

# Unequal Longevities and Lifestyles Transmission\*

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## Abstract

This paper studies an economy where unequal longevities are the unintended outcome of differences in lifestyles, and considers the optimal tax transfer policy in such a setting. For that purpose, we develop a three-period OLG model where the population, who ignores the negative impact of excessive work on longevity, is partitioned in two groups with different tastes for leisure, and follows an adaptation/imitation process *à la* Bisin and Verdier (2001). The existence, uniqueness and stability of steady-states are studied. It is shown that the optimal short-run and long-run Pigouvian taxes on wages differ, because the latter corrects not only agents's myopia, but, also, the unintended impact of leisure choices on future cohorts's tastes.

*Keywords:* longevity, OLG model, lifestyle, socialization, Pigouvian taxes.

*JEL codes:* I12, I18, Z13.

## 1 Introduction

Although longevity differentials can be partly explained by genetic and environmental factors, *how long* one lives depends also on *how* one lives. Actually, various dimensions of lifestyles are shown to affect longevity, so that unequal longevities reflect - among other things - the diversity of lifestyles adopted by people.<sup>1</sup>

The existence of longevity inequalities due to heterogeneous lifestyles raises the question of the necessity, for a government, to intervene in that context. In particular, should a utilitarian government do anything in front of such inequalities in longevity due to lifestyle differentials?

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<sup>1</sup>Longevity was shown to be affected, *ceteris paribus*, by smoking (Doll and Hill, 1950), by alcoholism (Poikolainen, 1982), by the quantity and quality of food (Bender *et al*, 1998; Stamler, 1973), and by physical activity (Paffenbarger *et al*, 1978).

The goal of this paper is to study the optimal taxation policy in an economy where unequal longevity result from differences in lifestyles. As in any optimal taxation problem, the form of the optimal policy under endogenous longevity depends not only on how ‘rational’ agents are in their decisions (i.e. on the extent to which one’s lifestyle maximizes one’s lifetime welfare), but, also, on how individual behaviours affect social welfare (i.e. on the extent to which one’s lifestyle affects others’s welfares). In order to do justice to those two determinants of the optimal public intervention, the specificity of this paper shall be twofold.

First, this paper focuses on an economy where differentials in longevity are the *unintended* outcome of differences in lifestyles, in the sense that agents suffer from a myopia preventing them from seeing the impact of their lifestyle on longevity. Hence, under such a myopia, the chosen lifestyle is non-optimal from the point of view of lifetime welfare maximization, and, thus, invites a correction by the State.

Second, this paper considers another unintended effect of lifestyles: their impact on the *taste composition* of future cohorts. Actually, the partition of a population into groups of agents with different lifestyles is not exogenous, but is affected by agents’s choices. Hence, given that agents’s lifestyles shape the tastes composition of next cohorts, the resulting unintended welfare effects invite also a correction.

At this stage, it should be stressed that the corrections required for the internalization, at the individual level, of the impact of one’s lifestyle decision on one’s own lifetime welfare may differ from the corrections required for the internalization of the ‘composition effects’ of one’s decisions on future cohorts’s welfare. Hence, it is not straightforward to see what a utilitarian government should do in this context.

This paper aims at casting a new light on that issue, by studying the optimal public policy in an economy where unequal longevity are the unintended outcome of differences in lifestyles, and where the partition of the population in distinct lifestyles is endogenous. That issue has not been studied so far, as studies on the optimal policy under endogenous longevity focussed on homogeneous populations (see Zhang *et al*, 2006), or involved some unexplained heterogeneity (see Chakraborty, 2004; Pestieau *et al*, 2008).<sup>2</sup> On the contrary, this paper studies optimal policy under unequal longevity due to a well-identified cause, the adopted lifestyle.

For that purpose, we set up a three-period OLG model, where the length of the third period depends on the lifestyle adopted during the second period. For simplicity, agents differ on a *single* dimension of lifestyle: the length of working time, following Nylen *et al*’s (2001) study showing that overwork leads, *ceteris paribus*, to a shorter life.<sup>3</sup> Thus, each cohort, who ignores the impact of work

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<sup>2</sup>By ‘unexplained heterogeneity’, I mean that nothing explains, in the model, why some members of a cohort live longer than others.

<sup>3</sup>See also Van Echtelt (2005) on the negative impact of overwork on longevity. Given the tendency of extra work to favor hart-diseases (see Hayashi *et al*, 1996), diabetes (see Kawakami *et al*, 1999) and obesity (see Nakamura *et al*, 1998), that relation is quite robust.

on longevity, is, at adulthood, divided into two types of agents, having a more or less strong taste for leisure, and choosing their working time accordingly.

To study the optimal public policy under an endogenous composition of the population, it is assumed that the adherence to a lifestyle is acquired, during childhood (first period), through an adaptation and imitation process *à la* Bisin and Verdier (2001), where parents's socialization efforts and the social environment interact.

The major contribution of this paper is to highlight that the policies required for the decentralizations of the short-run social optimum - i.e. under a fixed partition of the population - and of the long-run social optimum - i.e. under an endogenous partition of the population - differ significantly, as the latter involves not only a correction for agents's myopia, but, also, a correction for unintended 'composition' effects on future cohorts's tastes under the postulated socialization process.

This rest of this paper is organized as follows. Section 2 presents the model, describes the temporary equilibrium under laissez-faire, and examines the existence, uniqueness and stability of steady-state equilibria. Section 3 describes the short-run and long-run social optima, and studies their decentralization. Section 4 concludes.

## 2 The model

### 2.1 Environment

Let us consider a three-period OLG model. Each cohort is a continuum of agents, with a measure normalized to 1.

Period 1, which is of unitary length, is a period of childhood, during which one has no consumption, does not work, and is only subject to socialization.<sup>4</sup> During period 2, which is also of unitary length, individuals work, enjoy some leisure and consume. Period 3 is a period of retirement, whose length depends on the lifestyle adopted during the second period of life.

During their childhood (period 1), all members of a cohort are the same, and have no cultural type. However, once adult (in periods 2 and 3), a cohort becomes divided in two kinds of agents, depending on their preferences. We shall, in this paper, assume that the two kinds of agents differ only regarding the intensity of their taste for leisure with respect to consumption. Thus, each adult agent within a cohort is either of type  $C$ , i.e. 'consumption-lover', or of type  $L$ , i.e. 'leisure-lover'. Agents of type  $i \in \{C, L\}$  work during their second period of life, have one child, and are retired in period 3.

Throughout the paper, the number of agents born at time  $t$  who are of type  $C$  when being young adults (at time  $t+1$ ) shall be denoted by  $q_{t+1}$  ( $0 \leq q_{t+1} \leq 1$ ), while the number of young adults of type  $L$  is equal to  $1 - q_{t+1}$ .<sup>5</sup>

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<sup>4</sup>Hence, the model behaves like a two-period OLG model.

<sup>5</sup>Given that the population is of unitary size,  $q_{t+1}$  can also be interpreted as the proportion of the population of young adults at time  $t+1$  that is of type  $C$ .

**Longevity** Agents of type  $i \in \{C, L\}$  born at  $t$  live the first two periods, of length one. The third period, which is a period of retirement, has a length  $h_{t+2}^i$  ( $0 \leq h_{t+2}^i \leq 1$ ) depending on the lifestyle adopted during the second period.  $h_{t+2}^i$  depends on leisure time when being young adult,  $l_{t+1}^i$  ( $0 \leq l_{t+1}^i \leq 1$ ):

$$h_{t+2}^i = h(l_{t+1}^i) \quad (1)$$

It is assumed that  $h(0) = 0$ ,  $h'(l_{t+1}^i) > 0$ ,  $h''(l_{t+1}^i) < 0$ .<sup>6</sup>

For analytical convenience, we shall, in this paper, assume that:

$$h(l_{t+1}^i) = \frac{l_{t+1}^i}{1 + l_{t+1}^i} \quad (2)$$

That functional form, based on Chakraborty (2004), satisfies the above conditions, and allows the analytical solvability of the model.

**Production** For simplicity, production is assumed to be linear in labour:

$$y_t = wL_t \quad (3)$$

where  $y_t$  is the output,  $w$  is a productivity parameter, and  $L_t$  is the total labour force, defined as  $L_t = q_t(1 - l_t^C) + (1 - q_t)(1 - l_t^L)$ , where  $1 - l_t^i$  is the labour supplied by agents of type  $i \in \{C, L\}$ .<sup>7</sup> Factors are paid at their marginal productivity: each unit of labour supplied is paid by a wage  $w$ .

**Socialization** The population follows an adaptation and imitation process *à la* Bisin and Verdier (2001). The transmission of the cultural trait  $i \in \{C, L\}$  is modelled as a mechanism where socialization *inside* the family ('direct vertical' socialization) and socialization *outside* the family ('oblique' socialization) interact.

Families are composed of a single parent and a single child. Children are born at time  $t$  without any defined cultural trait, and are first exposed to their parent's lifestyle. Direct vertical socialization to the parent's trait  $i \in \{C, L\}$  occurs with a probability  $\rho_{t+1}^i$ . If the direct vertical socialization to the parent's trait does not take place, which happens with a probability  $1 - \rho_{t+1}^i$ , the child will then pick up the trait of a person - i.e. a 'role model' - chosen randomly in the population of young adults, and, thus, will pick the trait  $C$  with a probability  $q_t$ , and the trait  $L$  with a probability  $1 - q_t$ .

Hence, if  $p_{t+1}^{CC}$  and  $p_{t+1}^{CL}$  (resp.  $p_{t+1}^{LL}$  and  $p_{t+1}^{LC}$ ) denote the probabilities that a child born at  $t$  in a family with trait  $C$  (resp.  $L$ ) is socialized to, respectively, trait  $C$  and trait  $L$  (resp.  $L$  and  $C$ ), the transition probabilities are:

$$\begin{aligned} p_{t+1}^{CC} &= \rho_{t+1}^C + (1 - \rho_{t+1}^C) q_t & p_{t+1}^{CL} &= (1 - \rho_{t+1}^C) (1 - q_t) \\ p_{t+1}^{LL} &= \rho_{t+1}^L + (1 - \rho_{t+1}^L) (1 - q_t) & p_{t+1}^{LC} &= (1 - \rho_{t+1}^L) q_t \end{aligned} \quad (4)$$

<sup>6</sup> On the impact of work on longevity, see Nylen *et al* (2001) and Van Ectelt (2005).

<sup>7</sup> The labour from the two types of agents are equally productive.

By the Law of Large Numbers,  $p_{t+1}^{ij}$  is also equal to the proportion of children whose parents are of type  $i$  who have the cultural trait  $j$ . Hence, the proportion  $q_{t+1}$  of agents born at time  $t$  who become of type  $C$  follows the dynamic law:

$$q_{t+1} = [\rho_{t+1}^C + (1 - \rho_{t+1}^C) q_t] q_t + [(1 - \rho_{t+1}^L) q_t] (1 - q_t) \quad (5)$$

The first term of (5) is the probability to be socialized to trait  $C$  when having a family of type  $C$ , multiplied by the probability to belong to a family of type  $C$ , while the second term is the probability to acquire trait  $C$  when being born in a family of type  $L$ , multiplied by the probability to belong to a family of type  $L$ .

