; since recruiting in this model consumes time and resources, it seems natural to think of new jobs as high productivity jobs.[3]

At the beginning of each period, each filled job may receive an idiosyncratic productivity shock, e, which is drawn from a fixed and common knowledge differentiable distribution $F(e)$. $F(e)$ has e_u as its upper support and e_l as its lower support, with e_u as the only mass-point. Shocks arrive every period with probability λ. Hence, the idiosyncratic productivity shocks have persistence, with expected persistence given by $1/\lambda$, but no memory, since the new shocks, conditional on arrival, are independent from the previous shocks. More formally, the process governing the evolution of the idiosyncratic component e is described by:

$$e_{t+1} = \begin{cases} e_t, \text{with probability } (1-\lambda) \\ e, \text{with probability } \lambda F'(e) \end{cases}$$

The productivity of the job can be decomposed into two parts: one component common to all jobs in the economy, p, and one idiosyncratic component, e. More precisely, the productivity of the job is given by $p+\sigma e$. We may think of p as indexing the aggregate state of the economy, and of σ as indexing the dispersion of the idiosyncratic shocks. Note that the maximum productivity that a job may have is $p+\sigma e_u$. Once a match is created, it may be destroyed for exogenous reasons at the rate s per period or because the firm decides so. The matching process is described by a matching function $m(u,v)$ which gives the number of contacts between unemployed workers (u) and vacant jobs (v) in a given period. As is traditional in the literature, it is assumed that $m(u,v)$ is increasing and concave in both of its arguments and exhibits constant returns to scale in u and v. Also let $\theta \equiv v/u$ denote market tightness (from the firm's point of view). It follows that the per-period probability of an unemployed worker meeting with a vacant

[3] Note that the results presented throughout the paper do not hinge on the assumption that new jobs are the most productive jobs in the economy or, more generally, that they have a common level of idiosyncratic productivity. At the expense of more cumbersome algebra the model could feature stochastic job matchings.

job can be written as:

$$f(\theta) \equiv \frac{m(u,v)}{u} = m(1,\theta), \text{ with } f'(\theta) > 0$$

Similarly, the per-period probability of a firm with a vacant job meeting with an unemployed worker can be written as:

$$q(\theta) \equiv \frac{m(u,v)}{v} = m(\frac{1}{\theta},1), \text{ with } q'(\theta) < 0$$

Finally, note that $q(\theta)$ and $f(\theta)$ are related through the identity: $f(\theta) = q(\theta)\theta$.

2.3. The Firm's Problem

To solve the firm's problem, it is convenient to write the asset equations for the different jobs. Jobs are different in the sense that they may be in different states; for instance, firms may decide to invest in the productivity of the job or not. If the firm decides to invest in the productivity of the job, then the expected future productivity of the job will be higher. For that, the firm must pay a flow investment cost, which is given by the sum of a fixed cost and an opportunity cost, owing to jobs being less productive during the investment period. For simplicity, it is assumed that these investments last just one period. For concreteness' sake, we label this investment decision upgrading. The alternative to upgrading is not to invest in the productivity of the job, save on investment costs, and not improve the expected future productivity of the job. Hence, the firm must decide between upgrading and (simply) **p**roduction. It is convenient to define $J(e)$ as the asset value of a filled job with idiosyncratic productivity component e. If the firm decides to upgrade the job, the asset value of the job is $J^o(e)$ (where the superscript o stands for **o**rganization activities, one way of upgrading the job). If the firm decides to produce, the asset value of the job is $J^p(e)$ (where the superscript p stands for **p**roduction activities). The firm also has the power to destroy the match. For that the firm must bear the (implicit) cost T.[4] We can thus define the value of a destroyed job $J^d(e) \equiv -T$. Since the firm always chooses upgrading, production, or destruction depending on which brings the highest expected present discounted value, then $J(e)$ is defined as follows:

$$J(e) = \max \langle J^p(e), J^o(e), -T \rangle \tag{2.1}$$

[4] Here it is assumed that T is an implicit cost borne solely by the firm if it decides to terminate the match. A more comprehensive treatment of the interpretation of T is offered in section 4.

At the beginning of the period the firm observes if the job survived the exogenous quit process (natural turnover) or not. If the job survived, then the firm learns the aggregate state of the economy, p, and the idiosyncratic productivity of the job, e, that is, it observes the productivity of the job, $p+\sigma e$, and then decides to engage either in production or in organization or to destroy the job.[5]. If the firm decides to engage in production, then the relevant asset equation, $J^p(e)$, reads as:

$$J^p(e) = p + \sigma e - b + \beta(1-s)[\lambda \int_{e_l}^{e_u} \max \langle J^p(x), J^o(x), -T\rangle \, dF(x) + (1-\lambda)J^p(e)] + \beta s J^v \tag{2.2}$$

The current flow net benefits of engaging in production consist in the output of the job, $p + \sigma e$, minus a wage payment made to the worker which equals the flow benefit the worker receives if unemployed, b.[6] In the next period (discounted at rate $\beta \equiv 1/(1+r)$, with r as the exogenous real interest rate), the job survives the exogenous quit process with probability $(1-s)$. If the job survives, then it is hit by a new idiosyncratic productivity shock that arrives with probability λ per period and is drawn from $F()$. If a shock arrives, and since shocks, conditional on arrival, have no memory, the expected asset value of the job is given by $\int_{e_l}^{e_u} \max \langle J^p(x), J^o(x), -T\rangle \, dF(x) \equiv E[J(x)]$. If the job does not survive, the firm creates a vacancy if the asset value of a vacant job, J^v, is nonnegative. If the firm decides to engage in organization, then the relevant asset equation, $J^o(e)$, reads:

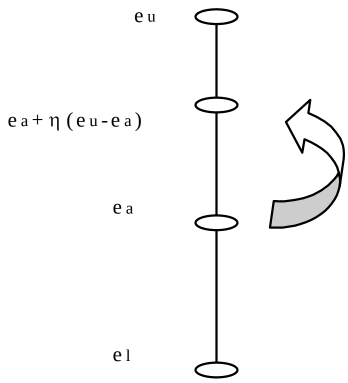
$$\begin{aligned} J^o(e) &= \phi(p+\sigma e) - b - K + \\ &+ \beta(1-s)[\lambda \int_{e_l}^{e_u} \max \langle J^p(x), J^o(x), -T\rangle \, dF(x) + (1-\lambda)J(e+\eta(e_u - e))] + \beta s J^v \end{aligned} \tag{2.3}$$

The flow net benefits of engaging in organization consist in the output of the job, $\phi(p+\sigma e)$, minus the wage paid to the worker, b, and the fixed cost K to engage in organization. ϕ is a parameter between 0 and 1 that captures the disruptive nature of engaging in organization. A low ϕ signifies a high opportunity cost of engaging in organization in terms of sacrificed current output. For instance, $\phi = 0$ may be appropriate if we have in mind (as organization activities) off-the-job training. A ϕ strictly greater than zero would be appropriate for (discretionary) on-the-job

[5] The firm observes p and e separately. This is not important in the present context. However, Menezes (2000) presents an extended version of the model of section 3 where the economy fluctuates between a high aggregate state and a low aggregate state, that is, p takes on two values, p^{high} and p^{low}. Then it is important to observe p and e separately since it allows solving the model using state-contingent reservation rules. In the present context, the timing of events is important to the extent that the firm observes both p and e before deciding what to do.

[6] A fuller treatment of wages is left for the next section.

training. In the next period, and if the job survives the exogenous quit process, the job may be hit by a new productivity shock. In this sense, the outcome of investing in the productivity of the job is stochastic, since the firm succeeds in improving the productivity of the job from its current level only with probability $(1-\lambda)$. Finally, it is assumed that by upgrading the job, the firm closes a fraction $\eta \in (0,1]$ of the gap between the current level of idiosyncratic productivity and the highest productivity possible, e_u, or the productivity of new jobs.[7,8] Figure 2 illustrates the upgrading technology.[9]



[7] The model could feature upgraded jobs being more productive than new ones. This could be achieved in a number of ways. For instance, $\eta > 1$ implies that upgraded jobs are more productive than new ones and would not substantively modify the model as long $0 < J^{o'} < J^{p'}$ holds. Note that this condition prevents most productive jobs to undergo successive rounds of upgrading (only interrupted by the uncertain arrival of a low productivity shock). Alternatively, we could consider stochastic job matchings. See footnote 3.

[8] An alternative way of thinking about the upgrading technology and motive is to assume a fixed current productivity level but allow the frontier productivity to increase (in a stochastic manner). Then the gap between current productivity and the frontier productivity would eventually increase, leading, thus, to a desire to upgrade the job. While conceptually similar, in terms of modeling strategy, it turns out to be simpler to fix the frontier productivity and let the current productivity change.

[9] Conditional on upgrading the job and surviving the exogenous quit process, the expected idiosyncratic productivity level in the next period for a job with current idiosyncratic productivity e reads:

$$(1-\lambda)(e+\eta(e_u-e))+\lambda E[x]$$

Conditional on not upgrading the job and surviving the exogenous quit process, the expected idiosyncratic productivity level in the next period for a job with current idiosyncratic productivity e reads:

$$(1-\lambda)e+\lambda E[x]$$

For there to be some upgrading in equilibrium, it is obvious that we must have must have $\lambda \in [0,1)$ and $\eta \in (0,1]$.

Figure 2: Upgrading Technology: the firm closes η% of the gap between e_a and e_u with probability $(1-\lambda)$.

We are now in position to solve the firm's problem. The solution to this problem has a reservation property. If the idiosyncratic productivity shock e is low enough then the firm destroys the job. Otherwise it keeps the job. If the firm decides to keep the job, then the firm either upgrades the job or simply engages in production. As it turns out, for high productivity jobs, production is preferred to upgrading. However, there will be (usually) a range of jobs where upgrading is preferred to production and to destruction. Hence, there will be two cut-offs for the idiosyncratic productivity component, e_d and e_o, that govern the firm's decision concerning destroying and upgrading the job, as illustrated in Figure 3 where the value functions $J^p(e)$, $J^o(e)$, and $J^d(e)$ are plotted in $(e, Value\ Functions)$ space. The differences in the slopes of the value functions lead to the reservation property of the problem. Note that Figure 3 implicitly assumes that all the margins–production, upgrading, and destruction–are active in the sense that one can find a range of idiosyncratic productivity levels where one of these margins is preferred. Of course, the world could be such that only two margins are active, say, firms never upgrade because of prohibitively high upgrading costs.[10]

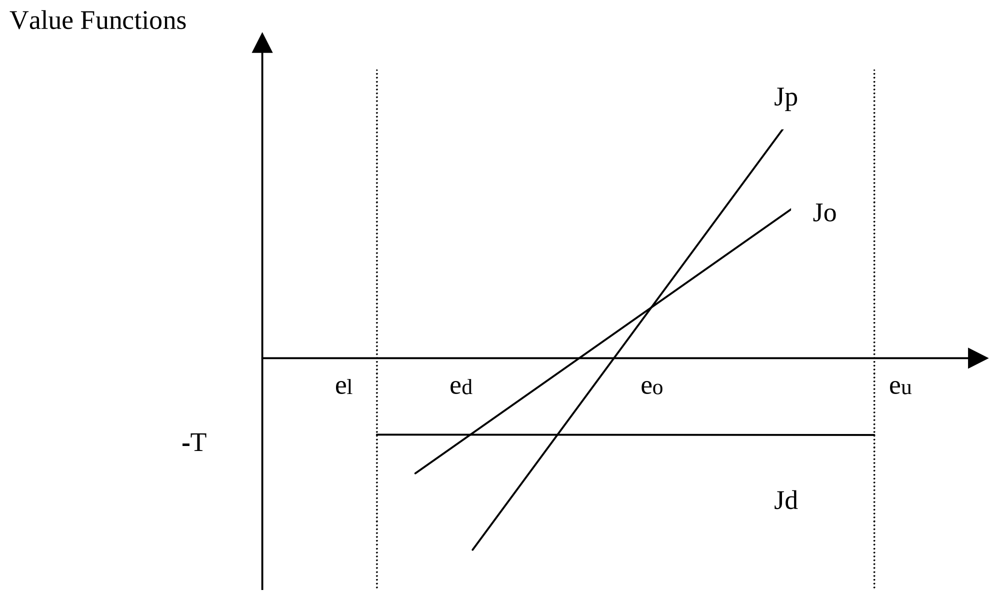


Figure 3: Value Functions.

