Capital Taxation and Local Labor Markets

Yasuhiro Sato*and Jacques François Thisse†

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Abstract

This paper develops a two-country model having imperfect matching between firms and workers, and investigates the effects of capital mobility and tax competition, via skill mismatch, on the spatial distribution of firms. Analysis shows that the difference in population between countries matters for the distribution of firms and for the consumers' welfare.

Keywords: fiscal competition, local labor markets, capital mobility

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*Graduate School of Environmental Studies, Nagoya University (Japan) and CORE, Université catholique de Louvain (Belgium).ysato@info.human.nagoya-u.ac.jp

†CORE, Université catholique de Louvain (Belgium), PSE (France) and CEPR.

1 Introduction

During the last two decades, OECD countries have experienced very high increases in foreign direct investments (FDI). Reporting the trends of FDI inflows and outflows as percentage of GDP based on the weighted average across all OECD countries from 1981 to 1999, Miyake and Sass (2000) find that both were around 0.5 percent in 1981 and rose to 2.5 percent in 1999. This internationalization of production has increased the real inward FDI position of the average OECD country, measured in constant 1996 purchasing power parities, from \$81 billion to \$158 billion over the 1990-2000 period (OECD, 2003). In such a context, the existence of differentials in corporate tax rates is likely to affect the location of economic activity and, indeed, empirical evidence shows that governments vastly use this instrument in the hope of influencing firms' locational choices (Mooij and Ederveen, 2003). Building on that observation, the literature on fiscal competition aims at studying how governments choose their tax rates in a strategic environment, typically by assuming that the productivity of capital is expressed through a standard neoclassical production function.(Wilson, 1999).

Because the outcome of fiscal competition crucially depends on the spatial mobility of production factors, a richer approach is to build on the microeconomic underpinnings that explain the location of firms and, thus, the international distribution of capital. This is what new economic geography (NEG) has accomplished by explaining how firms do interact to form clusters within a few regions (Fujita *et al.*, 1999; Baldwin *et al.*, 2003). It seems, therefore, natural to revisit the process of fiscal competition by incorporating the main forces uncovered by NEG, namely increasing returns, market size, and imperfect competition. This is the road taken recently by Anderson and Forslid (2003), Baldwin and Krugman (2004), and Ottaviano and van Ypersele (2005). However, very much like in NEG, all these authors have chosen to stress the role of the product market. Yet, recent empirical contributions suggest that labor market pooling is one of the main reasons that explain the existence of firms' clusters (Dumais *et al.*, 2002; Rosenthal and Strange, 2004). The specificity of human capital being itself the main reason for imperfect matching between firms and workers, we find it appealing to study how skill mismatch affects the spatial distribution of firms through both firms' locational choices and the working of local labor markets (Kim, 1989; Hamilton *et al.*, 2000; Amiti and Pissarides, 2005). It is worth noting that the existence of skill mismatch yields increasing returns with respect to the size of labor pool, thus implying that our approach concurs with NEG (Kim, 1989; Helsley and Strange, 1990).

In this paper, we study how fiscal competition affects firms' distribution and consumers' welfare across countries of different sizes in a setting where firms compete to hire heterogenous, but immobile, workers. In other words, as in most European economies, capital is mobile and labor is not. Because income redistribution is one of the main objectives of many EU-15 governments, we find it reasonable to abstract from the possible inefficiency of public goods provision. Finally, as most FDI take place in countries with similar technologies and factor endowments (think of the EU-15 members and the OECD countries), we also abstract from comparative advantage of both the Ricardian and Heckscher-Ohlin types. However, whereas the typical assumption in fiscal competition is that competing jurisdictions have the same size, countries involved in FDI vastly differ in terms of market sizes.[1] Size being out of the reach of harmonization, we focus on a setting in which countries are asymmetric in the size of their domestic capital and labor.