Following Bisin and Verdier, we shall assume that parents of type  $i \in \{C, L\}$  can socialize their children born at time  $t$  vertically, by educating them through a socialization effort  $e_t^i$  ( $0 \leq e_t^i \leq 1$ ). The socialization effort  $e_t^i$  constitutes an input in the cultural production of their children as adults:  $\rho_{t+1}^i = \rho(e_t^i)$ . A welfare loss  $C(e_t^i)$  is generated by a socialization effort  $e_t^i$ .<sup>8</sup>

Parents have a welfare gain from coexisting with children with the *same* type as themselves. The welfare derived by a parent of type  $i$  born at  $t - 1$  when he coexists with a child of type  $i$ , denoted by  $\varphi_{t+1}^{ii}$ , exceeds the welfare derived by a parent of type  $i$  when he coexists with a child of type  $j \neq i$ , denoted  $\varphi_{t+1}^{ij}$ . In this paper, welfare gains from having children of one's own type are independent from one's type and from time ( $\varphi_{t+1}^{ii} = \varphi_{t+1}^{jj} = \hat{\varphi}$  and  $\varphi_{t+1}^{ij} = \varphi_{t+1}^{ji} = \tilde{\varphi}$ ).

Hence, under additive lifetime welfare, the socialization effort  $e_t^i$  chosen by a parent of type  $i$  maximizes:

$$u_{t-1}^i = u(c_t^i, l_t^i) - C(e_t^i) + E(h_{t+1}^i) \left[ u(E(d_{t+1}^i), 1) + p_{t+1}^{ii}(e_t^i)\hat{\varphi} + p_{t+1}^{ij}(e_t^i)\tilde{\varphi} \right] \quad (6)$$

where  $u(c_t^i, l_t^i)$  is the utility of the second period, which involves a consumption  $c_t^i$ , and a leisure  $l_t^i$ , and  $E(h_{t+1}^i) u(E(d_{t+1}^i), 1)$  is the expected utility of the third period, which involves an (expected) annuitized consumption  $E(d_{t+1}^i)$ , and no work.  $E(h_{t+1}^i)$  is the expected length of period 3.<sup>9</sup>

Parents, when choosing  $e_t^i$ , weight the cost of socialization -  $C(e_t^i)$  - against its expected gains -  $p_{t+1}^{ii}(e_t^i)\hat{\varphi} + p_{t+1}^{ij}(e_t^i)\tilde{\varphi}$  -, which depend on the influence of their effort  $e_t^i$  on probabilities  $p_{t+1}^{ii}$  and  $p_{t+1}^{ij}$ , determined by the relation  $\rho_{t+1}^i = \rho(e_t^i)$ .

While there exist various ways to model that relation, we shall assume that the probability of direct vertical socialization to trait  $i$   $\rho_{t+1}^i$  equals parent's socialization effort  $e_t^i$ , in conformity with what Bisin and Verdier (2001) call the '*It's the family*' transmission technology:

$$\rho_{t+1}^i = e_t^i \quad (7)$$

That technology, which is most adequate for our purpose, exhibits what Bisin and Verdier call 'cultural substitution': a higher proportion of agents of type  $i$  in the population of reference makes parents of type  $i$  choose a *lower* effort.<sup>10</sup>

<sup>8</sup>We assume  $C(0) = 0$ ,  $C(e_t^i) > 0$  for  $e_t^i > 0$ ,  $C'(e_t^i) > 0$  and  $C''(e_t^i) > 0$ .

<sup>9</sup>For simplicity, there is no pure time preference here.

<sup>10</sup>As shown by Bisin and Verdier (2001), technologies with no cultural substitutability can

**Agents's decisions** In order to describe how agents of types  $C$  and  $L$  make their decisions, we need to make three additional assumptions.

First, temporal welfare  $u(c_t^i, l_t^i)$  is, for analytical convenience, assumed to take a Cobb-Douglas form in consumption and leisure time:

$$u(c^i, l^i) = (c^i)^{\theta^i} (l^i)^{1-\theta^i} \quad (8)$$

where  $0 < \theta^i < 1$ . Given that  $\theta^i$  reflects the importance of consumption with respect to leisure, we have here  $\theta^C > \theta^L$ .

Second, each agent - whatever his type - does not, when making his decisions, internalize its impact on longevity, but, rather, forms myopic anticipations, in the sense that he expects to have a life as long as the one of his 'model' adult:

$$E(h_{t+1}^i) = h_t^i \quad (9)$$

Third, the welfare cost from socialization efforts,  $C(e_t^i)$ , takes the following form:

$$C(e_t^i) = \frac{\xi(e_t^i)^2}{2} \quad (10)$$

where  $\xi$  accounts for the disutility of effort ( $\xi > 0$ ).

Substituting  $u(c^i, l^i)$ ,  $E(h_{t+1}^i)$  and  $C(e_t^i)$  in the function  $u_{i,t-1}$  allows us to rewrite the expected lifetime welfare of an agent of type  $i$  born at  $t-1$  as:<sup>11</sup>

$$u_{t-1}^i = (c^i)^{\theta^i} (l^i)^{1-\theta^i} - \frac{\xi(e_t^i)^2}{2} + h_t^i \left[ (d_{t+1}^i)^{\theta^i} (1)^{1-\theta^i} + p_{t+1}^{ii} \hat{\varphi} + p_{t+1}^{ij} \tilde{\varphi} \right] \quad (11)$$

## 2.2 Temporary equilibrium

Each agent of type  $i \in \{C, L\}$  chooses second and third-period consumptions, as well as socialization effort and leisure time, in such a way as to maximize his welfare subject to his budget constraint:

$$\begin{aligned} & \max_{c_t^i, d_{t+1}^i, e_t^i, l_t^i} (c_t^i)^{\theta^i} (l_t^i)^{1-\theta^i} - \frac{\xi(e_t^i)^2}{2} + h_t^i \left[ (d_{t+1}^i)^{\theta^i} + p_{t+1}^{ii} \hat{\varphi} + p_{t+1}^{ij} \tilde{\varphi} \right] \\ \text{s.t.} \quad & c_t^i + h_t^i d_{t+1}^i \leq w(1 - l_t^i) \end{aligned}$$

Assuming an interior solution, the FOCs of that problem are:

$$\begin{aligned} \theta^C (c_t^C)^{\theta^C - 1} (l_t^C)^{1-\theta^C} &= \lambda^C & \theta^L (c_t^L)^{\theta^L - 1} (l_t^L)^{1-\theta^L} &= \lambda^L \\ \theta^C (d_{t+1}^C)^{\theta^C - 1} &= \lambda^C & \theta^L (d_{t+1}^L)^{\theta^L - 1} &= \lambda^L \\ e_t^C &= \frac{h_t^C (1 - q_t) (\hat{\varphi} - \tilde{\varphi})}{\xi} & e_t^L &= \frac{h_t^L (q_t) (\hat{\varphi} - \tilde{\varphi})}{\xi} \\ (1 - \theta^C) (c_t^C)^{\theta^C} (l_t^C)^{-\theta^C} &= \lambda^C w & (1 - \theta^L) (c_t^L)^{\theta^L} (l_t^L)^{-\theta^L} &= \lambda^L w \end{aligned}$$

lead to stable equilibria without heterogeneity, which is inadequate for the purpose of this study.

<sup>11</sup>We abstract here from childhood.

where  $\lambda^C$  and  $\lambda^L$  denote the Lagrange multipliers associated with the budget constraints of agents of types  $C$  and  $L$ .<sup>12</sup>

Grouping the first two FOCs shows that, for all agents, first-period consumption is lower than second-period consumption. That result comes from the postulated utility function, under which the welfare return of consumption is higher the higher leisure time is, so that agents choose to consume more when retired.

Regarding socialization, one can notice that cultural substitution holds here:  $e_t^C$  is decreasing in  $q_t$ , and  $e_t^L$  is increasing in  $q_t$ . Note also that the expected longevity has a positive impact on the chosen socialization effort: the longer the third period is expected to be, the longer the expected period of coexistence with a child of one's type is. Hence, a larger expected longevity leads to a larger socialization effort. If  $h_t^C < h_t^L$ , one can expect that parents of type  $L$  choose, *ceteris paribus*, a larger socialization effort than agents of type  $C$ . As we shall see, this is not without consequences on the partition of the population at the steady-state.

As far as the choice of leisure is concerned, combining the first two FOCs with the fourth one and substituting the budget constraint yields:<sup>13</sup>

$$l_t^C = 1 - \theta^C(1 + h_t^C) \quad l_t^L = 1 - \theta^L(1 + h_t^L)$$

In the light of those expressions, it is not obvious to see whether agents with a lower taste for leisure will necessarily choose a lower leisure level, as there are two effects at work. If agents had the same expected longevity (i.e.  $h_t^C = h_t^L$ ), it would follow, under  $\theta^C > \theta^L$ , that agents with a low taste for leisure would choose a lower leisure ( $l_t^C < l_t^L$ ). However, the leisure decision depends also on the expected lifetime. Thus, under  $h_t^C < h_t^L$ , it might be the case that agents of type  $C$  choose, despite  $\theta^C > \theta^L$ , a higher leisure, because they expect to live a shorter life, which makes labour less useful. Note that, if  $\theta^C/\theta^L > 2$ , the first effect necessarily dominates the second, so that, whatever the prevailing group-specific longevity are, agents of type  $L$  choose necessarily a higher leisure, even though they expect to live longer than agents of type  $C$ . Our results are summarized in Proposition 1.<sup>14</sup>

**Proposition 1** *Under the laissez-faire,*

- $c_t^i \leq d_{t+1}^i \forall i \in \{C, L\}$ ,
- if  $h_t^C = h_t^L = \bar{h}_t$ , then  $l_t^C < l_t^L$ ,  $d_{t+1}^C > d_{t+1}^L$  and  $\frac{c_t^C}{l_t^C} > \frac{c_t^L}{l_t^L}$ ,
- if  $h_t^C \neq h_t^L$ , then  $l_t^C \leq l_t^L$ ,  $d_{t+1}^C \geq d_{t+1}^L$  and  $\frac{c_t^C}{l_t^C} \geq \frac{c_t^L}{l_t^L}$  iff  $\frac{\theta^C}{\theta^L} \geq \frac{1+h_t^L}{1+h_t^C}$ ,
- under  $h_t^C < h_t^L$  and  $\frac{\theta^C}{\theta^L} > 2$ , we have  $l_t^C < l_t^L$ ,  $d_{t+1}^C > d_{t+1}^L$  and  $\frac{c_t^C}{l_t^C} > \frac{c_t^L}{l_t^L}$ ,
- $e_t^C \geq e_t^L \iff \frac{1-q_t}{q_t} \geq \frac{h_t^L}{h_t^C}$ .

<sup>12</sup> Given that  $e_t^C$  and  $e_t^L$  lie, for any value of  $q_t$ , between 0 and 1, it must be assumed here that  $0 \leq h_t^C(\hat{\varphi} - \bar{\varphi})/\xi \leq 1$  and  $0 \leq h_t^L(\hat{\varphi} - \bar{\varphi})/\xi \leq 1$ .

<sup>13</sup> In order to guarantee a non-negative leisure level [i.e.  $l_t^i \geq 0 \forall i \in \{C, L\}$ ], we shall, in the rest of this paper, assume that  $\theta^i(1 + h_t^i) \leq 1 \forall h_t^i \in [0, 1]$ , which is equivalent to  $\theta^i \leq 1/2 \forall i \in \{C, L\}$ .

<sup>14</sup> See the Appendix for the proof.