[10] This would correspond to a version of Figure 3 with $J^o(e)$ below $-T$ for all e. Other cases are possible too.

Naturally, we focus on parameterizations of the model that leave all the margins active, as in Figure 3. More formally, the cut-offs e_d and e_o satisfy:

1. $J(e) = J^p(e)$ if $e > e_o$
2. $J(e) = J^o(e)$ if $e_d < e \leq e_o$
3. $J(e) = -T$ if $e \leq e_d$

If the firm decides to keep a job, then it must decide between upgrading and production. To better understand how a firm makes this decision we define $J^n(e) \equiv J^o(e) - J^p(e)$, the **n**et asset value of a job being upgraded. Using (2.2) and (2.3) we write $J^n(e)$ as follows:

$$J^n(e) = -(1-\phi)(p+\sigma e) - K + \beta(1-s)(1-\lambda)(J(e+\eta(e_u - e)) - J^p(e)) \tag{2.4}$$

This equation implicitly defines the reservation value of idiosyncratic productivity e_o such that for values above e_o production is preferred to upgrading and for values below e_o upgrading is preferred to production, that is:

1. $J^n(e_o) = 0$
2. $J^n(e) < 0$ for any $e > e_o$
3. $J^n(e) > 0$ for any $e < e_o$

To learn more about the determination of e_o we use the fact that for any non-trivial solution to the problem we must have $J(e_u) = J^p(e_u)$, since otherwise firms would never engage in production, and we assume that η is high enough so that $J(e+\eta(e_u - e)) = J^p(e+\eta(e_u - e))$ (η is close to one).[11] Note that e_o satisfies:

$$0 = -(1-\phi)(p+\sigma e_o) - K + \beta(1-s)(1-\lambda)(J^p(e_o+\eta(e_u - e_o)) - J^p(e_o)) \tag{2.5}$$

where we used the definition of e_o and (2.4). Using the linear structure of the problem we rewrite (2.5) to highlight the economics of the upgrading decision as follows:

$$0 = -(1-\phi)(p+\sigma e_o) - K + \beta(1-s)(1-\lambda)J^{p'}\eta(e_u - e_o) \tag{2.6}$$

According to (2.6), when e equals e_o the net benefits of upgrading equal the net costs of upgrading. The net costs of upgrading entail an opportunity cost and a fixed cost. The net benefits of upgrading are the product of three

[11] Intuitively, after upgrading the job and if the environment does not change, firms will engage in production. However, there is nothing in the problem that prevents firms from undergoing successive rounds of (re)organizations as long as firms are continuously hit by new and low enough productivity shocks.

terms: the increase in productivity obtained by upgrading, $\eta(e_u - e)$; the unit valuation of this increase, $J^{p'}$; and an appropriate or effective discount rate $\beta(1-s)(1-\lambda)$ (or $k \equiv \beta(1-s)(1-\lambda)$ for short). This basic decomposition of the effects influencing the upgrading decision carries over to more involved problem settings and is thus a convenient departure point for our understanding of the upgrading decision.[12] Using (2.2) we eliminate $J^{p'}$ from (2.6), which, after rearranging terms, reads:

$$e_o[(1-\phi)\sigma + \frac{\beta(1-s)(1-\lambda)\sigma}{1-\beta(1-s)(1-\lambda)}\eta] = -(1-\phi)p - K + \frac{\beta(1-s)(1-\lambda)\sigma}{1-\beta(1-s)(1-\lambda)}\eta e_u \tag{2.7}$$

2.4. Comparative Statics Exercises

We now have e_o defined by (2.7) as a function of parameters of the model, without reference to the other endogenous variables of the model (e_d, θ and u). The model thus exhibits a recursive nature that allows us to perform some comparative statics exercises at this stage. The following expression summarizes the possible comparative statics exercises:

$$e_o = e_o(\underset{(+)}{\phi}, \underset{(-)}{p}, \underset{(+)}{\sigma}, \underset{(-)}{K}, \underset{(-)}{r}, \underset{(-)}{s}, \underset{(-)}{\lambda}, \underset{(+)}{\eta}, \underset{(+)}{e_u})$$

For a given mass of jobs subject to being upgraded, a higher e_o means a higher number of jobs being upgraded in the economy–job upgrading rate, for short, or, alternatively, training incidence or rate. We now discuss the impact of changes in the parameters that affect the job upgrading rate directly through e_o.

The signs of ϕ and p follow from the opportunity cost associated with job upgrading. An improvement in the aggregate state of the economy (increase in p) leads to a decrease in e_o owing to a higher cost of the sacrificed output. Obviously, this is the more important the lower is ϕ.[13]

As expected, a higher arrival rate of new idiosyncratic shocks, λ, leads to a lower rate of job upgrading. Intuitively, a higher λ means more uncertainty with respect to the outcome of the upgrading investment. In the limit, when $\lambda = 1$, the expected future productivity of the job is independent of the firm's decision to upgrade the job, and hence firms will never upgrade. By the same token, a larger quit rate, s, decreases e_o. A higher e_u and higher η imply a better upgrading technology and lead to an increase in e_o. A higher fixed cost of upgrading, K, decreases e_o. The

[12] For instance, different wage agreements will affect $J^{p'}$ but not the essence of (2.6).

[13] In the present context, a change in p represents an unanticipated once-and-for-all change in aggregate conditions. To think of changes in p as indexing for the role of cyclical conditions is hence not entirely correct. See Section 4 and Menezes (2000).

impact of σ on e_o is positive: on the one hand, a high σ means a high opportunity cost to upgrading; on the other hand, a high σ means a higher expected return to job upgrading. It turns out that the second effect dominates.

2.5. Closing the Model

We still have to solve for the equilibrium level of the other endogenous variables of the model, e_d, θ and u. To close the model, we first specify the value of vacancies J^v:

$$J^v = -c + \beta[q(\theta)(J^p(e_u) - C) + (1 - q(\theta))J^v] \tag{2.8}$$

Vacancies cost c per period and are filled with probability $q(\theta)$. When the vacancy becomes filled, the match becomes productive at the beginning of the next period. New jobs enjoy an idiosyncratic productivity level of e_u.[14] The firm must incur set-up costs C, which could be interpreted as recruiting costs (including initial training). It is assumed that a free-entry condition and a consequent zero-profit condition in creating vacancies holds at all times:

$$J^v = 0 \tag{2.9}$$

We use (2.8) and (2.9) to solve for the value of new jobs, or the Job Creation (JC) condition:

$$J^p(e_u) = \frac{c}{\beta q(\theta)} + C \tag{2.10}$$

The expected value of a new job equals the expected cost of posting a vacancy ($\frac{c}{\beta q(\theta)}$), plus the recruiting costs (C). We rewrite $J^p(e_u)$ in terms of parameters of the model. This will give us an extra condition relating θ to parameters of the model and to the other endogenous variables of the model. To eliminate $E[J(x)]$ from (2.2) we note that:

$$\begin{aligned} E[J(x)] &= \int_{e_l}^{e_u} \max\left\langle J^p(x), J^o(x), J^d(x)\right\rangle dF(x) = \\ &= \int_{e_l}^{e_d} J^d(x)dF(x) + \int_{e_d}^{e_o} J^o(x)dF(x) + \int_{e_o}^{e_u} J^p(x)dF(x) \end{aligned}$$

After proper integration by parts, the RHS of the above expression becomes (recall $J^d(x) = -T$):

$$\begin{aligned} & -T * F(e_d) + J^o(e_o)F(e_o) - J^o(e_d)F(e_d) - \int_{e_d}^{e_o} J^{o'}(x)F(x)dx + \\ & + J^p(e_u)F(e_u) - J^p(e_o)F(e_o) - \int_{e_o}^{e_u} J^{p'}(x)F(x)dx \end{aligned}$$

[14] It is assumed that new jobs are not subject to the exogenous quit process. In other words, jobs are productive for at least one period, and all contacts between unemployed workers and vacancies result in productive matches. This assumption could easily be relaxed at the expense of more cumbersome algebra.

which implies that:

$$E[J(x)] = J^p(e_u) - \int_{e_d}^{e_o} J^{o'}(x)F(x)dx - \int_{e_o}^{e_u} J^{p'}(x)F(x)dx$$

since $J^o(e_o) = J^p(e_o)$, $F(e_l) = 0$ and $F(e_u) = 1$. Using (2.3) and (2.2) we learn that (recall $k \equiv \beta(1-s)(1-\lambda)$):

$$J^{o'}(x) = \phi\sigma + \frac{k\sigma(1-\eta)}{1-k}$$

and:

$$J^{p'}(x) = \frac{\sigma}{1-k}$$

To write $J^p(e_u)$ in terms of parameters of the model, we note that:

$$J^p(e_u) = J^p(e_o) + J^{p'}(e_u - e_o)$$

Since (by definition of e_o) $J^p(e_o) = J^o(e_o)$, we have:

$$J^p(e_o) = J^o(e_o) = J^o(e_d) + J^{o'}(e_o - e_d)$$

or:

$$J^p(e_u) = -T + J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o) \tag{2.11}$$

where we used the fact that $J^o(e_d) = -T$ by definition of e_d. Putting all the pieces together, we have the following expression for $E[J(x)]$:

$$E[J(x)] = -T + J^{o'} \int_{e_d}^{e_o} (1 - F(x))dx + J^{p'} \int_{e_o}^{e_u} (1 - F(x))dx \tag{2.12}$$

Also by definition of e_d we have (using (2.3)):

$$-T = \phi(p + \sigma e_d) - b - K + \beta(1-s)\lambda E[J(x)] + kJ^p(e_d + \eta(e_u - e_d))$$

Substituting out $E[J(x)]$ and $J^p(e_d + \eta(e_u - e_d))$ we obtain:

$$\begin{aligned} 0 = {} & [\phi(p + \sigma e_d) - b - K] + (1 - \beta(1-s))T + \beta(1-s)\{J^{o'}(e_o - e_d) + \\ & + J^{p'}(e_u - e_o) - \lambda[J^{o'} \int_{e_d}^{e_o} F(x)dx + J^{p'} \int_{e_o}^{e_u} F(x)dx]\} + k(1-\eta)J^{p'}(e_u - e_d) \end{aligned} \tag{2.13}$$

Equation (2.13) can be thought of as a Job Destruction (JD) condition.[15,16] The first term in square brackets is the net flow benefit associated with the marginal job. The term associated with T represents savings due to not incurring in the termination cost. The other terms represent the option value of keeping the job: Conditions may improve if there is upgrading or if a new shock arrives. We now use (2.11) to replace the left-hand side of (2.10) to obtain the useful expression for the JC condition:

$$-T + J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o) = \frac{c}{\beta q(\theta)} + C \tag{2.14}$$

Finally, to determine equilibrium unemployment, we use the following Beveridge Curve:

$$m(u, v) = s(1-u) + \lambda F(e_d)(1-s)(1-u) \tag{2.15}$$

or, in terms of u:

$$u[f(\theta) + s + \lambda F(e_d)(1-s)] = s + \lambda F(e_d)(1-s) \tag{2.16}$$

In equilibrium job creation equals job destruction. Since all contacts between unemployed workers and vacancies lead to productive matches, $m(u, v)$ represents the number of jobs created each period. At the beginning of each period, a fraction of the existing jobs are destroyed owing to the exogenous quit process s. A fraction $\lambda F(e_d)$ of the jobs that survive the exogenous quit process receive an idiosyncratic productivity shock below the cut-off e_d and hence are destroyed.