Our main results may be summarized as follows. First, we show that, in all configurations (autarchy, no-tax, fiscal competition outcomes), the large country's residents enjoy a higher utility level than do those of the small country, thus implying the "importance of being large". This is reminiscent of Ottaviano and van Ypersele (2005) who assume that countries have different product-market sizes. However, the two approaches lead to very different results regarding the

[1] Exceptions include Bucovetsky (1991) and Wilson (1991) who introduce asymmetric population sizes in a simple tax competition model. They show that the large region imposes higher capital tax rate than does the small region, and that the small region can be better off with than without tax competition. This result is known as "the importance of being small".

international distribution of firms. Though increasing returns to scale enables the large country to attract more firms than does the small country, competition on local labor markets hinders the large country to have a more than proportionate share of firms, thus showing that there is no home market effect in the present context. More precisely, whereas the large country attracts a more than proportionate share of firms in Ottaviano and van Ypersele, the large country attracts more firms than the small one, but it does so less than proportionally here.

Furthermore, the few existing studies on asymmetric tax competition predict that the large country has higher corporate tax rate than does the small one (Bucovetsky, 1991; Wilson, 1991; Ottaviano and van Ypersele, 2005). This prediction does not fit well the real world, however. For example, Devereux *et al.* (2002) report the effective corporate tax rates with respect to several OECD countries. Their Figure 7 reveals that, if the effective average tax rates in Germany, Japan, and the United States are higher than those set in Austria, Finland and Sweden, those prevailing in Belgium and in Greece are higher than those in France and in the United Kingdom. We show that the small country levies higher corporate tax rate than does the large country, thus providing an explanation for the fact that small countries have higher effective average tax rates than do large countries. By focussing on microeconomic underpinnings of firms' location, we are thus able to identify results that invite us to reconsider the impact of fiscal competition.

Finally, our analysis has two major redistributional implications. The first one is that tax competition leads to redistribution between countries, whereas the standard fiscal competition literature suggests the existence of a "race to the bottom" (Cremer and Pestieau, 2004). Indeed, under autarky, governments have no incentive to tax firms and to transfer the proceeds to their workers. The other implication is that the large country always gains and the small country always loses from tax competition, thus implying tax competition leads to redistribution from the small to the large country. This sharply differs from the result obtained in the existing studies where the small country typically gains from tax competition (Bucovetsky, 1991; Wilson, 1991).

The remaining of the paper is organized as follows. The model is presented in section 2. In section 3, we study the international distribution of capital in the no-tax case, whereas the process of fiscal competition is discussed in section 4. Section 5 concludes.

2 The model and preliminary results

Consider an economy formed by two countries, labeled 1 and 2, and a total mass L of consumers. Each consumer is endowed with one unit of labor and one unit of capital. Our modeling strategy thus abstracts from redistributional issues between capital-owners and workers. Let $\theta \in (0,1)$ denote the share of consumers in country 1, which implies that θ also measures that country's shares of labor and capital. Let $l_1 = \theta L$ and $l_2 = (1-\theta)L$ denote the mass of consumers in countries 1 and 2, respectively. Without loss of generality, we assume that $l_1 \geq l_2$. Unless explicitly mentioned, we consider here asymmetric countries with $\theta > 1/2$, thus implying that country 1 (2) is the large (small) country. Consumers are immobile and can supply labor only in the country in which they reside, so that labor markets are "local". By contrast, consumers are free to supply capital wherever they want.

2.1 Local labor markets

Workers have the same level of general human capital but heterogeneous skills. Let α be the common level of general human capital. Workers are heterogeneous in the type of work they are best suited for, but there is no ranking in any sense of these types of work. Workers' skill types are denoted by x. The characteristics of a worker relevant to firms are summarized by her skill.

The industry is formed by firms that supply a homogeneous good sold on a competitive market in each country. This good can be shipped at zero cost between the two countries so that its price is the same on both markets; we take this good as the numéraire. Hence, product market conditions do not influence firms' locational choices. In our setting, a firm is fully described by the amount of capital it uses as well as by the type of worker it needs. Let $f > 0$ be the fixed requirement of capital needed to be active on the market, so that the total number of firms in the economy is given by $N = L/f$. Each firm has a specific technology such that workers can produce only when they perfectly match the firm's skill needs. Since workers are heterogeneous, they have different matches with the firm's job offer. Thus, if firm k hires a worker whose skill differs from x_k, the worker must get trained and her cost of training to meet the firm's skill requirement is a function of the difference between the worker's skill x and the skill requirements x_k.