Regarding the last part of Proposition 1, note that whether agents of type  $C$  make a larger socialization effort than agents of type  $L$  depends not only on the prevailing partition of the population, but, also, on group-specific longevities.

### 2.3 Steady-state equilibria

The existence of a steady-state depends on the existence, in the  $(q_t, h_t^C, h_t^L)$  space, of an intersection of the  $qq$ ,  $hh^C$  and  $hh^L$  loci, along which respectively  $q_t$ ,  $h_t^C$  and  $h_t^L$  are constant. Let us define those loci formally.

Substituting for  $\rho_{t+1}^C = e_t^C$  and  $\rho_{t+1}^L = e_t^L$  in expression (5) yields:

$$q_{t+1} = q_t + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} (h_t^C(1 - q_t)^2 q_t - h_t^L(q_t)^2(1 - q_t))$$

Fixing  $q_{t+1} = q_t = q$  in that expression allows us to derive the  $qq$  locus, along which  $q$  is constant. Actually, there exist three values of  $q$  allowing the constancy of the composition of the population over time:

$$q = 0; \quad q = \frac{h^C}{h^L + h^C}; \quad q = 1$$

Thus, the  $qq$  locus is made of those three components.

At the steady-state, it must also be the case that agent-specific longevities are constant.  $h_{t+1}^C$  can be written as:

$$h_{t+1}^C = \frac{1 - \theta^C(1 + h_t^C)}{2 - \theta^C(1 + h_t^C)}$$

Hence,  $h_{t+1}^C = h_t^C$  defines the  $hh^C$  locus as follows:

$$h^C = \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}$$

Note that the steady-state value of  $h^C$  depends only on  $\theta^C$ , and is decreasing in  $\theta^C$ .

Similarly, from

$$h_{t+1}^L = \frac{1 - \theta^L(1 + h_t^L)}{2 - \theta^L(1 + h_t^L)}$$

one can derive the  $hh^L$  locus, along which  $h^L$  is constant over time:

$$h^L = \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}$$

That long-run value of  $h^L$  depends only on  $\theta^L$ , and is decreasing in  $\theta^L$ .

Regarding the existence of an intersection of the three loci  $qq$ ,  $hh^C$  and  $hh^L$  in the  $(q_t, h_t^C, h_t^L)$  space, it is clear that, if  $q$  equals 1 (resp. 0), the variable

$h^L$  (resp.  $h^C$ ) is not defined, so that the system becomes 2-dimensional, and the steady-state consists merely of the intersection of the  $hh^C$  locus (resp.  $hh^L$  locus) with the  $qq$  locus corresponding to  $q = 1$  (resp.  $q = 0$ ). The  $qq$  and  $hh^C$  loci (resp.  $hh^L$  locus) being non parallel lines, these must intersect only once. Thus, the economy admits two steady-state equilibria with a homogeneous population, entirely made either of agents of type  $C$  (i.e.  $q = 1$ ), or, alternatively, of agents of type  $L$  (i.e.  $q = 0$ ).

The issue of existence of a steady-state with a heterogeneous population can be reformulated as the question of the intersection of the  $qq$ ,  $hh^C$  and  $hh^L$  loci strictly within the  $(q_t, h_t^C, h_t^L)$  space. Under the postulated functional forms, it can be shown that the three loci intersect only once at a value of  $q \neq 0 \neq 1$ , so that there exists a *unique* steady-state with a heterogeneous population. Moreover, it is shown in the Appendix that the intermediate steady-state is, under mild conditions, (locally) stable, in the sense that any economy that does not lie initially at another steady-state equilibrium will converge towards the intermediate steady-state. Those results are summarized below.<sup>15</sup>

**Proposition 2** *There exist three steady-state equilibria:*

$$\begin{aligned} - q = 0, h^L &= \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}. \\ - q = \frac{h^C}{h^L + h^C}, h^C &= \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}, h^L = \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}. \\ - q = 1, h^C &= \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}. \end{aligned}$$

*The intermediate steady-state is, under mild conditions, locally stable.*

Given that the intermediate steady-state (i.e. involving heterogeneity in tastes) is the equilibrium that is the most likely to emerge - except if the population is initially uniform - we shall, in the rest of this paper, pay a particular attention to that steady-state equilibrium, which is described by Proposition 3.

**Proposition 3** *At the intermediate steady-state equilibrium, the laissez-faire yields:*

$$\begin{aligned} - h^C &< h^L, \\ - q &< 1/2, \\ - l^C &< l^L, \\ - c^i &< d^i \quad \forall i \in \{C, L\}, \\ - d^C &> d^L \text{ and } \frac{c^C}{l^C} > \frac{c^L}{l^L}, \\ - e^C &= e^L. \end{aligned}$$

The proof of those results goes as follows. Given that  $\theta^C > \theta^L$ , the steady-state longevity of agents of type  $L$  is larger than the one of agents of type  $C$ , as equilibrium longevity  $h^i$  is decreasing in  $\theta^i$  for  $i \in \{C, L\}$ . Another prediction of the model concerns the long-run composition of the population. Actually, given that  $h^L > h^C$ , the steady-state population will be composed by a larger proportion of ‘leisure-lovers’ than ‘consumption-lovers’.<sup>16</sup> Hence, if  $q_0 > \frac{h^C}{h^C + h^L}$ ,

<sup>15</sup>See the Appendix for the proof.

<sup>16</sup>This is so because  $q^* = \frac{h^C}{h^C + h^L} < 1/2$  when  $h^C < h^L$ .

the model predicts the rise of a ‘leisure society’ (a fall of  $q_t$  over time), on the grounds that parents of type  $L$ , by facing a longer expected lifetime - and thus higher pay-offs from having children like them - tend, *ceteris paribus*, to make a stronger effort to transmit their trait, which will lead, *in fine*, to the larger propagation of their own lifestyle. As agents of type  $L$  live longer at the steady-state, it must also be the case, under the postulated longevity production, that these work less, so that  $l^C < l^L$ ,  $d^C > d^L$  and  $\frac{c^C}{l^C} > \frac{c^L}{l^L}$ . Finally, given that, at the steady-state,  $q = \frac{h^C}{h^C + h^L}$ , it is easy to see from the FOC for optimal socialization effort that the two types of agents must do the same levels of socialization effort at the steady-state.

### 3 Social optimum and optimal policy

Let us now characterize the social optimum in the economy under study. For that purpose, social welfare shall be defined in the standard ‘Benthamite’ way, as the sum of utilities of each group of agents, weighted by their size.<sup>17</sup>

Given that this paper aims at emphasizing the importance of ‘composition effects’ for the definition of the optimal public intervention, we shall proceed here in two stages, and consider first the short-run social optimum, i.e. the optimum for a *given* partition  $q_t$ , and, then, the long-run social optimum. That two-stage procedure will also allow us, when studying the decentralization of the social optimum, to compare the optimal policies under exogenous and endogenous composition of the population in terms of tastes.

#### 3.1 The short-run first-best problem

**The short-run first-best solution** The short-run social optimum is the octuple  $\{c_t^C, c_t^L, d_{t+1}^C, d_{t+1}^L, e_t^C, e_t^L, l_t^C, l_t^L\}$  maximizing the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & (q_t) \left[ (c_t^C)^{\theta^C} (l_t^C)^{1-\theta^C} - \frac{\xi(e_t^C)^2}{2} + h_{t+1}^C \left[ (d_{t+1}^C)^{\theta^C} + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \tilde{\varphi} \right] \right] \\ & + (1 - q_t) \left[ (c_t^L)^{\theta^L} (l_t^L)^{1-\theta^L} - \frac{\xi(e_t^L)^2}{2} + h_{t+1}^L \left[ (d_{t+1}^L)^{\theta^L} + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \tilde{\varphi} \right] \right] \\ & + \lambda \left[ w (q_t(1 - l_t^C) + (1 - q_t)(1 - l_t^L)) - q_t c_t^C - (1 - q_t) c_t^L - q_t h_{t+1}^C d_{t+1}^C - (1 - q_t) h_{t+1}^L d_{t+1}^L \right] \end{aligned}$$

where  $\lambda$  denotes the Lagrange multiplier associated to the resource constraint of the economy. Note that the partition of the population  $q_t$  is here taken as given.

<sup>17</sup>At it is well-known, the aggregation of utilities of agents having different preferences makes sense only if individual utilities are interpersonally comparable. One way to achieve this is, as proposed by Mirrlees (1982, p. 78-80), by means of discussions between agents about utilities. Although agents of types  $C$  and  $L$  have different tastes, they may communicate about their life experiences, and reach an agreement as to how their welfare should be included in a social welfare function. We assume that such an agreement can be reached, so that the aggregation of utilities of agents of different types in a social welfare function makes sense.

The FOCs for optimal consumptions and socialization efforts are:

$$\begin{aligned}\theta^C (c_t^C)^{\theta^C - 1} (l_t^C)^{1 - \theta^C} &= \lambda = \theta^L (c_t^L)^{\theta^L - 1} (l_t^L)^{1 - \theta^L} & e_t^C &= \frac{h_{t+1}^C (1 - q_t) (\hat{\varphi} - \bar{\varphi})}{\xi} \\ \theta^C (d_{t+1}^C)^{\theta^C - 1} &= \lambda = \theta^L (d_{t+1}^L)^{\theta^L - 1} & e_t^L &= \frac{h_{t+1}^L (q_t) (\hat{\varphi} - \bar{\varphi})}{\xi}\end{aligned}$$

The first two FOCs involve the standard equalization of marginal utilities of consumption for the different types of agents at the two periods. Those FOCs imply that first-period consumption cannot exceed second-period consumption. These imply also that  $d_{t+1}^C$  exceeds  $d_{t+1}^L$ , and that  $c_t^C / l_t^C > c_t^L / l_t^L$ .

As indicated by the two FOCs concerning socialization efforts, the short-run social optimum requires socialization effort levels to be chosen on the basis of optimal - rather than expected - future (group-specific) longevities. Hence, the optimal socialization effort is larger than the one under laissez-faire if and only if the expected longevity is lower than the optimal longevity. Regarding how optimal socialization efforts differ across agents of different types, the FOCs suggest that this depends on whether  $(1 - q) / q$  exceeds  $h_{t+1}^L / h_{t+1}^C$  or not.

The FOCs for optimal leisure times are:

$$\begin{aligned}\frac{(c_t^C)^{\theta^C} (1 - \theta^C) (l_t^C)^{-\theta^C} + h_{t+1}^{C'} \left[ (d_{t+1}^C)^{\theta^C} + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \hat{\varphi} \right]}{w + h_{t+1}^{C'} d_{t+1}^C} &= \lambda \\ \frac{(c_t^L)^{\theta^L} (1 - \theta^L) (l_t^L)^{-\theta^L} + h_{t+1}^{L'} \left[ (d_{t+1}^L)^{\theta^L} + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \hat{\varphi} \right]}{w + h_{t+1}^{L'} d_{t+1}^L} &= \lambda\end{aligned}$$

Note that those FOCs differ from the FOCs for optimal leisure under laissez-faire. Clearly, both types of agents tend, under laissez-faire, to have a too low level of leisure with respect to what is socially optimal. The reason why this is the case lies in agents's tendency to take the length of the third period of life as fixed, whereas this depends on their working time during the second period.<sup>18</sup> This myopia implies that individual longevities under laissez-faire are lower than their first-best levels. It implies also that  $c_t^{iLF} / d_{t+1}^{iLF} < c_t^{iFB} / d_{t+1}^{iFB}$  for all agents of type  $i \in \{C, L\}$ , meaning that all agents tend to oversave with respect to what is optimal.