The solution of the model exhibits a recursive structure. First we solve for e_o using (2.7) and then for e_d using (2.13) and the solution for e_o. Armed with e_d and e_o we obtain θ using (2.14), and finally, with knowledge of e_o, e_d, and θ we solve for u using (2.16). Some geometry is useful to understand the nature of the solution of the problem. The upper panel of Figure 4 depicts the JC (2.14) and JD (2.13) conditions in (e_d, θ) space. The JC condition slopes down since a higher e_d leads to a lower value of new jobs, and hence θ must fall in order for the expected costs of filling a vacancy to fall. Since e_d is determined without reference to θ the JD schedule is a vertical line. In the bottom panel of Figure 4, the equilibrium value of θ is used with the aid of the Beveridge curve (2.15) to pin down the equilibrium values of v and u. Note that e_o shifts both the JC and the JD schedules.

[15] Implicit in the writing of (2.13) is the assumption that $e_d + \eta(e_u - e_d) > e_o$, which is satisfied if η is close enough to one. Intuitively, this amounts to the assumption that the marginal job if upgraded would be put into production.

[16] The model features labor hoarding in the sense that the current flow benefits associated with the marginal worker $(\phi(p+\sigma e_d)-b-K)$ are negative.

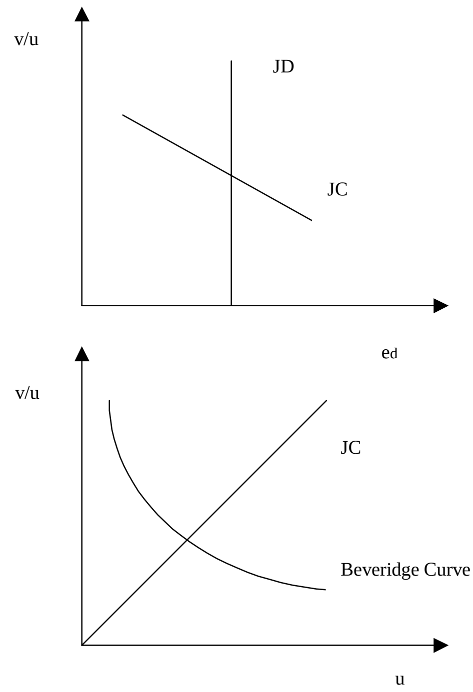


Figure 4: Geometry of Equilibrium Solution with Exogenous Wages.

3. An Extended Model

This section extends the model of the previous section in a number of substantive ways without altering its basic structure. The main departures consist in introducing wages that follow a Nash-Bargaining game and parameters that capture the role of labor market policy.

As is traditional in the literature, the wage formation process follows from a Nash-Bargaining game where the threat points are given by the participants' options outside the match (J^v for the firm and U, the value of unemployment, for the worker). Firms and workers are assumed to share the surplus associated with the match in fixed proportions at all times. The fraction that accrues to the worker is denoted by ψ (worker's bargaining power). As discussed in Mortensen and Pissarides (1999a, 1999b), the solution to the proposed wage determination process is a two-tier wage schedule. The wage is contingent on the idiosyncratic productivity of the match. Let $w(e)$ be the wage earned by a worker in a continuing (as opposed to new) match with idiosyncratic productivity e. Recall that new matches have idiosyncratic productivity e_u. However, new matches, which have by assumption idiosyncratic productivity e_u, do not necessarily pay $w(e_u)$. The wage associated with new matches, w_o, may not coincide with

$w(e_u)$, since when an unemployed worker meets with a vacant job, the firm must bear the creation costs, which will then become sunk after the match is created. Hence, the firm is willing to form the match only if it is assured of a positive gain. In addition, since it is assumed that termination costs fall solely upon firms, these costs (jointly with J^v) must be considered as the outside options to the firm. This also creates a wedge between w_o and $w(e_u)$. We return to this issue below.

A word on wages and training is in order. As mentioned in Sections 1 and 2, discretionary firm-specific training is a natural interpretation of the upgrading activities studied here. As is well-known, there is a large literature on who should pay for the costs of training, which goes back at least to the work of Becker (1964). Here, the assumption that workers and firms share the surplus of the match at all times, regardless of whether the worker is engaged in organization or in production, implies that wages adjust when there is training in a way that the surplus is (still) split in the proportions ψ and $(1-\psi)$ (for the worker and for the firm, respectively). This allows us to focus on the main objectives of the paper–the implications of labor market policy on training incidence and of training policy on equilibrium labor market outcomes–while treating the issue of training and wages following the traditional Nash-Bargaining game.[17]

3.1. The Firm's and the Worker's Problem

The asset value equations for the different types of jobs must be rewritten to reflect that wages now are endogenous and not necessarily equal to the unemployment benefit. We first rewrite (2.2):

$$J^p(e) = p + \sigma e - (1+t)w(e) - a + \beta(1-s)[\lambda E[J(x)] + (1-\lambda)J^p(e)] + \beta s J^v \tag{3.1}$$

Taxes on wages are introduced through a linear-tax schedule. The employer is required to pay a tax proportional to the wage at rate t and a lump-sum amount a, which is interpreted as an employment subsidy if negative and as an employment tax if positive. Similarly, (2.3) now reads:

$$J^o(e) = \phi(p + \sigma e) - (1+t)w(e) - a - K + \beta(1-s)[\lambda E[J(x)] + (1-\lambda)J^p(e + \eta(e_u - e))] + \beta s J^v \tag{3.2}$$

[17] Hashimoto (1996) offers an interpretation of training as a 'shared investment' between the worker and the firm similar in spirit to the present wage bargaining process.

$J^n(e)$ is defined as before:

$$J^n(e) = -(1-\phi)(p+\sigma e) - K + \beta(1-s)(1-\lambda)J^{p'}\eta(e_u - e) \tag{3.3}$$

Note that wages do not enter directly in (3.3). Consequently, the linear-tax schedule does not affect e_o in a direct manner. This will be the case as long as the taxes on wages do not depend directly on the job being upgraded or not. For instance, the existence of training-conditional wage subsidies would require modifying (3.3). Interestingly, $J^n(e)$ preserves the structure as in the minimalist model of the previous section. This does not, however, imply that e_o is unaffected by the extensions considered in the present model. From (3.1) it is clear that $J^{p'}$ depends on $(1+t)w'(e)$ and is yet to be determined.[18] Vacancies are valued as follows:

$$J^v = -c + \beta[q(\theta)(J_o - (C - H)) + (1 - q(\theta))J^v] \tag{3.4}$$

New jobs have value J_o. H is a hiring subsidy, and hence the relevant private creation costs for the firm are given by $(C - H)$. The asset value of new jobs is:

$$J_o = p + \sigma e_u - (1+t)w_o - a + \beta(1-s)[\lambda E[J(x)] + (1-\lambda)J_o] + \beta s J^v \tag{3.5}$$

New jobs have idiosyncratic productivity e_u and pay a gross wage of $((1+t)w_o + a)$.

Owing to the endogeneity of wage determination, we now write the asset equation for a worker in a continuing match with idiosyncratic productivity e, $W(e)$, as:

$$W(e) = w(e) + \beta(1-s)[\lambda \int_{e_l}^{e_u} \max \langle W(x), U\rangle \, dF(x) + (1-\lambda)W(e)] + \beta s U \tag{3.6}$$

where $\int_{e_l}^{e_u} \max \langle W(x), U\rangle \, dF(x) \equiv E[W(x)]$ illustrates that the worker may decide to quit and become unemployed at no cost. The worker's problem also obeys a reservation property since $W(e)$ is increasing in e and U does not depend on e. Hence we can define a reservation level for the idiosyncratic productivity component R_w that satisfies:

$$\int_{e_l}^{e_u} \max \langle W(x), U\rangle \, dF(x) = F(R_w)U + \int_{R_w}^{e_u} W(x)dF(x)$$

By virtue of the wage formation process, $R_w = e_d$, which implies jointly privately efficient match terminations. Unemployment has an asset value given by:

[18] We shall proceed and solve the model in the spirit of Figure 3.A. Then we show that in the equilibrium where all margins are active indeed we have $0 < J^{o'} < J^{p'}$.

$$U = b + \rho\varpi + \beta[f(\theta)W_o + (1 - f(\theta))U] \tag{3.7}$$

where $\rho\varpi$ represents the portion of total flow benefits of unemployment due to policy. More precisely, ρ is the replacement-ratio and ϖ is the average wage earned by employed workers. b would then be interpreted as, say, the value of household production. With probability $f(\theta)$, the unemployed worker meets with a vacancy and forms a new match, with an asset value of W_o:

$$W_o = w_o + \beta(1 - s)[\lambda E[W(x)] + (1 - \lambda)W_o] + \beta sU \tag{3.8}$$

3.2. Wages

The starting wage determined by the Nash-Bargaining game over the future income stream foreseen by the worker and the firm supports the outcome:

$$w_o = \arg\max\{[W_o - U]^{\psi}[S_o - (W_o - U)]^{1-\psi}\} \tag{3.9}$$

subject to the following definition of initial match surplus:

$$S_o \equiv J_o - (C - H) - J^v + W_o - U \tag{3.10}$$

By the same token, the continuing wage contract supports the outcome:

$$w(e) = \arg\max(W(e) - U)^{\psi}(J(e) - J^v)^{1-\psi} \tag{3.11}$$

subject to the following definition of continuing match surplus:

$$S(e) = \begin{cases} J^p(e) - J^v + T + W(e) - U, e > e_o \\ J^o(e) - J^v + T + W(e) - U, e_d < e \leq e_o \end{cases} \tag{3.12}$$

As Mortensen and Pissarides (1999a, 1999b) discuss, the difference between the initial wage bargain and subsequent renegotiation arises for two reasons. First, creation costs are sunk in the latter but 'on-the-table' in the former. Second, termination costs are avoided if no match is formed but must be paid if an existing match is terminated. The solutions to the above optimization problems satisfy the following first-order conditions:

$$\psi(J_o - J^v - (C - H)) = (1 - \psi)(1 + t)(W_o - U) \tag{3.13}$$

and:

$$\psi(J(e) - J^v + T) = (1-\psi)(1+t)(W(e) - U) \tag{3.14}$$

which imply the following sharing-rules:

$$\begin{cases} J(e) - J^v + T = \frac{(1-\psi)(1+t)}{(1-\psi)(1+t)+\psi} S(e) \\ J_o - J^v - (C-H) = \frac{(1-\psi)(1+t)}{(1-\psi)(1+t)+\psi} S_o \end{cases} \Leftrightarrow \begin{cases} W(e) - U = \frac{\psi}{(1-\psi)(1+t)+\psi} S(e) \\ W_o - U = \frac{\psi}{(1-\psi)(1+t)+\psi} S_o \end{cases}$$

Note that the workers' shares decrease in t since high tax rates induce decreases in wages in order to avoid high tax bills. Before proceeding to solve for the wage equations, we recall that by assumption competition drives the asset value of vacancies down to zero at all times. More formally:

$$J^v = 0 \tag{3.15}$$

and hence (using (3.5)):

$$J_o - (C - H) = \frac{c}{\beta q(\theta)} \tag{3.16}$$

We are now in a position to solve for the wage equations. Manipulating (3.5), (3.8), (3.7), (3.16), and the sharing rules we find:

$$w_o = (1-\psi)(b + \rho\varpi) + \frac{\psi}{1+t}[p + \sigma e_u - a + \theta c - (1-k)(C-H) - \beta(1-s)\lambda T] \tag{3.17}$$