In describing the heterogeneity of workers, we follow Kim (1989), Helsley and Strange (1990), Hamilton *et al.* (2000) and many others by assuming that the skill space is described by the circumference C of a circle with a length normalized to one. Individuals' skills are continuously and uniformly distributed along this circumference; the density is constant in country i and denoted by l_i. In this context, the density expresses the size of the local labor market. There are n_i firms in country i, with $n_1 + n_2 = N$. Firms' job requirements x_k are equally spaced along the circumference C so that $1/n_i$ is the distance between two adjacent firms in the skill space. The training cost function is $\beta |x - x_k|$, where β expresses the ability of a worker to learn how to adjust to a technology different from her skill. After training, all workers are identical from the firm's viewpoint since their ex post productivity is observable and equal to α by convention (thus, there is no moral hazard problem within firms).

In this paper, we assume that each worker's skill type is not observable for firms and firms know only the distribution of x (Hamilton *et al.*, 2000). Hence, the training costs are paid by workers. However, workers know their own types and observe the firms' skill needs. In order to induce the appropriate set of workers to take jobs with the most suitable firm, workers must pay at least some part of the training cost. In addition, since the supply of a worker is inelastic, firms cannot offer a wage menu so that the worker must pay for all the costs of training which are not observable to the firm (hence resolving the adverse selection problem). Consequently, each firm i offers the same wage to all its workers, conditional on the worker having been trained to the skill x_k. Each worker then compares the wage offers of firms and the required training costs; she simply chooses to work for the firm offering the highest wage net of training costs.

Suppose that firm k is located in country i. Assuming that the skill spaces are identical across countries, firms on each side of k offer wages $w_{i,k-1}$ and $w_{i,k+1}$, then firm k's labor pool consists of two subsegments whose outer boundaries are $\overline{x}_{i,k}$ and $\overline{x}_{i,k+1}$. The worker at $\overline{x}_{i,k}$ receives the same net wage from firm k and firm $k-1$, whereas the worker at $\overline{x}_{i,k+1}$ receives the same net wage from firm k and firm $k+1$. Because firm k knows the training cost function and all firms' skill requirements, it can determine $\overline{x}_{i,k}$ and $\overline{x}_{i,k+1}$ as the solutions to the two equations $w_{i,k}-\beta(x_{i,k}-\overline{x}_{i,k}) = w_{i,k-1}-\beta(\overline{x}_{i,k}-x_{i,k-1})$ and $w_{i,k}-\beta(\overline{x}_{i,k+1}-x_{i,k}) = w_{i,k+1}-\beta(x_{i,k+1}-\overline{x}_{i,k+1})$. Hence, we have

$$\begin{aligned} \overline{x}_{i,k} &= \frac{w_{i,k-1} - w_{i,k} + \beta(x_{i,k} + x_{i,k-1})}{2\beta} \\ \overline{x}_{i,k+1} &= \frac{w_{i,k} - w_{i,k+1} + \beta(x_{i,k} + x_{i,k+1})}{2\beta}. \end{aligned} \tag{1}$$

Firm k's profits are then given by

$$\begin{aligned} \pi_{i,k} &= \int_{\overline{x}_{i,k}}^{\overline{x}_{i,k+1}} l_i(\alpha - w_{i,k})dx - r_i f \\ &= l_i(\alpha - w_{i,k})(\overline{x}_{i,k+1} - \overline{x}_{i,k}) - r_i f \end{aligned} \tag{2}$$

where r_i is the price of capital and $w_{i,k}$ the wage firm k pays when it is located in country i. Note that countries have different wages because labor is immobile between countries. In the rest of this section, we suppose that capital is immobile too. Hence, the number of firms in each country is given and each country is in autarky. The amount of capital available in country i being l_i, the fact that a firm needs f units of capital to be active implies that the number of firms in country i under autarky is given by

$$n_i^a = \frac{l_i}{f} \tag{3}$$

where the superscript a stands for the autarky case. This in turn implies that the large country has a larger number of firms than does the small ($n_1^a > n_2^a$). However, the capital-labor ratio is the same across countries

$$\frac{n_1^a}{l_1} = \frac{n_2^a}{l_2} \tag{4}$$

because the per capita endowment of capital is the same in the two countries.