In the light of this, we can describe the short-run social optimum as follows.<sup>19</sup>

**Proposition 4** *The short-run social optimum is such that:*

$$\begin{aligned}- c_t^{iFB} &\leq d_{t+1}^{iFB} \quad \forall i \in \{C, L\}, \\ - d_{t+1}^{CFB} &> d_{t+1}^{LFB}, \quad \frac{c_t^{CFB}}{l_t^{CFB}} > \frac{c_t^{LFB}}{l_t^{LFB}}, \\ - e_t^{iLF} &\leq e_t^{iFB} \iff h_t^{iLF} \leq h_{t+1}^{iFB} \quad \forall i \in \{C, L\}, \\ - e_t^{CFB} &\leq e_t^{LFB} \iff \frac{1 - q}{q} \leq \frac{h_{t+1}^{LFB}}{h_{t+1}^{CFB}},\end{aligned}$$

<sup>18</sup>Note that the fact that agents choose a too low leisure level - independently from the expected welfare gains from coexistence with children - is here due to the postulated functional form, which implies the 'fear of ruin' property  $u'(d)d < u(d)$  [as  $\theta^i d^{\theta^i - 1} d < d^{\theta^i}$  under  $\theta^i < 1$ ] (see Eeckhoudt and Pestieau, 2007).

<sup>19</sup>See the Appendix for a proof of those statements.

$$\begin{aligned}
& - l_t^{iLF} < l_t^{iFB} \quad \forall i \in \{C, L\}, \text{ so that } \frac{c_t^{iLF}}{d_{t+1}^{iLF}} < \frac{c_t^{iFB}}{d_{t+1}^{iFB}} \quad \forall i \in \{C, L\}, \\
& - h_{t+1}^{iLF} < h_{t+1}^{iFB} \quad \forall i \in \{C, L\}.
\end{aligned}$$

Let us now examine whether that short-run social optimum can be decentralized.

**Decentralization of the short-run social optimum** This subsection studies the decentralization of the short-run social optimum by means of group-specific tax/transfers instruments. Note that such instruments suppose that the government can observe the type of agents.<sup>20</sup> Moreover, this analysis assumes also that agents cannot change their type.<sup>21</sup>

Comparing the FOCs under laissez-faire with the ones describing the short-run social optimum shows that the decentralization of the short-run optimum requires a lump sum transfer system equalizing marginal utilities of consumption across agents and periods. However, because of the presence of myopia at the individual level, such transfers do not suffice to decentralize the optimum. Actually, the decentralization of the optimal leisures for agents  $C$  and  $L$  requires group-specific Pigouvian taxes on wages taking the impact of work on longevity into account:

$$\tau^C = h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1 - \theta^C) + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C - 1} w} > 0 \quad (12)$$

$$\tau^L = h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1 - \theta^L) + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L - 1} w} > 0 \quad (13)$$

Those Pigouvian taxes  $\tau^C$  and  $\tau^L$  are positive. Agents, whatever their type is, tend, by ignoring the negative impact of work on longevity, to work too much, so that a positive tax on wages is required to decentralize the social optimum. Naturally, those taxes depend on the impact of leisure on longevity.<sup>22</sup> Note that  $\tau^C$  and  $\tau^L$  depend also positively on the expected welfare gains from coexistence with one's child: the larger these are, the larger is the welfare loss due to myopia, inviting a bigger correction.

Whereas those Pigouvian taxes, coupled with lump-sum transfers, are necessary to decentralize the social optimum, these are not sufficient. Actually, comparing the FOCs for optimal socialization efforts with the ones describing agents's decentralized choices reveals that the former are based on *actual* longevity levels, whereas the latter are based on *myopic anticipations*. If, for instance, agents of type  $C$  overestimate their longevity (i.e.  $h_t^C > h_{t+1}^C$ ), they invest too much in the socialization of their child, so that the decentralization

<sup>20</sup>The observability of types is in conformity with the model: the fact that one derives welfare from coexisting with children of one's type requires the observability of types.

<sup>21</sup>As described in Section 2, the trait  $i \in \{C, L\}$  is taken *once-and-for-all* during the socialization process, so that there can be no 'adaptation' to public policy.

<sup>22</sup>If  $h_{t+1}^{C'}$  and  $h_{t+1}^{L'}$  equal 0, the myopia is benign, so that  $\tau^C$  and  $\tau^L$  equal zero.

of the short-run optimum requires some instruments making them choose lower socialization efforts.

However, efforts - which are here of purely physical nature (i.e. non-monetary and non-temporal) - can hardly be taxed or subsidized, as these are personal, non-monitored, variables. Hence, it follows that transfers  $T^C$  and  $T^L$  and Pigouvian taxes  $\tau^C$  and  $\tau^L$  on wages do not suffice, under myopia, to decentralize the short-run optimum. Those results are summarized below.<sup>23</sup>

**Proposition 5** *The decentralization of the short-run social optimum requires:*

- lump sum transfers  $T^C$  and  $T^L$  equalizing the marginal utilities of consumption for all agents at all periods,
- group-specific Pigouvian taxes  $\tau^C$  and  $\tau^L$  on wages. Those taxes are equal to:

$$\begin{aligned}\tau^C &= h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1 - \theta^C) + p_{t+1}^{CC} \hat{\varphi} + p_{t+1}^{CL} \tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C - 1} w} > 0 \\ \tau^L &= h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1 - \theta^L) + p_{t+1}^{LL} \hat{\varphi} + p_{t+1}^{LC} \tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L - 1} w} > 0\end{aligned}$$

*However, the optimal socialization effort levels cannot be decentralized.*

Thus, the fact that the government cannot interfere with the socialization decision, which relies on mistaken expectations on future longevity, prevents the decentralization of the short-run social optimum. Note that, at the steady-state, myopic anticipations yield a perfect anticipation of longevity (as  $E(h_{t+1}^i) = h_t^i = h_{t+1}^i$ ), so that this problem will disappear. However, as we shall now see, the decentralization of the long-run social optimum will also require to deal with the - so far neglected - effects of agent's decisions on the composition of future cohorts.

### 3.2 The long-run first-best problem

**The long-run first-best solution** The above analysis, which studied the decentralization of the social optimum, was carried out under a fixed proportion of the two types of agents. However, that postulate is a simplification, as the partition of the population is generally not constant over time, and is likely to be influenced by agents's decisions.

Actually, it is only in the special case of a homogeneous population - i.e.  $q_t$  is 0 or 1 -, that the tastes composition of the population remains the same over time, so that there is no difference between the short-run and the long-run social optimum.

Nevertheless, in the general case where some heterogeneity exists initially, the steady-state level of  $q$  depends on the longevity of each group, and, thus,

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<sup>23</sup>See the Appendix for the proof.

on their leisure. Hence, there may be a large difference between what is optimal under a *given* partition of the population and what is optimal under an *endogenous* composition.

The first-best long-run problem can be stated as the search for the octuple  $\{c^C, c^L, d^C, d^L, e^C, e^L, l^C, l^L\}$  maximizing the Lagrangian:<sup>24</sup>

$$\begin{aligned} \mathcal{L} = & (q) \left[ (c^C)^{\theta^C} (l^C)^{1-\theta^C} - \frac{\xi(e^C)^2}{2} + h^C \left[ (d^C)^{\theta^C} + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi} \right] \right] \\ & + (1-q) \left[ (c^L)^{\theta^L} (l^L)^{1-\theta^L} - \frac{\xi(e^L)^2}{2} + h^L \left[ (d^L)^{\theta^L} + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi} \right] \right] \\ & + \lambda \left[ w (q(1-l^C) + (1-q)(1-l^L)) - qc^C - (1-q)c^L - qh^C d^C - (1-q)h^L d^L \right] \end{aligned}$$

where  $q$  takes its steady-state value  $\frac{h^C}{h^C+h^L}$ .

The FOCs for optimal consumptions are:

$$\begin{aligned} \theta^C (c^C)^{\theta^C-1} (l^C)^{1-\theta^C} &= \lambda = \theta^L (c^L)^{\theta^L-1} (l^L)^{1-\theta^L} \\ \theta^C (d^C)^{\theta^C-1} &= \lambda = \theta^L (d^L)^{\theta^L-1} \end{aligned}$$

These coincide with the standard conditions of equalization of marginal utilities of consumption across periods and agents. Once again, first-period consumption should not exceed second-period consumption. As in the short-run social optimum, agents of type  $C$  have a larger second-period consumption than agents of type  $L$ .

The FOCs for optimal socialization efforts are:

$$\begin{aligned} e^C &= \frac{h^C(1-q)(\hat{\varphi} - \tilde{\varphi})}{\xi} \\ e^L &= \frac{h^L(q)(\hat{\varphi} - \tilde{\varphi})}{\xi} \end{aligned}$$

where  $h^C$ ,  $h^L$  and  $q$  are at their optimal levels. Note that, given that  $q = \frac{h^C}{h^C+h^L}$  at the steady-state, optimal socialization efforts are equal for all agents.

Finally, the FOCs for the optimal leisure time for agents of types  $C$  and  $L$  are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h^C} &= q'_h h^{C'} [u^C - u^L] + q(c^C)^{\theta^C} (1-\theta^C)(l^C)^{-\theta^C} \\ &+ qh^{C'} \left[ (d^C)^{\theta^C} (1-\theta^C) + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi} \right] - \lambda qw \\ &+ \lambda q'_h h^{C'} [(1-l^C)w - (1-l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \end{aligned}$$

<sup>24</sup>For simplicity, indices are omitted here.

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial h^L} &= q'_{h^L} h^{L'} [u^C - u^L] + (1 - q) (c^L)^{\theta^L} (1 - \theta^L) (l^L)^{-\theta^L} \\
&\quad + (1 - q) h^{L'} \left[ (d^L)^{\theta^L} (1 - \theta^L) + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi} \right] - \lambda (1 - q) w \\
&\quad + \lambda q'_{h^L} h^{L'} [(1 - l^C) w - (1 - l^L) w - c^C + c^L - h^C d^C + h^L d^L] \\
&= 0
\end{aligned}$$

Unlike the FOCs characterizing the short-run social optimum, those FOCs capture the impact of agent's lifestyle (i.e. leisure times) on the long-run composition of the population. The intuition behind those *intergenerational composition effects* goes as follows. When an agent chooses his leisure time, this determines his own longevity, which, under myopic anticipations on longevity, will also affect the socialization efforts of agents of the same type (taking him as a 'model') at the *next* generation (through its impact on agents' temporal horizons). Thus the leisure decision of an agent affects indirectly the taste composition of the next cohort, which, when making its own decisions, will also affect the composition of the next cohort, etc., until the steady-state partition of the population is reached. That infinite chain linking all generations's lifestyles is taken into account by the social planner in the long-run first-best problem, but not in the short-run first-best problem.

Note that it is only in the special case where the partition of the population is fixed -  $q'_{h^C} = q'_{h^L} = 0$  - that those FOCs would collapse to the ones describing the short-run optimum. In comparison with the FOCs characterizing the short-run optimum, each of those FOC includes two additional terms.