Similarly, using (3.1), (3.6), (3.7), (3.16), and the sharing rules we obtain:

$$w^p(e) = (1-\psi)(b + \rho\varpi) + \frac{\psi}{1+t}[p + \sigma e - a + \theta c + (1 - \beta(1-s))T], e \geq e_o \tag{3.18}$$

Wages increase with the flow benefits of being unemployed and with market tightness. A tighter market leads to a higher probability of finding a match and hence increases the value of the worker's outside options. Creation costs lower initial wages but do not affect continuing wages. Intuitively, this results from creation costs being sunk for the latter but on the table for the former. Termination costs lower initial wages since they lower the expected value of forming a match. However, once the match is formed, termination costs lead to a deterioration of the employer's position, pushing the wage up in a way that dominates the previous effect. As expected, in the absence of creation and termination costs, $w_o = w^p(e_u)$. We also solve for wages when the firm chooses upgrading. For that we use

(3.2), (3.6), (3.7), (3.16), and the sharing rules to find:

$$For\ e_o \geq e > e_d,\ w^o(e) = (1-\psi)(b+\rho\varpi) + \frac{\psi}{1+t}[\theta c - a - (1-\beta(1-s)]T + \tag{3.19}$$
$$+\frac{\psi}{(1+t)(1-k\psi)}[((1-k)\phi + k(1-\psi))p + ((1-k)\phi + k(1-\psi)(1-\eta))\sigma e +$$
$$+ k(1-\psi)\eta\sigma e_u - (1-k)K]$$

When a job is upgraded the wage reflects the benefits of upgrading, captured by the term in e_u, and the costs, captured by the term in K and the presence of ϕ in the terms in p and e. Note that the model in the last section employed a wage that can be thought of as a special case of the wage schedules presented in this section, since the latter reduce to the flow benefits of being unemployed $(b+\rho\varpi)$ when $\psi = 0$. Finally, we check that the wage equations indeed guarantee $0 < J^{o'} < J^{p'}$. This is easily done using (3.1), (3.18), (3.2), and (3.19), which imply:

$$J^{p'} = \frac{(1-\psi)\sigma}{1-k} \tag{3.20}$$

$$J^{o'} = \frac{(1-\psi)\sigma}{1-k} * \frac{\phi(1-k) + k(1-\psi)(1-\eta)}{1-k\psi} \tag{3.21}$$

and simple inspection shows that in fact $\frac{J^{o'}}{J^{p'}} < 1$.

3.3. The Upgrading Decision

We revisit the determination of e_o. We already showed that the essence of J^n survived to the extensions introduced in this section (see (3.3)). We now discuss how these extensions modify the solution for e_o. Using (3.1) and (3.18) we solve (3.3) for e_o, which, after some manipulation, yields:

$$e_o[(1-\phi)\sigma + \frac{(1-\psi)k\sigma}{1-k}\eta] = -(1-\phi)p - K + \frac{(1-\psi)k\sigma}{1-k}\eta e_u \tag{3.22}$$

(of which (2.7) is a special case corresponding to $\psi = 0$). Now ψ has a direct impact on e_o, with a higher ψ leading to lower e_o: the firm's benefits of upgrading decrease with ψ through $J^{p'}$, the unit valuation of productivity. The conspicuous absence of t from (3.22) is explained as follows. On the one hand, a higher t leads to a decrease in $J^{p'}$ for a given wage. On the other hand, a higher t leads to a deterioration of the worker's share of the surplus and hence to a lower wage. The nature of the Nash-Bargaining game ensures that these effects cancel each other out, that is, $J^{p'}$ does not depend on t.

3.4. Closing the Model

As in the previous section, the model features a recursive structure. In fact, we solve for e_o using (3.22) without reference to other endogenous variables. To (3.22) we add a JD condition, a JC condition and, finally, a Beveridge curve. However, since the algebra is at times cumbersome, the details of the derivations are left to the Appendix.

We now turn to the JD condition. The starting point recognizes that whenever the value of a filled job falls below the termination costs, job destruction is triggered. More formally, the marginal job has an idiosyncratic productivity level that satisfies the following condition:

$$J^o(e_d) = -T \tag{3.23}$$

After manipulating some of the equations and rules discussed so far, the above condition translates into (see Appendix):

$$\begin{aligned} 0 = {} & \phi(p + \sigma e_d) - a - (1+t)(b + \rho\varpi) - K - \frac{\psi}{1-\psi}\theta c + (1 - \beta(1-s))T + \\ & + \frac{\beta(1-s)\lambda}{1-\psi}[J^{o'} \int_{e_d}^{e_o} (1 - F(x))dx + J^{p'} \int_{e_o}^{e_u} (1 - F(x))dx] + \\ & + k[J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o) - (1-\eta)J^{p'}(e_u - e_d)] \end{aligned} \tag{3.24}$$

Turning to the JC condition, the proper starting point follows from the free-entry-condition:

$$J_o = \frac{c}{\beta q(\theta)} + (C - H) \tag{3.25}$$

Straightforward algebra leads to the JC condition (see Appendix):

$$J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o) = \frac{c}{\beta q(\theta)} + (1-\psi)[(C - H) + T] \tag{3.26}$$

Given a solution e_o to (3.22), (3.24) and (3.26) yield solutions to e_d and θ. The recursive nature of the extended model differs from that of the minimalist model in that wages depend on workers' outside options, which in turn depend on θ. Hence, e_d (which depends on $w(e_d)$) cannot be determined independently of θ . A different recursive structure yields a different geometry of the problem, informally illustrated in Figure 5. The JD schedule (2.13) slopes up because at higher θ the opportunity cost of employment is higher, so there is more job destruction. In fact, the slope of JD in (e_d, θ) space is:

$$\frac{d\theta}{de_d}_{|JD,\rho=0} = \frac{\frac{1-\beta(1-s)(1-\lambda F(e_d))}{1-\psi}\sigma J^{o'}}{\frac{\psi c}{1-\psi}} > 0$$

The JC (2.14) slopes down because at higher e_d job destruction is more likely, so there is less creation. The slope of JC in (e_d, θ) space is:

$$\frac{d\theta}{de_{d\,|JC}} = \frac{J^{o'}}{\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}} < 0$$

From the intersection of JD and JC we find the equilibrium θ that with the aid of the Beveridge curve pins down equilibrium v and u:

$$m(u, v) = s(1-u) + \lambda F(e_d)(1-s)(1-u) \tag{3.27}$$

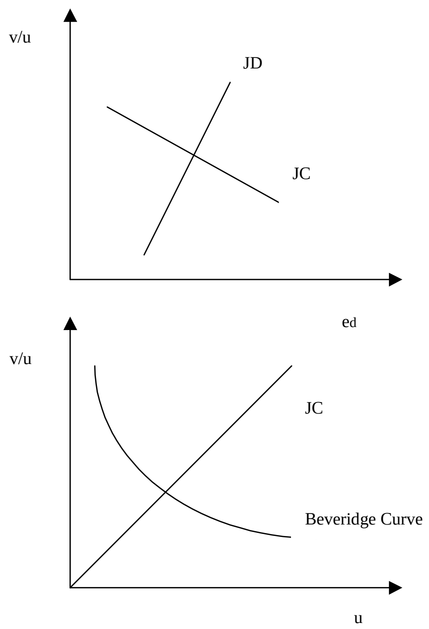


Figure 5: Geometry of Equilibrium Solution with Endogenous Wages.

Note that the JD slope was derived under the assumption that $\rho = 0$. In this case, equations (3.22), (3.24), (3.26), and (3.27) form a system, which we label *Final*, that determines the endogenous variables of the model. If $\rho \neq 0$, then ϖ enters in the JD condition, and since ϖ is endogenous, we must find an expression that relates ϖ to the endogenous variables and parameters of the model and modify the JD condition accordingly. Since this substantially complicates the algebra we work under the assumption that $\rho = 0$, unless otherwise stated. However, ϖ is a variable whose interest goes beyond its role as a modeling device to introduce unemployment benefits since it provides a feel for the welfare of employed workers in this world of risk-neutral agents. Hence it is useful to have an expression for ϖ:

$$\varpi = w_o F(e_d) + \int_{e_d}^{e_o} w^o(x)dF(x) + \int_{e_o}^{e_u} w^p(x)dF(x) \tag{3.28}$$

where $F(e_d)$ is the fraction of new matches.[19] In the Appendix it is shown that ϖ reads:

$$\begin{aligned}(1-\rho)\varpi = b &+ \frac{\psi}{(1-\psi)(1+t)}\theta c + [-\frac{\psi(1-k)}{1+t}((C-H)+T)]F(e_d) + \\ &+ [J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o)]\frac{\psi(1-k)}{(1+t)(1-\psi)}F(e_d) + \\ &+ \frac{(1-\beta(1-s))\psi}{(1+t)(1-\psi)}[J^{o'}\int_{e_d}^{e_o}(1-F(x))dx + J^{p'}\int_{e_o}^{e_u}(1-F(x))dx]\end{aligned} \tag{3.29}$$

Another measure of interest is aggregate income per participant, the sum of market output net of recruiting and job creation costs plus unemployment output:

$$\begin{aligned}y &= [(p+\sigma e_u)F(e_d) + \int_{e_d}^{e_o}\phi(p+\sigma x)dF(x) + \int_{e_o}^{e_u}(p+\sigma x)dF(x)](1-u) + \\ &+ bu - cv - Cm(v,u) \\ &= [(p+\sigma e_u)F(e_d) + \int_{e_d}^{e_o}\phi(p+\sigma x)dF(x) + \int_{e_o}^{e_u}(p+\sigma x)dF(x)](1-u) + [b - c\theta - Cf(\theta)]u\end{aligned} \tag{3.30}$$

3.5. Steady-State Flows

A novelty of the present paper is an endogenous job upgrading rate, or the fraction of jobs in the economy that are upgraded. A natural interpretation of this measure is training incidence. We are interested in the response of the steady-state job upgrading rate to changes in the parameters of the model.

Let $prod_t$ and upg_t be the fraction of filled jobs in period t engaged in production and in upgrading respectively. Then for any two periods t and $t+1$ we have:

$$prod_{t+1} = (1-\lambda)prod_t + (1-\lambda)upg_t + \lambda(F(e_u) - F(e_o))(prod_t + upg_t) + newmatches_t \tag{3.31}$$

$$upg_{t+1} = \lambda(F(e_o) - F(e_d))(prod_t + upg_t) \tag{3.32}$$

Equation (3.31) states that jobs can be engaged in production in period $t+1$ because: they were engaged in production in period t and did not receive a new shock ($(1-\lambda)prod_t$); they were engaged in upgrading in period

[19] Even though there is movement of the idiosyncratic productivity levels endogenous to the model and not simply because jobs are hit by new shocks (draws from $F()$), if we control for time, or look at steady-states, the integrals on the right-hand side of (3.29) capture the correct masses of jobs under the different states (upgrading and production).

t and did not receive a new shock (and hence will be put into production since they will enjoy an idiosyncratic productivity level greater than e_o) ($(1-\lambda)upg_t$); they received a new shock greater than e_o; or they are newly formed.[20] According to (3.32), jobs can be engaged in upgrading in period $t+1$ because they received a shock between e_d and e_o. Since we focus on steady-state values we drop the subscripts t and $t+1$ and solve (3.31) and (3.32) to obtain:

$$prod = 1 - \lambda(F(e_o) - F(e_d)) \tag{3.33}$$

$$upg = \lambda(F(e_o) - F(e_d)) \tag{3.34}$$

where we used the fact that in steady-state new matches equal $\lambda F(e_d)$. However, since not all jobs are filled, $\lambda(F(e_o)-F(e_d))$ represents the job upgrading rate for a given employment level, while $\lambda(F(e_o)-F(e_d))(1-s)(1-u)$ is the (overall) job upgrading rate.

4. Comparative Statics Exercises and Computational Experiments

This section offers and discusses some comparative statics exercises. These exercises are complemented with computational experiments whenever the latter provide an indispensable numerical feel for the questions at hand.

4.1. Deterioration in the State of Aggregate Conditions

Job upgrading in the present context can be thought of as a (disruptive) 'Productivity Improving Activity' (PIA), in the words of Aghion and Saint-Paul (1998), who study the effects of changes in aggregate conditions on firms' incentives to invest in this sort of PIAs. Aghion and Saint-Paul (1998) conclude from their theoretical work that recessions may be the optimal time to invest in PIAs owing to the cyclicality of the opportunity cost of the PIAs. They also suggest that discretionary off-the-job training could be an example of such PIAs. However, Aghion and Saint-Paul do not study the microfoundations of the PIAs they consider, unlike the present paper. Hence it seems interesting to investigate if the present model lends theoretical support to the results found in Aghion and Saint-Paul

[20] The assumption that new matches are engaged in production follows from new matches being relatively productive. This does not imply that there is no training associated with new jobs since it takes a period for jobs to become productive and that there are recruiting costs which could be interpreted as training costs. It is hence fruitful to bear in mind the distinction between initial training and on-going training (simply upgrading). We focus chiefly on the latter.

(1998) by asking the following question: What is the impact of a deterioration in the aggregate state of the economy on the rate of job upgrading, $\lambda(F(e_o)-F(e_d))(1-s)(1-u)$? To answer this question we must examine the response of the cut-offs, e_o and e_d, and of u to a deterioration in the state of the economy. The relevant experiment is to consider a decrease in p.[21] This exercise, like other comparative static exercises, can be analyzed in the context of Figure 3, where p acts as a shift factor of the value functions J^p and J^o; when p decreases, both J^p and J^o decrease. According to Figure 3, this leads to an unambiguous increase in e_d and, consequently, to an increase in the rate of job destruction, $\lambda F(e_d)$. What happens to e_o is not clear since it depends on the relative magnitude of the shifts in J^p and J^o. Hence, what happens to $(e_o - e_d)$ is not clear either. It should also be noted that even if we are able to pin down what happens to $[e_o - e_d]$ after a change in p, the response of the job upgrading rate would depend on the properties of $F(e)$ as well. This feature of the model is reminiscent of models where owing to (ex post) heterogeneity at the individual agent level, the distribution of agents across state-space (in this case, different e's), combined with non-linear adjustment of the agents, has the potential to cause time-varying elasticities of aggregates to the shocks. To abstract from this latter difficulty, it is assumed that $F(e)$ is well approximated by a uniform distribution, at least in the relevant range. However, even under this assumption, it does not suffice to determine which cut-off increases more in response to a decrease in p in order to learn what happens to the rate of job upgrading in the wake of a deterioration in the aggregate state of the economy, since u is endogenous. A sufficiently large fall in employment following a deterioration in the state of aggregate conditions may lead to a procyclical job upgrading rate even if $(F(e_o) - F(e_d))$ is countercyclical.

In the problem depicted in Figure 3, e_o solves (3.22). As a consequence, we are guaranteed that e_o decreases with p as long as opportunity costs exist (ϕ strictly below 1). However, this result is not robust to modifications in the way that the idiosyncratic productivity component and the common productivity component interact to form the current flow benefit. For the sake of argument, suppose for the moment that the current flow benefit for a firm when producing is $p\sigma e$ and when upgrading is $\phi p\sigma e$. Then it is not difficult to show that e_o is defined as:

$$e_o[(1-\phi)p\sigma + \frac{k\sigma p}{1-k}\eta] = -K + \frac{k\sigma p}{1-k}\eta e_u$$

[21] In the present context, a change in p represents an unanticipated once-and-for-all change in aggregate conditions.In Menezes (2000) I consider explicit cycles.

and hence:

$$\frac{de_o}{dp} = \frac{(\phi - 1)e_o + \frac{k}{1-k}\eta(e_u - e_o)}{[(1-\phi) + \frac{k}{1-k}\eta]p} \lesseqgtr 0$$

where $\lesseqgtr 0$ is there to remind us that $\frac{de_o}{dp}$ may be negative, positive, or zero depending on the parameters. For instance, and as expected, if the opportunity cost is low enough (ϕ close to 1), then $\frac{de_o}{dp} > 0$. To better understand this point, recall that the net benefit to job upgrading is the product of three terms: a discount rate (k), a distance in productivity space that the firm travels by upgrading ($\eta(e_u - e)$), and the (unit) valuation of this distance, ($J^{p'}()$):

$$k * \eta(e_u - e) * J^{p'}()$$

If the correct specification of the flow benefit is additive in p and e, then p does not matter for the net benefit, but it does matter for the net cost because of the opportunity cost. If the correct specification is multiplicative in p and e then $J^{p'}$ increases in booms, leading to a procyclical net benefit to upgrading that may outweigh the opportunity cost effect. In the remainder of the paper, we use the additive specification since if the multiplicative specification were followed then an increase in p would lead to an increase in the dispersion of the idiosyncratic productivity components, which would be at odds with Geroski and Gregg (1997), who suggest that the cross-section dispersion of relative profitability is countercyclical. This is especially relevant in light of MP's interpretation of the shocks as (relative) price shocks. It is important, however, to bear in mind that an additive specification biases the model to predict a countercyclical job upgrading rate.

Can we say anything about the increase in e_o following the decrease in p being larger or smaller than the increase in e_d? We first take a closer look at $\frac{de_o}{dp}$:

$$\frac{de_o}{dp} = -\frac{1}{\sigma}\frac{(1-\phi)(1-k)}{(1-\phi)(1-k) + (1-\psi)k\eta} < 0$$

and establish that the increase in e_o when p decreases is larger for high opportunity costs (low ϕ). To further investigate the cyclicality of the job upgrading rate, we must differentiate the system *Final* with respect to p. In terms of Figure 5, following a decrease in p, the JD schedule shifts to the right and the JC schedule shifts to the left. Hence, θ decreases and the impact on e_d may be ambiguous, since the lower θ decreases the value of being unemployed and, consequently, the wage. However, this wage effect is not strong enough to outweigh the direct effect of p on e_d and of e_o on e_d since a higher e_o decreases the option value of keeping a job, which by itself increases destruction. In sum:

	e_o	e_d	θ	u
$p\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$

Returning to the impact on the rate of job upgrading, and since employment falls with p, we look for the conditions under which the rate of job upgrading for a given level of employment increases with p since they are sufficient for a procyclical rate of job upgrading. For that we focus on the relative sensitivity of the cut-offs e_o and e_d to changes in p, that is, on the sign of:

$$\frac{de_o}{dp} - \frac{de_d}{dp}$$

In the Appendix it is shown that the sign of $(\frac{de_o}{dp} - \frac{de_d}{dp})$ depends on parameters and on equilibrium values of the endogenous variables of the model, which in turn depend on labor market policy. Hence, to learn about the importance of parameters we need to take into account that changes in $(\frac{de_o}{dp} - \frac{de_d}{dp})$ depend on: 1) the changes in the parameters and 2) the resulting equilibrium adjustments in the endogenous variables. One parameter that deserves to be singled out is ϕ, since measures of it are particularly hard to come by and hence computational experiments become ever the more informative. More specifically, we ask the following questions: How strong should the opportunity cost motive be for the model to predict countercyclical upgrading? What is the role of labor market policy for the cyclicality of upgrading? Computational experiments described below shed light on these questions.

4.1.1. Model Specification and calibration

Specification The following standard specification of the c.d.f $F()$ is adopted:

$$F(e) = \frac{e - e_l}{e_u - e_l}$$

that is, $F() - U[e_l, e_u]$. For the matching function we follow den Haan et al. (1997) and adopt:

$$m(u, v) = \frac{uv}{(u^\gamma + v^\gamma)^{1/\gamma}}$$

As den Haan et al. (1997) discuss, the above specification is motivated by considering how the matching technology operates on individual workers and firms and has the convenient property of ensuring bounded probabilities $q(\theta)$ and $f(\theta)$, unlike the traditional Cobb-Douglas specification.

Parameterization Table 1 summarizes the choice of parameters. The policy parameters a (the lump-sum employment tax), t (the employment tax rate), H (the job creation subsidy), and ρ (the replacement ratio) were set to zero for the sake of parsimony. The parameters were chosen to deliver reasonable equilibrium values for unemployment (about ten percent or less) and for the expected mean duration of unemployment (about three to four months). In accordance with the analytical results, we restrict our attention to parameterizations of the model that deliver $e_o > e_d$. For that, T (the implicit firing cost) was set at 3.6 (slightly less than six months worth of production for the most productive job), K to 2, and C to 1, all in the same order of magnitude. The remaining parameters are taken from Mortensen and Pissarides (1999a) (with the exceptions of e_l, η, and γ).

Table 1

Notation	Value	Meaning
p	1	common productivity
σ	1	dispersion of idiosyncratic shocks
$[e_l, e_u]$	$[-1, 1]$	support of idiosyncratic shocks
η	1	fraction of productivity gap $(e_u - e)$ closed by upgrading
λ	0.1	arrival rate of idiosyncratic shocks
r	0.02	real interest rate
s	0.05	natural turnover rate
γ	2	matching function parameter
ψ	0.5	worker's share
C	1	creation costs
K	2	training fixed costs
c	0.3	vacancy posting costs
b	0.3	unemployment benefit

Results Figure 6 plots on the y-axis the equilibrium values of $\frac{de_o - de_d}{dp}$ (the response of the upgrading or training 'window', $(e_o - e_d)$), e_o, e_d, and u when we let ϕ vary between 0.1 and 1. Note that $\frac{de_o - de_d}{dp}$ is always positive, which is to say that upgrading incidence is procyclical. Obviously, in the absence of opportunity costs ($\phi = 1$), then $\frac{de_o}{dp} = 0$ and $\frac{de_o - de_d}{dp} > 0$. As opportunity costs increase (ϕ decreases, approaching its natural lower bound of 0), then $\frac{de_o}{dp}$ becomes increasingly negative but not negative enough to lead to a countercyclical upgrading rate. In other words, the destruction margin reacts relatively more to changes in p than the upgrading margin, despite significant firing costs ($T = 3.6$), which would suggest a low tendency to resort to adjustments through the destruction margin. In fact, an increase in firing costs leads to a vertical drop in the $\frac{de_o - de_d}{dp}$ schedule and hence to a less procyclical upgrading rate *ceteris paribus*. However, an increase in firing costs also leads to a decrease in job destruction or to a lower e_d (for a given ϕ). It is clear from the picture that for firing costs to be large enough to induce a countercyclical upgrading rate (graphically, to induce a drop in $\frac{de_o - de_d}{dp}$ large enough such that it crosses the zero line), then it will be optimal to never fire workers, especially, for low opportunity cost values (graphically, e_d would be lower than $e_l = -1$ in Figure 6). Hence, it is very hard to find parameterizations of the model that predict 1) countercyclical upgrading rates (given employment), and 2) that all margins are active,[22] despite assuming an additive specification, which, and as discussed above, biases the model to predict a countercyclical upgrading rate. On top of that, and neglected in the work of Aghion and Saint-Paul, we must bear in mind that the countercyclical nature of unemployment also acts to turn the (relevant) job upgrading rate into a procyclical variable.