The first term captures the impact, in welfare terms, of the change in the partition of the population resulting from a variation in the agent's leisure time. If, for instance, the lifetime welfare of an agent of type  $C$  - denoted by  $u^C$  - exceeds the one of an agent of type  $L$ , the first term of the FOC for  $l^C$  is positive, implying, *ceteris paribus*, a higher  $l^C$ , in such a way as to raise the proportion of agents of type  $C$  at the steady-state. On the contrary, under  $u^C > u^L$ , the first term of the FOC for  $l^L$  becomes negative, implying a lower leisure  $l^L$  for agents of type  $L$ , in such a way as to reduce their proportion in the long-run. Hence, the first term has opposite signs for the two types of agents.

The second additional term consists of the impact of a change in the partition of the population at the steady-state on the resources of the economy. If, for instance, agents of type  $C$  are net contributors, then the factor in brackets is positive, inviting, *ceteris paribus*, a higher proportion of agents of that type, and, thus, a higher leisure level for them. Here again, it is not difficult to see that those additional terms are of opposite signs in the two FOCs, as the derivatives  $q'_{h^C}$  and  $q'_{h^L}$  multiplying the factors in brackets are of opposite signs in the two FOCs.

Comparing the FOCs for optimal leisure with the ones under laissez-faire suggests that the leisure of one type of agent at the social optimum may be superior or inferior to its level under laissez-faire, depending on whether the sum of those two additional terms with the myopia term of the FOC is positive

or negative. The following proposition summarizes the above findings.<sup>25</sup>

**Proposition 6** *The long-run social optimum is such that:*

$$\begin{aligned}
& - c^{iFB} \leq d^{iFB} \quad \forall i \in \{C, L\}, \\
& - d^{CFB} > d^{LFB}, \quad \frac{c^{CFB}}{l^{CFB}} > \frac{c^{LFB}}{l^{LFB}}, \\
& - e^{CFB} = e^{LFB} \\
& - l^{CLF} \underset{\geq}{\leq} l^{CFB} \quad \text{iff} \quad \Xi \underset{\geq}{\leq} 0,
\end{aligned}$$

$$\begin{aligned}
& \text{where } \Xi \equiv qh^C \left[ (d^{CFB})^{\theta^C} (1 - \theta^C) + p^{CCFB} \hat{\varphi} + p^{CLFB} \tilde{\varphi} \right] + q'_{h^C} h^{C'} [u^{CFB} - u^{LFB}] \\
& + \lambda q'_{h^C} h^{C'} [(1 - l^{CFB})w - (1 - l^{LFB})w - c^{CFB} + c^{LFB} - h^{CFB} d^{CFB} + h^{LFB} d^{LFB}],
\end{aligned}$$

$$- l^{LLF} \underset{\geq}{\leq} l^{LFB} \quad \text{iff} \quad \Psi \underset{\geq}{\leq} 0,$$

$$\begin{aligned}
& \text{where } \Psi \equiv (1-q)h^L \left[ (d^{LFB})^{\theta^L} (1 - \theta^L) + p^{LLFB} \hat{\varphi} + p^{LCFB} \tilde{\varphi} \right] + q'_{h^L} h^{L'} [u^{CFB} - u^{LFB}] \\
& + \lambda q'_{h^L} h^{L'} [(1 - l^{CFB})w - (1 - l^{LFB})w - c^{CFB} + c^{LFB} - h^{CFB} d^{CFB} + h^{LFB} d^{LFB}].
\end{aligned}$$

Note that, if there was no myopic terms in  $\Xi$  and  $\Psi$ , we would have  $\Xi \underset{\geq}{\leq} 0$  if and only if  $\Psi \underset{\geq}{\leq} 0$ , because the two (remaining) terms constituting  $\Xi$  and  $\Psi$  would be of opposite signs. Hence, in that case, if the leisure level of one type of agents under laissez-faire exceeds its socially optimal level, it must necessarily be the case that the leisure level of the other type of agents is lower than its first-best level. However, the terms  $\Xi$  and  $\Psi$  include also a myopic term, so that one cannot exclude *a priori* the case where both types of agents tend, under laissez-faire, to overwork with respect to what is socially optimal. In that general case, the extent to which agents would work excessively under laissez-faire would depend not only on their myopia, but, also, on which partition of the population is socially optimal.

**Decentralization of the long-run social optimum** Comparing the FOCs for optimal long-run consumption with the ones at the laissez-faire shows that the decentralization of the long-run social optimum requires, as in the short-run case, lump sum transfers equalizing marginal utilities of consumption across agents and periods.

Regarding the decentralization of the optimal socialization efforts levels, the comparison of the FOCs of the laissez-faire with the ones characterizing the social optimum shows that, as long as leisure times - and, through  $h^C$  and  $h^L$ , the composition of the population  $q$  - take their optimal values for all types of agents, the optimal effort levels will also prevail.<sup>26</sup>

Contrasting the FOCs relative to the choice of leisure times reveals that the Pigouvian taxes designed for the decentralization of the short-run social optimum are no longer appropriate here, because of the endogeneity of the partition of the population. From a long-run perspective, agents choose their leisure time *without* considering its impact on the composition of future cohorts.

<sup>25</sup>See the Appendix for the proof.

<sup>26</sup>Note that, given the constancy of longevity at the steady-state, myopic anticipations are benign at steady-state for the choice of socialization efforts, so that lumpsum transfers do not need to be complemented by - hardly implementable - instruments on efforts.

Hence, besides the correction for agents's myopia, the decentralization of the long-run social optimum requires also some correction for those uninternalized 'composition effects'.

The following proposition presents the Pigouvian taxes required for the decentralization of the long-run social optimum.<sup>27</sup>

**Proposition 7** *The long-run social optimum can be decentralized by means of:*

- lump sum transfers  $T^C$  and  $T^L$  yielding an equalization of marginal utilities of consumption across agents and periods;
- Pigouvian taxes  $\tau^C$  and  $\tau^L$  on wages equal to:

$$\begin{aligned}\tau^C &= h^{C'} \frac{(d^C)^{\theta^C} (1 - \theta^C)^{[1]} + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi}}{\lambda w} + \frac{q'_{h^C} h^{C'} u^C - u^L}{q} \frac{[2]}{\lambda w} \\ &\quad + \frac{q'_{h^C} h^{C'} w(1 - l^C) - w(1 - l^L)^{[3]} - c^C + c^L - h^C d^C + h^L d^L}{q w} \\ \tau^L &= h^{L'} \frac{(d^L)^{\theta^L} (1 - \theta^L)^{[1']} + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi}}{\lambda w} + \frac{q'_{h^L} h^{L'} u^C - u^L}{1 - q} \frac{[2']}{\lambda w} \\ &\quad + \frac{q'_{h^L} h^{L'} w(1 - l^C) - w(1 - l^L)^{[3']} - c^C + c^L - h^C d^C + h^L d^L}{1 - q} \frac{[3']}{w}\end{aligned}$$

The comparison of those Pigouvian taxes  $\tau^C$  and  $\tau^L$  allowing the decentralization of the long-run optimum with the ones in the short-run first-best problem suggests that the long-run optimal tax is equal to short-run optimal tax [i.e. term [1]] - internalizing the impact of myopia on one's own lifetime welfare - *plus* the correction for the unintended 'composition effects' of individual leisure decisions on the composition of future populations [i.e. terms [2] and [3]].

Note first that, if each agent's longevity had no impact on the composition of the population - i.e.  $q'_{h^C} = q'_{h^L} = 0$  - Pigouvian taxes  $\tau^C$  and  $\tau^L$  would be equal to their short-run levels. However, given that, for an initial  $q_0 \in ]0, 1[$ , the steady-state  $q$  equals  $\frac{h^C}{h^C + h^L}$ , the longevity of each group influences the long-run composition of the population. Parents with a longer temporal horizon have larger welfare gains from coexisting with their child, and, thus, choose higher effort levels, leading to a larger long-run proportion of agents of that kind. Note also that, if leisure time had no impact on longevity, i.e.  $h^{C'} = h^{L'} = 0$ , the optimal tax rates would be zero, as agents's myopia and ignorance of the impact of their decisions on the composition of the future cohorts would then be benign.

Let us now interpret the two additional components of the optimal tax rates.<sup>28</sup>

<sup>27</sup>See the Appendix for the proof.

<sup>28</sup>For that purpose, we shall concentrate on terms [2] and [3] in  $\tau^C$ .

The term [2] accounts for the welfare gains, at the social level, from changing the composition of the population  $q$ . That term is a product of three factors: the first is the marginal effect, on  $q$ , of a rise of agents  $C$ 's longevity; the second is the marginal effect, on agents  $C$ 's longevity, of a higher leisure  $l^C$ ; the third factor,  $u^C - u^L$ , is the difference between lifetime welfares of agents of types  $C$  and  $L$ . The sign of that term depends on which lifestyle brings the highest lifetime welfare.<sup>29</sup>

The term [3] accounts for the marginal effect, on the available resources, of a rise of the proportion of agents of type  $C$  as a result of a higher  $l^C$ . The sign of that term is also ambiguous: it is not clear that a rise of  $q$  resulting from a higher  $l^C$  increases or reduces the amount of available resources in the economy.

Note that, contrary to the decentralization of the short-run optimum, it is not easy to see which *signs* Pigouvian taxes  $\tau^C$  and  $\tau^L$  have: this depends on the signs and sizes of the three terms. If, for instance, the lifestyle of agents of type  $C$  brings a higher lifetime welfare than the one of agents of type  $L$  [i.e. [2] > 0], and if increasing the proportion of agents of type  $C$  tends also to raise the resources in the economy [i.e. [3] > 0], the internalization of the myopia and of unintended composition effects go in the same directions, and require agents of type  $C$  to work less. Hence, a positive tax on the labour income of agents of type  $C$  is required to restore the social optimum by making them work less than under the *laissez-faire*. If, on the contrary, agents of type  $C$  have a lower lifetime welfare than agents of type  $L$ , and if [1] + [3] < |[2]|, it follows, despite the myopia of agents of type  $C$ , that the decentralization of the long-run social optimum requires to subsidize their labour, as these tend to work too little from a social perspective.

Finally, it should be stressed that those Pigouvian taxes and the lump sum transfers allow the decentralization of the long-run social optimum, as socialization efforts are, under that tax and transfer system, optimally chosen, unlike what used to prevail in the short-run. Actually, agents, when choosing how much to socialize their child, still rely here on expected longevity rather than actual longevity, but given the constancy of longevity over time, this does not prevent the optimal level of effort to be chosen. Thus the decentralization of the long-run social optimum can, despite the physical nature of efforts levels, be carried out.

**Comparison of short-run and long-run Pigouvian taxes** A major corollary of the above analysis is that the optimal *short-run* and *long-run* Pigouvian taxes on wages differ significantly. The reason for this is that, in the short-run problem, the maximization of social welfare of the cohort can be achieved without being concerned with the impact of individual decisions on the welfare of future cohorts. However, from a long-run perspective, agents's decisions are imperfect not only from the point of view of individual lifetime welfare maximization, but, also, from the perspective of the composition of future cohorts in

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<sup>29</sup>If the  $C$  lifestyle brings a higher lifetime welfare than the  $L$  lifestyle, increasing  $q$  is socially desirable. If, however, it is the opposite, then it is desirable to reduce  $q$ .

terms of tastes. This double imperfection of leisure decisions invites a distinct governmental correction.