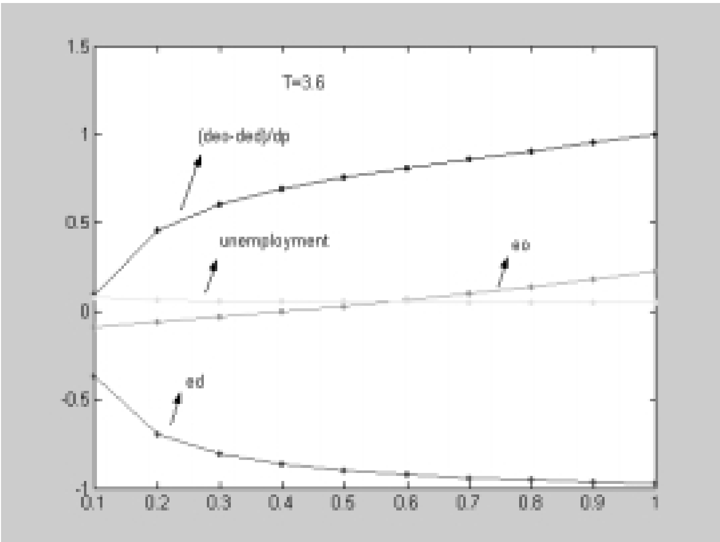


Figure 6. Impact of varying opportunity cost ($\phi = 1$ means no opportunity cost to upgrading)

[22] This second condition rules out the trivial case of prohibitively large firing costs or no endogenous job destruction. If all job destruction is exogenous, then $\frac{de_d}{dp} = 0$ and an additive specification coupled with opportunity costs would suffice for the model to deliver a countercyclical upgrading rate.

4.2. Labor Market Policy

4.2.1. Stronger Employment Protection Legislation

The cost that the firm must bear in order to destroy the match, T, may be thought of as an indicator of the strength of Employment Protection. Conventional wisdom suggests that an increase in T leads to less job destruction. In addition, Garibaldi (1998) shows that the strength of Employment Protection is positively correlated with equilibrium job reallocation (the sum of job destruction and job creation). Hence, some authors hypothesize that strong Employment Protection leads to low levels of productivity growth since job reallocation is seen as a crucial ingredient to productivity growth (see Burgess 1995 or papers that deal with 'Eurosclerosis,' for instance, Blanchard and Portugal 1998). Employment Protection may also be related to job upgrading. In fact, Christensen et al. (1997) argue that stronger Employment Protection leads to more investment in workers' productivity. These papers suggest that the impact of stronger Employment Protection on average productivity across firms is theoretically ambiguous and that a framework like the one presented here, where both job upgrading and job reallocation effects of stronger Employment Protection are analyzed in a unified manner, points the way to the framework necessary to shed further light on this interesting issue.

To consider the impact of an increase in the strength of Employment Protection on the rate of job upgrading, we differentiate the system *Final* with respect to T. Note that T does not affect e_o since it changes neither the net cost nor the net benefit of engaging in upgrading vis à vis engaging in job production (see (3.22)). Figure 5 helps us in understanding the effects of an increase in T. The JC schedule shifts to the left since a given e_d an increase in T lowers the value of new jobs, and by the free-entry-condition the expected costs of new jobs must also fall, which comes about by a fall in θ (which in turn leads to a higher vacancy filling rate and lower vacancy posting costs). Turning to the JD schedule, we find that T shifts JD to the left since, for a given e_d, a higher θ is needed to keep the value of the marginal job engaged in upgrading equal to the value of destruction, $-T$. As θ increases, the value of upgrading decreases since the wage paid to the worker increases and, therefore, the job destruction condition–$J^o(e_d)+T=0$–holds (for a given e_d and higher T). Hence, a graphical inspection of the problem suggests a decrease in e_d and an ambiguous change in θ. However, differentiation of the system *Final* with respect to T

reveals (see Appendix):

$$\begin{array}{cccc} & e_o & e_d & \theta \\ T\uparrow & = & \downarrow & \downarrow \end{array}$$

and thus both e_d and θ fall with an increase in T, as intuition would suggest. As a consequence, job upgrading rises for a given level of employment. Intuitively, there is a range of jobs that would be destroyed with low T but upgraded with high T. Now we must consider the impact of T on u to complete the exercise. For that we turn to the bottom graph of Figure 5 for guidance. We use the result that θ falls and hence JC rotates clockwise. At the same time, and for a given u, the decrease in e_d leads to less job destruction, and hence v must decrease so that job creation also decreases, and hence the Beveridge curve falls. What happens to u is not clear: One the one hand unemployment duration is higher; on the other hand unemployment incidence is lower.[23] This is shown more formally by differentiating the Beveridge curve:

$$\frac{du}{dT}[m(1,\theta)+s+\lambda F(e_d)(1-s)] = -um_v \underset{(-)}{\frac{d\theta}{dT}} + \lambda F'(e_d)(1-s)(1-u)\underset{(-)}{\frac{de_d}{dT}} \lesseqgtr 0$$

where m_v is the partial derivative of $m(u,v)$ with respect to v–a positive number. In the event that a not so large increase in unemployment obtains, perhaps because m_v is small and the duration effect is weak, we find that an increase in T leads to an increase in the job upgrading rate.

To derive aggregate productivity implications of an increase in T we would also consider the effects on equilibrium job reallocation, the sum of equilibrium job destruction and equilibrium job creation $m(u,v)$. Since theory tells us that u and v may move either way, to pin down what happens to equilibrium job creation it would be convenient to think in terms of an 'iso-creation' curve (in (u,v) space, in the bottom graph of Figure 5), a curve along which equilibrium job creation is constant and equal to the initial level, $m(u_0,v_0)$. Using this iso-creation curve, in all similar to an isoquant curve, would allow us to conclude about the sign of the variation in equilibrium job creation, and, concomitantly, in equilibrium job reallocation. While the mechanisms at work are clear, the final results depend heavily on the parameters of the model, including the ones characterizing the relative productivity of upgraded jobs vis à vis new jobs, and the matching function and distribution function $F()$. Nevertheless, the present analysis highlights the potential trade-off between labor market flexibility and training and points to more informative empirical exercises.

[23] This result is found elsewhere in the literature (see Garibaldi (1998) and references therein).

4.2.2. Unemployment Benefit

The productivity effects of unemployment insurance have been studied by Acemoglu and Shimer (1999), among others. In that line of research, the focus lies on the moral-hazard effects of unemployment insurance, on the incentives for unemployed workers to seek high quality jobs and for firms to offer these high quality jobs. Obviously, the present study cannot capture these dimensions of the effects of unemployment insurance on labor productivity. However, and unlike the cited line of research, the present study does capture the importance of unemployment insurance for the firm's investment decisions in the productivity of *existing* jobs in the presence of idiosyncratic and aggregate shocks. Hence it adds to this literature.

The flow benefit of being unemployed is given by $b + \rho\varpi$. Recall that b is interpreted as the part of the unemployment benefit that is not necessarily related to policy (say, the value of household production). We start by looking at the impact of an increase in b.

A higher value of b does not affect e_o (from (3.22)) and leads to a shift of JD to the right, since it pushes the wage up at a given θ. The equilibrium moves down along the JC schedule. Hence, e_d increases, θ decreases, and u increases (see Appendix):

$$\begin{array}{ccccc} & e_o & e_d & \theta & u \\ b\uparrow & = & \uparrow & \downarrow & \uparrow \end{array}$$

A more sophisticated treatment of unemployment insurance may involve looking at the effect of varying the replacement ratio ρ. As discussed in the previous section, for that both JD and JC must be modified using (3.29), which renders analytical exercises relatively uninformative. However, with the aid of a simple computational experiment, we confirm that an increase in ρ has the same qualitative effects as an increase in b, leading to an increase in the average wage, ϖ, at the expense of higher unemployment incidence (higher e_d) and higher unemployment duration (lower θ), mirroring results found in Mortensen and Pissarides (1999a).

In sum, a higher unemployment benefit (either through an increase in b or through an increase in the replacement ratio ρ) leads to a lower job upgrading rate owing to a compound effect of a lower job upgrading rate for a given employment level and to a lower employment level.

4.2.3. Creation Costs

A higher creation subsidy acts as to lower private creation costs $(C - H)$, which does not affect e_o (from (3.22)). However, it shifts JC to the right; for a given e_d (and e_o) the value of new jobs does not change, so a decrease in private creation costs must be accompanied by an increase in the expected cost of posting vacancies, which is brought about by an increase in θ. Hence, we move along the JD schedule to a new equilibrium where both e_d and θ are higher. The rise in e_d leads to a rise in the Beveridge curve. The increase in θ leads to a lower expected duration of being unemployed and, hence, to an ambiguous effect on equilibrium unemployment:

$$\begin{array}{lcccc} & e_o & e_d & \theta & u \\ (C-H)\downarrow & = & \uparrow & \uparrow & ? \end{array}$$

Therefore, as long as the decrease in unemployment duration is not too strong, job upgrading falls. An implication of these results is that new jobs promoted by Active Labor Markets Policies (ALMPs) such as hiring subsidies may come at the expense of investments in the productivity of existing jobs.

4.3. Training Policy

4.3.1. Lump-Sum Subsidies

To study the effects of training policy on equilibrium labor market outcomes and welfare, we analyze the role of policies that affect e_o in a direct manner, that is, that require modifying (3.22). A simple way to introduce training policy and a natural starting point is to consider a lump-sum subsidy conditional on the job being upgraded, reflected by a decrease in K. Such policy increases e_o, pushing training incidence up for a given destruction rate and employment level. The fall in K leads to an improvement in the upgrading margin vis à vis the destruction margin, which pushes destruction down given e_o and θ. However, the increase in e_o pushes destruction up because an increase in e_o leads to a decrease in the option value of keeping the job. The net effect is a decrease in destruction, which translates into a shift of the JD schedule to the left. The increase in e_o also leads to a shift in the JC schedule to the left; given e_d, an increase in e_o causes a fall in the value of new jobs, and for the JC condition to hold the expected cost of new jobs must fall, which is brought about by a lower θ and consequent lower expected duration of vacancies. While graphical analysis suggests that θ may move either way, it can be shown that θ rises. Intuitively, the value of jobs increases, including that of new ones, and by the free-entry-condition the cost of new jobs must also increase, which is accomplished by a rise in θ and, consequently, in the expected duration (and cost) of vacancies ($\frac{c}{\beta q(\theta)}$). Hence, unemployment incidence and unemployment duration decrease. Summing up:

	e_o	e_d	θ	u
$K \downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$

The job upgrading rate for a given employment level unambiguously increases since not only does e_o increase (the upgrading margin improves vis à vis the production margin) but also e_d decreases (the upgrading margin improves vis à vis the destruction margin). In addition, employment increases, further increasing the training rate. Despite an increase in wages for all levels of idiosyncratic productivity–either through a lower K or through a higher θ–computational experiments (not reported) reveal that the average wage tends, if anything, to decrease slightly owing to the decrease in e_d.

Finally, note that we can find differential lump-sum employment subsidies (or taxes) conditional on the job being upgraded or producing in all equivalent, in terms of their impact on labor market equilibrium outcomes, to the lump-sum training subsidy. This is easily seen by replacing a in (3.1) and (3.2) with a^p and a^o respectively (with $a^o < a^p$), and with a^o in (2.13).

4.3.2. Subsidizing Lost Output

An alternative policy to encourage training may encompass compensating the firm for a fraction of the output lost due to the worker receiving training. This would correspond to an increase in ϕ (a decrease in the opportunity cost of training). It can be shown that an increase in ϕ leads to an increase in e_d, and to a shift of JC to the right and to an ambiguous shift of JD. Given e_d and θ, a higher e_o decreases the option value of keeping a job, which by itself pushes destruction up. However, a higher ϕ increases the current flow benefits of the marginal job, thus pushing destruction down. In practice, and for all parameter combinations tried, computational experiments indicate that an increase in ϕ has the same qualitative effects as a lump-sum training subsidy with the exception of the effects on the average wage, which are now nonmonotonic but still small.

4.4. An Empirical Application: Rationalizing Cross-Country Differences in Labor Market Policy and in Equilibrium Labor Market Outcomes

In this section we answer the following question: Do the model's predictions with respect the relation between labor market policy and labor market equilibrium outcomes hold in the data? Since a formal econometric test of the model's predictions is clearly beyond the scope of the paper, we address this question by assessing how well the model's predictions fit the stylized facts on cross-country differences in labor market policy and in labor market equilibrium outcomes. Like Mortensen and Pissarides (1999c), we consider the differences between the US economy and an 'European' economy, as described in Table 2.[24]

Table 2: Stylized facts of US and European labor markets

Object	Symbol	Europe (EU) vs. US
Unemployment rate	u	higher in EU
Expected duration of unemployment	$1/f(\theta)$	higher in EU
Unemployment incidence	$\lambda F(e_d)$	higher in US
Training incidence	$\lambda(F(e_o) - F(e_d))$	higher in EU

Borrowing the characterization of US and EU from Mortensen and Pissarides (1999c) as (*low* b, *low* T) and (*high* b, *high* T) respectively, calibrated versions of the model are consistent with the stylized facts of Table 2. This success is readily explained in light of the comparative statics performed above: a low T decreases training incidence through an increase in unemployment incidence, while a low b has little affect on training incidence since it works in an indirect manner through reservation wages. However, a low b tightens the market, decreasing, thus, unemployment duration, and combines with the low T to deliver a lower unemployment rate.[25]

[24] To conserve on space we refer the reader to Mortensen and Pissarides (1999b) for a discussion on data on unemployment rates, unemployment incidences, and on unemployment durations, and to Lynch (1994, pp. 11), for a discussion on data on international differences in training incidences (job upgrading).

[25] Brunello and Medio (2000) offer an alternative explanation of international differences in education and workplace training. Their model does not feature endogenous job destruction and job creation, and emphasizes instead the role of education. The international differences considered in Brunello and Medio, however, are the same as in Table 2.

5. Conclusions and Future Research

This paper has illustrated the suitability of the class of models of à la Mortensen and Pissarides (1994, 1999a, 1999b) to investigate the firm's decision to invest in the productivity of its workforce in the presence of idiosyncratic and aggregate shocks. The flexibility and transparency of the framework proposed and developed here motivated a number of analytical and numerical exercises that were used to address both the relation between labor market policy and job upgrading, and the relation between training policy and labor market equilibrium outcomes. On a more empirical note, the model was used to rationalize cross-country differences in labor market policy and in labor market equilibrium outcomes.

Of particular interest among the several theoretical results derived is that the model implies that a firm specific training subsidy not only has the desired effect of increasing future productivity but also has the power to reduce unemployment, via lower unemployment incidence and lower unemployment duration, while increasing wages. Hence training subsidies insure the employed against unemployment risk but not at the expense of the unemployed since training subsidies tighten the market and decrease unemployment duration. This result raises other questions, which, while left out in the present work, surely warrant further investigation. For instance: To what extent can training policy be used to undo the effects of Employment Protection, in the spirit of second best theory where distortions are used (read training policy) to undo the impact of some other 'unremovable' distortions (read Employment Protection)? Future research should also pay attention to the financing side of policies, a necessary condition for drawing conclusions about welfare, and to workers' behavior towards risk, a necessary condition for judging the insurance role provided by training subsidies.

The model also served as a new lens through which to study the effects of aggregate shocks on productivity. Computational experiments suggest that training rates are likely to fall in the aftermath of a permanent deterioration in the state of aggregate conditions. The higher sensitivity of the destruction margin to changes in macro conditions relative to the upgrading margin, even in environments with strong Employment Protection, leave the training rate a procyclical variable despite procyclical (opportunity) costs of training. A fuller treatment of cycles and training is, however, beyond the scope of this paper and is given in Menezes (2000). Nevertheless, it is clear that the model paves the way for a novel study of the importance of the upgrading margin in the context of the paradigm propagation-amplification so commonly found in the business cycles literature. The model also motivates the question: Does adding the upgrading dimension to the models à la Mortensen and Pissarides improve their ability to explain (some aspects of) employment dynamics?

A different research path would be to incorporate capital to study the interactions between upgrading labor and upgrading capital, a path also suggested by Cooper, Haltiwanger, and Power (1999), who look at the capital upgrading side of the story and are a valuable source of insights.

6. Appendix

6.1. Closing the Extended Model and Measures of Interest

6.1.1. Job Destruction

Assuming $J^v = 0$ the appropriate starting point is:

$$J^o(e_d) + T = 0 \tag{6.1}$$

or when we recall the definition of $J^o(e)$ (see (3.16)):

$$0 = \phi(p + \sigma e_d) - a - (1+t)w(e_d) - K + (1 - \beta(1-s)\lambda)T + \beta(1-s)\lambda(E[J(x)] + T]) + kJ^p(e_d + \eta(e_u - e_d)) \tag{6.2}$$

$w(e_d)$ is endogenous and must be eliminated. For that, we first rewrite (3.6) as follows:

$$(1-k)(W(e) - U) = w(e) + \beta(1-s)\lambda(E[W(x)] - U) - (1-\beta)U \tag{6.3}$$

Using (3.7) we substitute out $(1-\beta)U$:

$$(1-k)(W(e) - U) = w(e) + \beta(1-s)\lambda(E[W(x)] - U) - (b + \rho\varpi) - \beta f(\theta)(W_o - U) \tag{6.4}$$

Recall that the free-entry-condition implies:

$$J_o - (C - H) = \frac{c}{\beta q(\theta)}$$

which can be used in (6.4) since $(W_o - U)$ and $J_o - (C - H)$ are related through the sharing rules. Thus (6.4) when evaluated at e_d becomes (recall $f(\theta) = q(\theta)\theta$):

$$(1-k)(W(e_d) - U) = w(e_d) + \beta(1-s)\lambda(E[W(x)] - U) - (b + \rho\varpi) - \frac{\psi}{(1-\psi)(1+t)}\theta c$$

or:

$$0 = w(e_d) + \beta(1-s)\lambda(E[W(x)] - U) - (b + \rho\varpi) - \frac{\psi}{(1-\psi)(1+t)}\theta c \tag{6.5}$$

since $W(e_d) - U = 0$ by (jointly) efficient terminations. Using (6.5) we can substitute out $w(e_d)$ from (6.2) to obtain the final expression for the Job Destruction condition:

$$\begin{aligned} 0 = {} & \phi(p + \sigma e_d) - a - (1+t)(b + \rho\varpi) - K - \frac{\psi}{1-\psi}\theta c + (1 - \beta(1-s))T + \\ & + \frac{\beta(1-s)\lambda}{1-\psi}[J^{o'} \int_{e_d}^{e_o} (1 - F(x))dx + J^{p'} \int_{e_o}^{e_u} (1 - F(x))dx] + \\ & + k[J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o) - (1-\eta)J^{p'}(e_u - e_d)] \end{aligned} \tag{6.6}$$

where we used the following equalities:

$$E[W(x)] - U = \frac{\psi}{(1-\psi)(1+t)+\psi}E[S(x)] = \frac{\psi}{(1-\psi)(1+t)}(E[J(x)] + T)$$

$$E[J(x)] = -T + J^{o'} \int_{e_d}^{e_o} (1 - F(x))dx + J^{p'} \int_{e_o}^{e_u} (1 - F(x))dx$$

$$J^p(e_d + \eta(e_u - e_d)) = -T + J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o) - (1-\eta)J^{p'}(e_u - e_d)$$

The first two of the above three equalities were derived in sections 2 and 3. The last one simply explores the linear nature of the value functions.

6.1.2. Job Creation

The starting point follows from the free-entry-condition:

$$J_o = \frac{c}{\beta q(\theta)} + (C - H) \tag{6.7}$$

To evaluate the left-hand-side of (6.7) in terms of parameters of the model and of endogenous variables we use the wage equations (3.17) and (3.18) to rewrite the value functions J_o and $J^p(e)$:

$$(1-k)J_o = (1-\psi)[p + \sigma e_u - a - (1+t)(b + \rho\varpi)] + \psi[\beta(1-s)\lambda T + (1-k)(C - H) - \theta c] + \beta(1-s)\lambda E[J(x)] \tag{6.8}$$

$$(1-k)J^p(e) = (1-\psi)[p+\sigma e - a - (1+t)(b+\rho\varpi)] - \psi[(1-\beta(1-s))T + \theta c] + \beta(1-s)\lambda E[J(x)], e \geq e_o \quad (6.9)$$

Now rewrite the left-hand-side of (6.7) as:

$$J_o = (J_o - J^p(e_u)) + J^p(e_u)$$

which is convenient since $(J_o - J^p(e_u))$ reduces to (using (6.8) and (6.9)):

$$J_o - J^p(e_u) = \psi[(C-H)+T]$$

and $J^p(e_u)$ is simply (recall (2.11)):

$$J^p(e_u) = -T + J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o)$$

Putting all the pieces together we have the Job Creation condition:

$$J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o) = \frac{c}{\beta q(\theta)} + (1-\psi)[(C-H)+T]$$

6.1.3. Average Wage

The average wage is defined as:

$$\varpi = w_o F(e_d) + \int_{e_d}^{e_o} w^o(x) dF(x) + \int_{e_o}^{e_u} w^p(x) dF(x) \quad (6.10)$$

In steady state, the fraction of jobs corresponding to new matches equals the fraction of destroyed matches, $F(e_d)$, and these jobs pay w_o. The second and third terms capture the wages earned by workers engaged in upgrading $(w^o(x))$ and in production $(w^p(x))$ in continuing matches. After proper integration by parts we rewrite the right-hand-side of (6.10) as:

$$((w_o - w^p(e_u)) + w^p(e_u))F(e_d) + \{w^o(e_o)F(e_o) - w^o(e_d)F(e_d) - w^{o'} \int_{e_d}^{e_o} F(x)dx\} +$$
$$+ \{w^p(e_u)F(e_u) - w^p(e_o)F(e_o) - w^{p'} \int_{e_o}^{e_u} F(x)dx\}$$

which simplifies to:

$$-\frac{\psi(1-k)}{1+t}[(C-H)+T]F(e_d) + w^p(e_u)F(e_d) - w^o(e_d)F(e_d) - w^{o'} \int_{e_d}^{e_o} F(x)dx + w^p(e_u) - w^{p'} \int_{e_o}^{e_u} F(x)dx$$

since $w^o(e_o) = w^p(e_o)$, $F(e_u) = 1$ and $(w_o - w^p(e_u))$ is evaluated using (3.17) and (3.18). We substitute out $w^p(e_u)$ using the following equality:

$$w^p(e_u) = w^o(e_d) + w^{o'}(e_o - e_d) + w^{p'}(e_u - e_o)$$

and obtain:

$$[-\frac{\psi(1-k)}{1+t}((C-H)+T) + w^{o'}(e_o - e_d) + w^{p'}(e_u - e_o)]F(e_d) +$$
$$+ w^o(e_d) + w^{o'}\int_{e_d}^{e_o}(1-F(x))dx + w^{p'}\int_{e_o}^{e_u}(1-F(x))dx$$