Regarding the extent to which the short-run Pigouvian taxes differ from the long-run ones, let us first notice that the terms [2] and [2'] must be of opposite signs: if agents  $C$ 's lifestyle brings a higher lifetime welfare than the one of agents of type  $L$ , then agents  $L$ 's lifestyle must also bring a lower lifetime welfare than the one of agents of type  $C$ . Terms [3] and [3'] are also of opposite signs: if agents of type  $C$  are net contributors, agents of type  $L$  must be net beneficiaries. From all this, one can also deduce the following result, concerning the differential between short-run and long-run Pigouvian taxes.<sup>30</sup>

**Proposition 8** *Let  $\tau^{iSR}$  and  $\tau^{iLR}$  denote the Pigouvian taxes on agent  $i$ 's wage required for the decentralization of the short-run and long-run social optima. We have:  $\tau^{CLR} - \tau^{CSR} \begin{matrix} \geq \\ \leq \end{matrix} 0 \iff \tau^{LLR} - \tau^{LSR} \begin{matrix} \leq \\ \geq \end{matrix} 0$ .*

This proposition states that the correction for composition effects raises the tax on the labour of one type of agents, and reduces the tax on the labour of the other type. Hence, the optimal long-run tax on labour is, for one group, higher than its optimal short-run level, whereas the opposite holds for the other group. In other words, shifting from the maximization of short-run social welfare to the maximization of long-run social welfare tends, for one type of agents, to reinforce the correction required by the myopia - and make those agents live more than what maximizes their lifetime welfare -, whereas that shift weakens the correction required by the myopia for the other type of agents, and makes those other agents live less than what maximizes their lifetime welfare.

Although such a result might seem quite counter-intuitive, this comes from the fact that, from a utilitarian point of view, the tendency of agents to work too much with respect to what is the best for them is *not* the unique determinant of public intervention. A utilitarian government must also, in its intervention, take the composition effects of agents's decisions into account. When agents decide how much to work - and unconsciously choose their length of life -, they also affect the temporal horizon of the next generation taking them as 'models' (either as a parent or as a 'role model'). The members of the next generation will then choose, on the basis of their 'models', their socialization efforts, determining the composition of the next generation, which, in itself, will also make its decisions on the basis of its own 'model', and so forth. Hence, correcting those - non internalized - intergenerational composition effects involves making agents work more or less - and live less or more - than what maximizes their *own* lifetime welfares.

## 4 Conclusions

This paper aimed at studying the optimal public policy in an economy where unequal longevities are the unintended outcome of differences in lifestyles. For

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<sup>30</sup>See the Appendix for the proof.

that purpose, we developed a three-period OLG model, where the length of the third period depends on second period leisure. The population, who ignores the impact of work on longevity, is partitioned in two groups with different tastes for leisure, and follows an imitation/adaptation process *à la* Bisin and Verdier (2001).

We showed that the long-run social optimum can be decentralized by means of (1) inter-group lump sum transfers; (2) group-specific Pigouvian taxes on wages, which internalize not only the unintended effects of leisure on individual lifetime welfare, but, also, its impact on the composition of future cohorts. This paper shows also that the shift from the maximization of short-run social welfare to the maximization of long-run social welfare involves a change in the optimal policy, in the sense that taking intergenerational composition effects into account raises the optimal tax on labour for one type of agents, but reduces it for the other type.

Thus, the endogeneity of the population's preferences is far from neutral for the optimal public intervention. Therefore, while it is tempting to discuss the optimal public policy in a static framework - for a *given* partition of the population in terms of tastes -, the policy recommendations from such a static study may differ significantly from what would best serve the long-run interests of the economy.

Note that our conclusions, which rely on the standard utilitarian framework, are far from uncontroversial. After all, the government, by its fiscal policy, affects here what future people will *be*, which is, from a liberal point of view, questionable, as such an intervention may match the behaviour of a uniformizing totalitarian State.<sup>31</sup> However, it should be stressed that the interferences studied here, which shape the tastes of (future) *non existing* people, differ from influences on the tastes of *existing* people, so that one cannot accuse the social planner of totalitarianism.<sup>32</sup>

At the end of the day, the major question raised by the results of this paper concerns the arbitrages to be made between the interests of existing people *versus* the ones of future, non-existing people. Correcting a myopia may well improve the welfare of current people, but, by the composition effects it generates, may also reduce long-run welfare, so that an arbitrage is to be made between the welfare of current cohorts and the welfare of future cohorts. Note that this kind of problem, raised, for instance, by Malthus (1798) in the context of poverty reduction, goes far beyond the scope of this paper.<sup>33</sup> But the complexity of solving intergenerational trade-offs should not make us do *as if* such arbitrages did not exist. Tensions between short-run and long-run social welfare maximization do exist, and governments have no other reasonable choices than to take these into account.

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<sup>31</sup>The tensions between utilitarianism and liberalism in the context of a heterogenous society are best summarized in Mill's (1859) work on the conditions required for State intervention.

<sup>32</sup>On the latter, see Hahn (1982).

<sup>33</sup>Malthus's argument against the Poor Laws was that those laws, although reducing poverty in the short-run, would, by favouring population growth, lead to a rise in the proportion of poor people in the long-run.

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## 6 Appendix

### 6.1 Laissez-faire

#### 6.1.1 Temporary equilibrium

The expression  $\frac{c_t^i}{d_{t+1}^i} = l_t^i$  is obtained from rearranging the first two FOCs. From this, it follows that, under  $0 \leq l_t^i \leq 1$ , it must be the case that  $c_t^i \leq d_{t+1}^i \forall i \in \{C, L\}$ .

Actually, if the budget constraint is binding, we have  $c_t^i + h_t^i d_{t+1}^i = w(1 - l_t^i)$ , so that, given the above expression, we can deduce that:  $c_t^i = \frac{l_t^i w(1 - l_t^i)}{l_t^i + h_t^i}$ .

From the first and third FOCs, we have:  $l_t^i = \frac{1 - \theta^i}{\theta^i} \frac{c_t^i}{w}$ . Substituting for  $c_t^i$  in this yields:  $l_t^i = 1 - \theta^i(1 + h_t^i)$ . From that expression, it is easy to see that, if  $h_t^L = h_t^C = \bar{h}_t$ , we have also, given  $\theta^L < \theta^C$ ,  $l_t^C < l_t^L$ . Moreover, given that  $d_{t+1}^i = \frac{c_t^i}{l_t^i} = \frac{w(1 - l_t^i)}{l_t^i + h_t^i}$ , it follows that  $l_t^C < l_t^L$  and  $h_t^L = h_t^C = \bar{h}_t$  imply also  $d_{t+1}^C > d_{t+1}^L$ . Given that  $\frac{c_t^i}{d_{t+1}^i} = l_t^i$ , this implies also  $\frac{c_t^C}{l_{t+1}^C} > \frac{c_t^L}{l_{t+1}^L}$ .

Under the general case where  $h_t^L \neq h_t^C$ , we know that agents of type  $C$  choose a lower leisure than agents of type  $L$  if and only if:  $1 - \theta^C(1 + h_t^C) < 1 - \theta^L(1 + h_t^L)$ . This is true if and only if:  $\frac{\theta^C}{\theta^L} > \frac{1 + h_t^L}{1 + h_t^C}$ .

Given that  $h_t^L, h_t^C \in [0, 1]$  and  $h_t^L > h_t^C$ , that condition is satisfied for all longevities if and only if  $\frac{\theta^C}{\theta^L} > 2$ . Note that if  $l_t^C < l_t^L$ , we have  $d_{t+1}^C > d_{t+1}^L$ , as  $\frac{w(1 - l_t^C)}{l_t^C + h_t^C} > \frac{w(1 - l_t^L)}{l_t^L + h_t^L}$  under  $h_t^L > h_t^C$ . Given that  $\frac{c_t^i}{d_{t+1}^i} = l_t^i$ , this implies also  $\frac{c_t^C}{l_{t+1}^C} > \frac{c_t^L}{l_{t+1}^L}$ .

The inequality condition on efforts levels follows from comparing the two efforts levels in the above FOC:  $\frac{h_t^C(1 - q_t)(\hat{\varphi} - \tilde{\varphi})}{\xi} \geq \frac{h_t^L(q_t)(\hat{\varphi} - \tilde{\varphi})}{\xi} \iff \frac{h_t^C}{h_t^L} \geq \frac{q_t}{1 - q_t}$ .

#### 6.1.2 Steady-state equilibrium

**Existence of steady-states** The dynamics of the economy is given by:

$$\begin{aligned} q_{t+1} &= F(q_t, h_t^C, h_t^L) = q_t + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} (h_t^C(1 - q_t)^2 q_t - h_t^L(q_t)^2(1 - q_t)) \\ h_{t+1}^C &= G(h_t^C) = \frac{1 - \theta^C(1 + h_t^C)}{2 - \theta^C(1 + h_t^C)} \\ h_{t+1}^L &= H(h_t^L) = \frac{1 - \theta^L(1 + h_t^L)}{2 - \theta^L(1 + h_t^L)} \end{aligned}$$

Imposing  $h_{t+1}^C = h_t^C$  and  $h_{t+1}^L = h_t^L$  yields:

$$h^C = \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C}; h^L = \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L}$$

Those two levels are independent from each others, and independent from  $q$ . Substituting for these in the dynamic law for  $q$  and defining  $\phi(q_t) \equiv q_{t+1}$  yields:

$$\phi(q_t) = q_t + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \left( \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C} (1 - q_t)^2 q_t - \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L} (q_t)^2 (1 - q_t) \right)$$

The existence of a steady-state equilibrium amounts to find a fixed point for  $\phi(q_t)$ , that is, a value of  $q_t$  such that  $\phi(q_t) = q_t$ . It is obvious that  $q_t = 0$  and  $q_t = 1$  are such fixed points, as  $\phi(0) = 0$  and  $\phi(1) = 1$ . Hence, the homogenous society is a steady-state equilibrium.

The existence of an intermediate steady-state equilibrium - i.e. an equilibrium involving some heterogeneity - amounts to find a value of  $q_t \in ]0, 1[$  such that  $\phi(q_t) = q_t$ .

Note that  $\phi(q_t)$  satisfies the following conditions:

- (i)  $\phi(0) = 0$
- (ii)  $\phi(1) = 1$
- (iii)  $0 \leq \phi(q_t) \leq 1$
- (iv)  $\lim_{q \rightarrow 0} \phi'(q_t) = 1 + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{1 - \sqrt[2]{1 - \theta^C(1 - \theta^C)}}{\theta^C} > 1$
- (v)  $\lim_{q \rightarrow 1} \phi'(q_t) = 1 + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{1 - \sqrt[2]{1 - \theta^L(1 - \theta^L)}}{\theta^L} > 1$

Thus, given that  $\phi(q_t)$  crosses the 45° line at  $q_t = 0$  and  $q_t = 1$ , and has a slope that exceeds 1 when  $q_t$  tends towards 0 and when  $q_t$  tends towards 1, it must be the case, given the continuity of  $\phi(q_t)$ , that  $\phi(q_t)$  intersects the 45° line for  $q_t^* \in ]0, 1[$ .

The uniqueness of that intermediate steady-state can be shown by *reductio ad absurdum*. Suppose that such another intermediate equilibrium  $\tilde{q} \in ]0, 1[$  exists, and differs from  $\frac{h^C}{h^L + h^C}$ . This amounts to assuming that equals  $\tilde{q} = \frac{h^C}{h^L + h^C} + \varepsilon$ , with  $\varepsilon \neq 0$ ,  $\varepsilon \neq \frac{h^L}{h^L + h^C}$  and  $\varepsilon \neq \frac{-h^C}{h^L + h^C}$ .