Now recall from (6.5) that $w^o(e_d)$ reads:

$$0 = w(e_d) + \beta(1-s)\lambda(E[W(x)] - U) - (b + \rho\varpi) - \frac{\psi}{(1-\psi)(1+t)}\theta c$$

which is solved for $w^o(e_d)$ after we use the sharing rules to replace $(E[W(x)] - U)$ by $\frac{\psi(E[J(x)]+T)}{(1-\psi)(1+t)}$ since the latter is already familiar (see (2.12)). Finally, note that the slopes of the $w()$ functions are as follows (using (3.18) and (6.5)):

$$\begin{cases} w^{p'} = \frac{\psi\sigma}{1+t} \\ w^{o'} = \frac{\psi\sigma}{(1+t)(1-k\psi)}[\phi(1-k) + k(1-\psi)(1-\eta)] \end{cases}$$

and hence we can write $w^{p'}$ and $w^{o'}$ in terms of the more familiar $J^{p'}$ and $J^{o'}$:

$$w^{p'} = \frac{\psi(1-k)}{(1+t)(1-\psi)}J^{p'}$$

$$w^{o'} = \frac{\psi(1-k)}{(1+t)(1-\psi)}J^{o'}$$

which allows us to obtain the final expression for ϖ:

$$(1-\rho)\varpi = b + \frac{\psi}{(1-\psi)(1+t)}\theta c + [-\frac{\psi(1-k)}{1+t}((C-H)+T)]F(e_d) + \tag{6.11}$$
$$+ [J^{o'}(e_o - e_d) + J^{p'}(e_u - e_o)]\frac{\psi(1-k)}{(1+t)(1-\psi)}F(e_d) +$$
$$+ \frac{(1-\beta(1-s))\psi}{(1+t)(1-\psi)}[J^{o'}\int_{e_d}^{e_o}(1-F(x))dx + J^{p'}\int_{e_o}^{e_u}(1-F(x))dx]$$

6.2. Comparative Statics Exercises

6.2.1. p

We now differentiate the system *Final* with respect to p. Due to the recursive structure of the system, we obtain $\frac{de_o}{dp}$ solely from (3.22):

$$\frac{de_o}{dp} = -\frac{1}{\sigma}\frac{(1-\phi)(1-k)}{(1-\phi)(1-k)+(1-\psi)k\eta} < 0 \tag{6.12}$$

To better handle the algebra we introduce $k_o \equiv \beta(1-s)(1-\lambda F(e_o))$ and $k_d \equiv \beta(1-s)(1-\lambda F(e_d))$ and rewrite the Job Destruction condition (3.24) as:

$$0 = JD(e_o, e_d, \theta, .)$$

Then we have:

$$0 = \frac{\partial JD}{\partial p} + \frac{\partial JD}{\partial e_d}\frac{de_d}{dp} + \frac{\partial JD}{\partial \theta}\frac{d\theta}{dp} + \frac{\partial JD}{\partial e_o}\frac{de_o}{dp}$$

where:

$$\begin{cases} \frac{\partial JD}{\partial p} = \phi \\ \frac{\partial JD}{\partial e_d} = \frac{1-k_d}{1-\psi}J^{o'} \\ \frac{\partial JD}{\partial \theta} = -\frac{\psi c}{1-\psi} \\ \frac{\partial JD}{\partial e_o} = \frac{k_o - k\psi}{1-\psi}(J^{o'} - J^{p'}) \end{cases}$$

and $\frac{de_o}{dp}$ is given by (6.12). Similarly, differentiating the Job Creation condition we find:

$$(J^{o'} - J^{p'})\frac{de_o}{dp} - J^{o'}\frac{de_d}{dp} = -\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}\frac{d\theta}{dp}$$

which is solved for $\frac{d\theta}{dp}$:

$$\underbrace{(J^{o'} - J^{p'})}_{(-)}\underbrace{[\frac{1-k_d+k_o-k\psi}{1-k_d}]}_{(+)}\underbrace{\frac{de_o}{dp}}_{(-)} + \underbrace{\frac{1-\psi}{1-k_d}}_{(+)}\phi = [-\underbrace{\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}}_{(+)} + \underbrace{\frac{\psi c}{1-k_d}}_{(+)}]\frac{d\theta}{dp}$$

Hence, $\frac{d\theta}{dp} > 0$. Similarly, we can show that $\frac{de_d}{dp} < 0$. However, the algebra for $\frac{de_d}{dp}$ becomes intractable unless we assume $\psi = 0$. In this case, $\frac{\partial JD}{\partial \theta} = 0$ and we have:

$$\frac{de_d}{dp} = \underbrace{(\frac{\partial JD}{\partial e_d})^{-1}}_{(+)}[-\phi + \underbrace{(J^{o'} - J^{p'})}_{(-)}k_o\underbrace{\frac{de_o}{dp}}_{(-)}] < 0$$

A decrease in p leads to an increase in e_o, which shifts the Job Creation schedule to the left and the Job Destruction schedule to the right. Hence, graphical analysis shows that θ decreases and e_d may move either way. After differentiating the system, we find as expected that e_d increases with the fall in p. When $\psi = 0$ the wage is given by b, the reservation wage, and the Job Destruction schedule is vertical since e_d does not depend on θ. Then the shift to the right of the Job Destruction schedule leads to an increase in e_d. When $\psi > 0$ the fall in θ leads to a lower value of being unemployed and consequently to a lower wage which decreases destruction. However, this effect is not strong enough to lead to a procyclical destruction rate.

6.2.2. T

Since T does not enter in (3.22) we establish $\frac{de_o}{dT} = 0$. Then we obtain from differentiating the Job Destruction and Job Creation conditions the following system on $\frac{de_d}{dT}$ and $\frac{d\theta}{dT}$:

$$0 = \frac{\partial JD}{\partial e_d}\frac{de_d}{dT} + \frac{\partial JD}{\partial \theta}\frac{d\theta}{dT} + (1 - \beta(1-s))$$

$$-J^{o'}\frac{de_d}{dT} = -\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}\frac{d\theta}{dT} + (1-\psi)$$

which yields:

$$[\underbrace{-\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}}_{(+)} + \underbrace{\frac{\psi c}{1-k_d}}_{(+)}]\frac{d\theta}{dT} = \underbrace{\frac{-(1-\psi)\beta(1-s)\lambda F(e_d)}{1-k_d}}_{(-)} \Longrightarrow \frac{d\theta}{dT} < 0$$

$$\frac{de_d}{dT} = \underbrace{(\frac{\partial JD}{\partial e_d})^{-1}}_{(+)}[-\underbrace{\frac{\partial JD}{\partial \theta}}_{(-)}\underbrace{\frac{d\theta}{dT}}_{(-)}\underbrace{-(1-\beta(1-s))}_{(-)}] \Longrightarrow \frac{de_d}{dT} < 0$$

6.2.3. b

Since b does not enter in (3.22) we establish $\frac{de_o}{db} = 0$. Then we obtain from differentiating the Job Destruction and Job Creation conditions the following system on $\frac{de_d}{db}$ and $\frac{d\theta}{db}$:

$$0 = \frac{\partial JD}{\partial e_d}\frac{de_d}{db} + \frac{\partial JD}{\partial \theta}\frac{d\theta}{db} - (1+t)b$$

$$-J^{o'}\frac{de_d}{db} = -\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}\frac{d\theta}{db}$$

which yields:

$$[\underbrace{-\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}}_{(+)} + \underbrace{\frac{\psi c}{1-k_d}}_{(+)}]\frac{d\theta}{db} = \underbrace{\frac{-(1-\psi)(1+t)}{1-k_d}}_{(-)} \Longrightarrow \frac{d\theta}{db} < 0$$

$$\frac{de_d}{db} = \underbrace{(\frac{\partial JD}{\partial e_d})^{-1}}_{(+)}(1+t)[\underbrace{\frac{-cq'(\theta)(1-k_d)}{-cq'(\theta)(1-k_d)+\psi c\beta q(\theta)^2}}_{(+)}] \Longrightarrow \frac{de_d}{db} > 0$$

6.2.4. $(C-H)$

Since $(C-H)$ does not enter in (3.22) we establish $\frac{de_o}{d(C-H)} = 0$. Then we obtain from differentiating the Job Destruction and Job Creation conditions the following system on $\frac{de_d}{d(C-H)}$ and $\frac{d\theta}{d(C-H)}$:

$$0 = \frac{\partial JD}{\partial e_d}\frac{de_d}{d(C-H)} + \frac{\partial JD}{\partial \theta}\frac{d\theta}{d(C-H)}$$

$$-J^{o'}\frac{de_d}{d(C-H)} - (1-\psi) = -\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}\frac{d\theta}{d(C-H)}$$

which yields:

$$[\underbrace{-\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}}_{(+)} + \underbrace{\frac{\psi c}{1-k_d}}_{(+)}]\frac{d\theta}{d(C-H)} = \underbrace{-(1-\psi)}_{(-)} \Longrightarrow \frac{d\theta}{d(C-H)} < 0$$

$$\frac{de_d}{d(C-H)} = \underbrace{(\frac{\partial JD}{\partial e_d})^{-1}}_{(+)}[\underbrace{\frac{\beta q(\theta)^2(1-k_d)}{-cq'(\theta)(1-k_d)+\psi c\beta q(\theta)^2}}_{(+)}][\underbrace{-(1-\psi)}_{(-)}] \Longrightarrow \frac{de_d}{d(C-H)} < 0$$

6.2.5. Training Policy

Fall in K WLOG we assume $\eta = 1$. From (3.22) we find:

$$\frac{de_o}{dK} = -\frac{1}{\sigma}\frac{1-k}{(1-\phi)(1-k)+(1-\psi)k} < 0$$

and from the Job Destruction and Job Creation conditions:

$$\frac{de_d}{dK} = (\frac{\partial JD}{\partial e_d})^{-1}\{1 - \frac{\partial JD}{\partial \theta}\frac{d\theta}{dK} - \frac{\partial JD}{\partial e_o}\frac{de_o}{dK}\} \tag{6.13}$$

$$(J^{o'} - J^{p'})\frac{de_o}{dK} - J^{o'}\frac{de_d}{dK} = -\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}\frac{d\theta}{dK} \tag{6.14}$$

The fall in K and consequent rise in e_o lead to a horizontal shift of Job Destruction to the left (set $\frac{d\theta}{dK} = 0$ in (6.13)):

$$\begin{aligned}\frac{de_d}{dK} &= (\frac{\partial JD}{\partial e_d})^{-1}\{1 - \frac{\partial JD}{\partial e_o}\frac{de_o}{dK}\} = \\ &= (\frac{\partial JD}{\partial e_d})^{-1}\frac{(1-k_o)[1-(k\psi+\phi(1-k))]}{(1-k\psi)((1-\phi)(1-k)+k(1-\psi))} > 0\end{aligned}$$

The increase in e_o leads to a horizontal shift of Job Creation to the left (set $\frac{d\theta}{dK} = 0$ in (6.14)):

$$J^{o'}\frac{de_d}{dK} = (\underset{(-)}{J^{o'} - J^{p'}})\frac{de_o}{dK}$$

Therefore e_d falls and θ may move either way. Solving (6.13) and (6.14) for $\frac{d\theta}{dK}$ we find:

$$[\underbrace{-\frac{c\beta q'(\theta)}{(\beta q(\theta))^2}}_{(+)} + \underbrace{\frac{\psi c}{1-k_d}}_{(+)}]\frac{d\theta}{dK} = \frac{1-\psi}{1-k_d}\frac{(k_d-k_o)[\overbrace{(k\psi+\phi(1-k))-1}^{(-)}]}{(1-k\psi)((1-\phi)(1-k)+k(1-\psi))} < 0$$

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