We have, by definition of the steady-state,

$$\tilde{q} = \tilde{q} + \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} (h^C(1 - \tilde{q})^2 \tilde{q} - h^L(\tilde{q})^2(1 - \tilde{q}))$$

Given that  $\frac{\hat{\varphi} - \tilde{\varphi}}{\xi} > 0$ , this simplifies to:  $h^C(1 - \tilde{q})^2 \tilde{q} - h^L(\tilde{q})^2(1 - \tilde{q}) = 0$ . As  $\tilde{q} \in ]0, 1[$ , we can divide by  $\tilde{q}$  and  $(1 - \tilde{q})$ , and we obtain:  $\tilde{q} = \frac{h^C}{h^L + h^C}$ , implying  $\varepsilon = 0$ , contrary to what was assumed. Thus the intermediate steady-state is unique.

**Stability of the intermediate steady-state** In order to study the (local) stability of the intermediate steady-state equilibrium, let us first notice that the present system is non-linear, so that the conventional analysis of the Jacobian matrix (composed of the first-order derivatives of dynamic equations with respect to state variables) can only inform us on the stability of equilibria *provided* these are hyperbolic. Actually, if a fixed-point is hyperbolic, the Hartman-Grobman Theorem states that the stability of the linearized system (or its non-stability) implies the local stability of the non-linear system (or its non-stability) (see Medio and Lines, 2001). However, if the fixed-point is not hyperbolic, then the analysis of the linearized system does not allow us to draw any conclusion on the local stability of the non-linear system.

As stated in Medio and Lines (2001), fixed-points are, in discrete-time systems, hyperbolic if none of the eigenvalues of the Jacobian matrix, evaluated at the equilibrium, is equal to 1 in modulo.

The Jacobian matrix is defined as follows.

$$J \equiv \begin{pmatrix} \frac{\partial F}{\partial q_t} & \frac{\partial F}{\partial h_t^C} & \frac{\partial F}{\partial h_t^L} \\ \frac{\partial G}{\partial q_t} & \frac{\partial G}{\partial h_t^C} & \frac{\partial G}{\partial h_t^L} \\ \frac{\partial H}{\partial q_t} & \frac{\partial H}{\partial h_t^C} & \frac{\partial H}{\partial h_t^L} \end{pmatrix}$$

We have, at the intermediate steady-state:

$$\begin{aligned} \frac{\partial F}{\partial q_t} &= 1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} < 1 \text{ and positive (as } \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} h^C \leq 1). \\ \frac{\partial F}{\partial h_t^C} &= \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \left( \frac{h^L}{h^C + h^L} \right)^2 \frac{h^C}{h^C + h^L} < 1 \text{ and positive (as } \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} h^C \leq 1). \\ \frac{\partial F}{\partial h_t^L} &= -\frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \left( \frac{h^C}{h^C + h^L} \right)^2 \frac{h^L}{h^C + h^L} > -1 \text{ and negative.} \\ \frac{\partial G}{\partial q_t} &= 0 \\ \frac{\partial G}{\partial h_t^C} &= \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} > -1 \text{ and negative.} \\ \frac{\partial G}{\partial h_t^L} &= 0 \\ \frac{\partial H}{\partial q_t} &= 0 \\ \frac{\partial H}{\partial h_t^C} &= 0 \\ \frac{\partial H}{\partial h_t^L} &= \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2} > -1 \text{ and negative.} \end{aligned}$$

Note first that the trace of the Jacobian matrix is:

$$1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} + \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2}$$

which lies, under our assumptions on  $\frac{\hat{\varphi} - \tilde{\varphi}}{\xi}$ ,  $\theta^C$  and  $\theta^L$ , between -1 and 1.

The determinant of the Jacobian matrix is:

$$\left[ 1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{-\theta^C}{[2 - \theta^C(1 + h_t^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h_t^L)]^2}$$

which lies between 0 and 1.

We would like the linearized system to be stable. As this is well-known, a necessary and sufficient condition for the stability of the linearized system is that all eigenvalues of the Jacobian matrix estimated at the equilibrium are magnitudes less than 1. That condition implies also that the equilibrium is hyperbolic, leading to the (local) stability of the equilibrium.

Following Brooks (2004)'s study of stability in first-order three-dimensional dynamic systems, we know that all eigenvalues of a 3x3 Jacobian matrix are lower than 1 in modulo provided the following three conditions are satisfied:

- (i)  $|\det(J)| < 1$
- (ii)  $1 > [\sum M_i(J)] - [tr(J)] [\det(J)] + [\det(J)]^2$
- (iii)  $-\left[\sum M_i(J) + 1\right] < tr(J) + \det(J) < \left[\sum M_i(J) + 1\right]$

where  $\det(J)$ ,  $tr(J)$  and  $\sum M_i(J)$  denote respectively the determinant, the trace and the sum of the principal minors of the Jacobian matrix.

Condition (i) is satisfied (see above).

Regarding condition (ii), note first that  $\sum M_i(J)$  is here equal to:

$$\begin{aligned} & \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] + \frac{-\theta^C}{[2 - \theta^C(1 + h_i^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} + \\ & \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{-\theta^C}{[2 - \theta^C(1 + h^C)]^2} + \frac{-\theta^C}{[2 - \theta^C(1 + h_i^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} + \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} + \\ & \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{-\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} \end{aligned}$$

This simplifies to

$$\begin{aligned} & \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \left[1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1 + h^C)]^2}\right] + \frac{-\theta^C}{[2 - \theta^C(1 + h_i^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} \\ & + \left[2 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{-\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} \end{aligned}$$

Hence, condition (ii) requires:  $1 > [\sum M_i(J)] - [tr(J)] [\det(J)] + [\det(J)]^2$

$$\begin{aligned} 1 & > \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \left[1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1 + h^C)]^2}\right] + \frac{-\theta^C}{[2 - \theta^C(1 + h_i^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} \\ & + \left[2 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{-\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2} \\ & - \left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} - \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} - \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2}\right] \left[\left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2}\right] \\ & + \left[\left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \frac{\theta^C}{[2 - \theta^C(1 + h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1 + h^L)]^2}\right]^2 \end{aligned}$$

That condition holds for low values of  $\theta^C$  and  $\theta^L$ , in conformity with our assumptions.

Condition (iii) requires:  $-\left[\sum M_i(J) + 1\right] < tr(J) + \det(J) < \left[\sum M_i(J) + 1\right]$

$$-\left[1 - \frac{\hat{\varphi} - \tilde{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L}\right] \left[1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1 + h^C)]^2}\right] - \frac{-\theta^C}{[2 - \theta^C(1 + h_i^C)]^2} - \frac{-\theta^L}{[2 - \theta^L(1 + h^L)]^2}$$

$$\begin{aligned}
& - \left[ 2 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{-\theta^C}{[2 - \theta^C(1+h^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1+h^L)]^2} - 1 \\
& < 1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} + \frac{-\theta^C}{[2 - \theta^C(1+h^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1+h^L)]^2} + \left[ 1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{-\theta^C}{[2 - \theta^C(1+h^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1+h^L)]^2} \\
& < \left[ 1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \left[ 1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1+h^C)]^2} \right] + \frac{-\theta^C}{[2 - \theta^C(1+h^C)]^2} + \frac{-\theta^L}{[2 - \theta^L(1+h^L)]^2} \\
& + \left[ 2 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{-\theta^C}{[2 - \theta^C(1+h^C)]^2} \frac{-\theta^L}{[2 - \theta^L(1+h^L)]^2} + 1
\end{aligned}$$

That condition simplifies to

$$\begin{aligned}
& - \left[ 1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \left[ 1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1+h^C)]^2} \right] - \left[ 2 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{\theta^C}{[2 - \theta^C(1+h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1+h^L)]^2} - \\
& < \left[ 1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \left[ 1 + \frac{\theta^C}{[2 - \theta^C(1+h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1+h^L)]^2} \right] \\
& < \left[ 1 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \left[ 1 - \frac{\theta^C + \theta^L}{[2 - \theta^C(1+h^C)]^2} \right] + \left[ 2 - \frac{\hat{\varphi} - \bar{\varphi}}{\xi} \frac{h^C h^L}{h^C + h^L} \right] \frac{\theta^C}{[2 - \theta^C(1+h^C)]^2} \frac{\theta^L}{[2 - \theta^L(1+h^L)]^2} +
\end{aligned}$$

It is straightforward to see that, under our assumptions on  $\frac{\hat{\varphi} - \bar{\varphi}}{\xi}$ ,  $\theta^C$  and  $\theta^L$ , those two inequalities are satisfied, so that condition (iii) holds.

Under those three conditions, all eigenvalues of the Jacobian matrix are less than 1 in modulo, so that the equilibrium is stable. Moreover, those conditions suffice also to guarantee that no eigenvalue equals 1 in modulo, and, thus, the hyperbolicity of the equilibrium. Hence, from the Hartman-Grobman Theorem, it can be concluded that, under conditions (i)-(iii), the steady-state is locally stable.

## 6.2 Social optimum

### 6.2.1 Short-run first-best problem

**The first-best short-run solution** From the first two FOCs, we have  $\frac{c_t^i}{d_{t+1}^i} = l_t^i \forall i \in \{C, L\}$ . Note also that  $\theta^C (d_{t+1}^C)^{\theta^C - 1} = \lambda = \theta^L (d_{t+1}^L)^{\theta^L - 1}$ . Given that  $\theta^C > \theta^L$ , we have  $(d_{t+1}^C)^{\theta^C - 1} < (d_{t+1}^L)^{\theta^L - 1}$ . Thus, given that  $\theta^C, \theta^L < 1$  and  $\theta^C > \theta^L$ , this implies  $d_{t+1}^C > d_{t+1}^L$ . Combining this with the above expression yields:  $\frac{c_t^C}{l_t^C} > \frac{c_t^L}{l_t^L}$ .

The FOC for optimal leisure can be written as

$$\frac{1 - \theta^i}{\theta^i} \frac{c_t^i}{l_t^i} + h^{i'} \frac{d_{t+1}^{i\theta^i} (1 - \theta^i) + p_{t+1}^{ii} \hat{\varphi} + p_{t+1}^{ij} \tilde{\varphi}}{\theta^i d_{t+1}^{i(\theta^i - 1)}} = w$$

Comparing this with the FOC under laissez-faire,  $\frac{1 - \theta^i}{\theta^i} \frac{c_t^i}{l_t^i} = w$ , implies that we must have  $l_t^{iLF} < l_t^{iFB} \forall i \in \{C, L\}$ .

The optimal efforts levels are given by:  $e_t^{CFB} = \frac{h_{t+1}^C(1-q_t)\varphi}{\xi}$  and  $e_t^{LFB} = \frac{h_{t+1}^L(q_t)\varphi}{\xi}$ . Comparing these with the ones under laissez-faire:  $e_t^{CLF} = \frac{h_t^C(1-q_t)\varphi}{\xi}$  and  $e_t^{LLF} = \frac{h_t^L(q_t)\varphi}{\xi}$  implies that  $e_t^{iFB}$  exceeds  $e_t^{iLF}$  if and only if the socially optimal longevity  $h_{t+1}^i$  exceeds the prevailing longevity  $h_t^i$ , which may or may not be the case.<sup>34</sup>

**Decentralization of short-run optimum** The decentralized FOCs are:

$$\begin{aligned} \theta^C (c_t^C)^{\theta^C-1} (l_t^C)^{1-\theta^C} &= \lambda^C & \theta^L (c_t^L)^{\theta^L-1} (l_t^L)^{1-\theta^L} &= \lambda^L \\ \theta^C (d_{t+1}^C)^{\theta^C-1} &= \lambda^C & \theta^L (d_{t+1}^L)^{\theta^L-1} &= \lambda^L \\ (1-\theta^C)(c_t^C)^{\theta^C} (l_t^C)^{-\theta^C} &= \lambda^C w & (1-\theta^L)(c_t^L)^{\theta^L} (l_t^L)^{-\theta^L} &= \lambda^L w \\ e_t^C &= \frac{h_{t+1}^C(1-q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} & e_t^L &= \frac{h_{t+1}^L(q_t)(\hat{\varphi}-\tilde{\varphi})}{\xi} \end{aligned}$$

Actually, if transfers  $T^C$  and  $T^L$  are such that, after redistribution:

$$\begin{aligned} \theta^C (c_t^C)^{\theta^C-1} (l_t^C)^{1-\theta^C} &= \theta^L (c_t^L)^{\theta^L-1} (l_t^L)^{1-\theta^L} = \lambda \\ \theta^C (d_{t+1}^C)^{\theta^C-1} &= \lambda = \theta^L (d_{t+1}^L)^{\theta^L-1} \end{aligned}$$

as required in the social optimum, substituting for  $\lambda$  in the FOCs for optimal leisure yields:

$$\frac{(1-\theta^C)c_t^C}{\theta^C l_t^C} = w \quad \frac{(1-\theta^L)c_t^L}{\theta^L l_t^L} = w$$

Comparing these with the FOCs for socially optimal leisures:

$$\begin{aligned} \frac{(1-\theta^C)c_t^C}{\theta^C l_t^C} + h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1-\theta^C) + p_{t+1}^{CC}\hat{\varphi} + p_{t+1}^{CL}\tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C-1}} &= w \\ \frac{(1-\theta^L)c_t^L}{\theta^L l_t^L} + h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1-\theta^L) + p_{t+1}^{LL}\hat{\varphi} + p_{t+1}^{LC}\tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L-1}} &= w \end{aligned}$$

suggests that taxes

$$\begin{aligned} \tau^C &= h_{t+1}^{C'} \frac{(d_{t+1}^C)^{\theta^C} (1-\theta^C) + p_{t+1}^{CC}\hat{\varphi} + p_{t+1}^{CL}\tilde{\varphi}}{\theta^C (d_{t+1}^C)^{\theta^C-1} w} \\ \tau^L &= h_{t+1}^{L'} \frac{(d_{t+1}^L)^{\theta^L} (1-\theta^L) + p_{t+1}^{LL}\hat{\varphi} + p_{t+1}^{LC}\tilde{\varphi}}{\theta^L (d_{t+1}^L)^{\theta^L-1} w} \end{aligned}$$

on wages allow the satisfaction of above FOCs.

Note that, provided the socialization efforts took their optimal values, those taxes, if combined with the lump sum transfers defined above, would allow the

<sup>34</sup>Note that, although agents tend to overwork, the socialization decision is based on the expected - rather than actual - longevity, so that nothing guarantees an underinvestment in socialization.

decentralization of the social optimum. However, efforts are not optimally chosen, because agents, when choosing how much to socialize their child, rely on expected longevities rather than actual longevities. Given the purely physical nature of socialization efforts, the decentralization of the short-run social optimum under myopia cannot be carried out.

## 6.2.2 Long-run first-best problem

**The first-best long-run solution** From the first two FOCs, we have  $\frac{c^i}{d^i} = l^i \forall i \in \{C, L\}$ . Note also that  $\theta^C (d^C)^{\theta^C - 1} = \lambda = \theta^L (d^L)^{\theta^L - 1}$ . Given that  $\theta^C > \theta^L$ , we have  $(d^C)^{\theta^C - 1} < (d^L)^{\theta^L - 1}$ . Thus, given that  $\theta^C, \theta^L < 1$  and  $\theta^C > \theta^L$ , this implies  $d^C > d^L$ . Combining this with the above expression yields:  $\frac{c^C}{l^C} > \frac{c^L}{l^L}$ .

The equality of socialization efforts levels follows from the FOCs and the fact that  $q = \frac{h^C}{h^C + h^L}$  at the steady-state equilibrium.

Regarding leisure times, the FOCs are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h^C} &= q'_h h^{C'} [u^C - u^L] + q (c^C)^{\theta^C} (1 - \theta^C) (l^C)^{-\theta^C} \\ &\quad + q h'^C [(d^C)^{\theta^C} (1 - \theta^C) + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi}] - \lambda q w \\ &\quad + \lambda q'_h h^{C'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \\ \frac{\partial \mathcal{L}}{\partial h^L} &= q'_{h^L} h^{L'} [u^C - u^L] + (1 - q) (c^L)^{\theta^L} (1 - \theta^L) (l^L)^{-\theta^L} \\ &\quad + (1 - q) h^{L'} [(d^L)^{\theta^L} (1 - \theta^L) + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi}] - \lambda (1 - q) w \\ &\quad + \lambda q'_{h^L} h^{L'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \end{aligned}$$

Comparing those FOCs with the ones under laissez-faire shows that the laissez-faire level of  $l^C$  is inferior to its socially optimum level if and only if:

$$\begin{aligned} & q'_h h^{C'} [u^C - u^L] + q h'^C [(d^C)^{\theta^C} (1 - \theta^C) + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi}] \\ & + \lambda q'_h h^{C'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ & > 0 \end{aligned}$$

Similarly, the laissez-faire level of  $l^L$  is inferior its socially optimum level if and only if:

$$\begin{aligned} & q'_{h^L} h^{L'} [u^C - u^L] + (1 - q) h^{L'} [(d^L)^{\theta^L} (1 - \theta^L) + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi}] \\ & + \lambda q'_{h^L} h^{L'} [(1 - l^C)w - (1 - l^L)w - c^C + c^L - h^C d^C + h^L d^L] \\ & > 0 \end{aligned}$$

These inequalities are the ones in Proposition 6.

**Decentralization of the long-run social optimum** The FOCs are:

$$\begin{aligned} \theta^C (c^C)^{\theta^C - 1} (l^C)^{1 - \theta^C} &= \lambda^C & \theta^L (c^L)^{\theta^L - 1} (l^L)^{1 - \theta^L} &= \lambda^L \\ \theta^C (d^C)^{\theta^C - 1} &= \lambda^C & \theta^L (d^L)^{\theta^L - 1} &= \lambda^L \\ (1 - \theta^C) (c^C)^{\theta^C} (l^C)^{-\theta^C} &= \lambda^C w & (1 - \theta^L) (c^L)^{\theta^L} (l^L)^{-\theta^L} &= \lambda^L w \\ e^C &= \frac{h^C (1 - q) (\hat{\varphi} - \bar{\varphi})}{\xi} & e^L &= \frac{h^L (q) (\hat{\varphi} - \bar{\varphi})}{\xi} \end{aligned}$$

Actually, transfers  $T^C$  and  $T^L$  are such that, after redistribution:

$$\begin{aligned} \theta^C (c^C)^{\theta^C - 1} (l^C)^{1 - \theta^C} &= \theta^L (c^L)^{\theta^L - 1} (l^L)^{1 - \theta^L} = \lambda \\ \theta^C (d^C)^{\theta^C - 1} &= \lambda = \theta^L (d^L)^{\theta^L - 1} \end{aligned}$$

Substituting for  $\lambda$  in the FOCs for optimal leisure yields:

$$\frac{(1 - \theta^C) c^C}{\theta^C l^C} = w \quad \frac{(1 - \theta^L) c^L}{\theta^L l^L} = w$$

Comparing these with the FOCs for socially optimal leisures:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h^C} &= q'_{h^C} h^{C'} [u^C - u^L] + q (c^C)^{\theta^C} (1 - \theta^C) (l^C)^{-\theta^C} \\ &\quad + q h^{C'} [(d^C)^{\theta^C} (1 - \theta^C) + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi}] - \lambda q w \\ &\quad + \lambda q'_{h^C} h^{C'} [(1 - l^C) w - (1 - l^L) w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \\ \frac{\partial \mathcal{L}}{\partial h^L} &= q'_{h^L} h^{L'} [u^C - u^L] + (1 - q) (c^L)^{\theta^L} (1 - \theta^L) (l^L)^{-\theta^L} \\ &\quad + (1 - q) h^{L'} [(d^L)^{\theta^L} (1 - \theta^L) + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi}] - \lambda (1 - q) w \\ &\quad + \lambda q'_{h^L} h^{L'} [(1 - l^C) w - (1 - l^L) w - c^C + c^L - h^C d^C + h^L d^L] \\ &= 0 \end{aligned}$$

suggests that taxes

$$\begin{aligned} \tau^C &= \frac{q'_{h^C} h^{C'} \frac{u^C - u^L}{\lambda w} + h^{C'} \frac{(d^C)^{\theta^C} (1 - \theta^C) + p^{CC} \hat{\varphi} + p^{CL} \tilde{\varphi}}{\lambda w}}{\frac{q'_{h^C} h^{C'} \frac{w(1 - l^C) - w(1 - l^L) - c^C + c^L - h^C d^C + h^L d^L}{w}}}{q} \\ \tau^L &= \frac{q'_{h^L} h^{L'} \frac{u^C - u^L}{\lambda w} + h^{L'} \frac{(d^L)^{\theta^L} (1 - \theta^L) + p^{LL} \hat{\varphi} + p^{LC} \tilde{\varphi}}{\lambda w}}{\frac{q'_{h^L} h^{L'} \frac{w(1 - l^C) - w(1 - l^L) - c^C + c^L - h^C d^C + h^L d^L}{w}}}{1 - q} \end{aligned}$$

on wages allow the satisfaction of above FOCs.

Those taxes, if combined with the lump sum transfers defined above, allow the decentralization of the social optimum, as efforts are, under that tax and transfer system, now optimally chosen, unlike what used to prevail in the short-run.

**Comparison of short-run and long-run Pigouvian taxes** To see why [2] + [3] and [2'] + [3'] must be of opposite signs, it suffices to notice that:

$$\frac{q'_{h^C}}{q} = \frac{h^L}{h^C(h^C+h^L)} > 0 \quad \frac{q'_{h^L}}{1-q} = \frac{-h^C}{h^L(h^C+h^L)} < 0$$

Moreover, the factors  $\frac{u^C-u^L}{\lambda w}$  and  $\frac{w(1-l^C)-w(1-l^L)-c^C+c^L-h^C d^C+h^L d^L}{w}$  are present in the formulae of  $\tau^C$  and  $\tau^L$ . Hence, it is easy to see that, whatever the signs and sizes of  $\frac{u^C-u^L}{\lambda w}$  and  $\frac{w(1-l^C)-w(1-l^L)-c^C+c^L-h^C d^C+h^L d^L}{w}$  are, [2] + [3] and [2'] + [3'] are necessary of opposite signs. Thus, if  $\tau^{CLR} - \tau^{CSR} \geq 0$ , it must be the case that the composition effect component of the tax is positive for  $C$ , and thus negative for  $L$ , so that  $\tau^{LLR} - \tau^{LSR} \leq 0$ . Moreover, if  $\tau^{LLR} - \tau^{LSR} \leq 0$ , it must also be the case that  $\tau^{CLR} - \tau^{CSR} \geq 0$ .