2.2 Equilibrium factor prices

We now show how $w_{i,k}$ and r_i are determined. We find the equilibrium wages by taking the first-order condition for $\pi_{i,k}$ with respect to $w_{i,k}$:

$$\frac{\partial \pi_{i,k}}{\partial w_{i,k}} = -(\overline{x}_{i,k+1} - \overline{x}_{i,k}) + (\alpha - w_{i,k})\left(\frac{\partial \overline{x}_{i,k+1}}{\partial w_{i,k}} - \frac{\partial \overline{x}_{i,k}}{\partial w_{i,k}}\right) = 0. \tag{5}$$

Focusing on a symmetric equilibrium, it follows from (1), (5), and $\overline{x}_{i,k+1} - \overline{x}_{i,k} = 1/n_i$ that

$$w_{i,k}^* = \alpha - \frac{\beta}{n_i} \equiv w_i^*. \tag{6}$$

Hence, the equilibrium wage is equal to the marginal productivity of labor after training, minus a premium that local firms are able to levy because workers cannot move costlessly from one firm located in country i to another. Note that this premium decreases as the number of firms located in this country rises.

Substituting $w_{i,k}^*$ into (2), we get

$$\pi_{i,k}^* = \frac{\beta l_i}{n_i^2} - r_i f. \tag{7}$$

Note that output per capita in country i is given by

$$\left\{\int_{\overline{x}_{i,k}}^{x_{i,k}} l_i \left[\alpha - \beta(x_{i,k} - x)\right] dx + \int_{x_{i,k}}^{\overline{x}_{i,k+1}} l_i \left[\alpha - \beta(x - x_{i,k})\right] dx\right\}$$
$$\times \left(\int_{\overline{x}_{i,k}}^{\overline{x}_{i,k+1}} l_i dx\right)^{-1} = \alpha - \frac{\beta}{4n_i}$$

thus implying that the industry exhibits increasing returns to scale with respect to the number of firms.

It remains to describe how the price of capital is determined in each country. Following a well-established tradition in this strand of literature, we assume that there is free entry in the industry

(Helpman and Krugman, 1985). Consequently, competition for capital drives profits down to zero, thus implying that r_i must be such that $\pi^*_{i,k} = 0$. This yields the equilibrium price of capital in country i:

$$r_i^* = \frac{\beta l_i}{f n_i^2}. \tag{8}$$

Under autarky, the price of capital is

$$r_i^* = \frac{\beta}{n_i^a}.$$

Hence, the price of capital is larger in the small country than in the large one.

Expressions (7) and (8) encapsulate the main forces at work in the present setting. First, as the size of the labor force l_i increases, country i becomes more profitable to firms because a larger labor pool allows them to hire more workers and, hence, to produce and sell more. We refer to that as the *labor-market pooling effect.* Second, when the number of firms n_i rises, there is more competition on the labor market, the size of which is fixed because workers are immobile. This leads firms to pay higher wages, thus making country i less attractive. We call this force the *labor-market crowding effect.* Last, because firms must break even, the price of capital goes down once more firms are located in country i. We refer to that as the *capital price effect.*

The (indirect) utility of an individual of skill type x working for firm k in country i is given by

$$V_{i,k}(x) = w_i^* - \beta\,|x - x_{i,k}| + r_i^*$$

which is equal to

$$V_{i,k}(x) = \alpha - \frac{\beta}{n_i} - \beta\,|x - x_{i,k}| + \frac{\beta l_i}{f n_i^2}$$

in the autarky case. The average utility of firm k's employees is then

$$\begin{aligned} V_{i,k} \;&=\; \left\{ \int_{\overline{x}_{i,k}}^{x_{i,k}} l_i \left[\alpha - \frac{\beta}{n_i} - \beta(x_{i,k} - x) + r_i^*\right] dx \right. \\ &\quad \left. + \int_{x_{i,k}}^{\overline{x}_{i,k+1}} l_i \left[\alpha - \frac{\beta}{n_i} - \beta(x - x_{i,k}) + r_i^*\right] dx \right\} \\ &\quad \times \left(\int_{\overline{x}_{i,k}}^{\overline{x}_{i,k+1}} l_i dx \right)^{-1} . \end{aligned}$$

Because $\overline{x}_{i,k+1} = x_{i,k} + 1/2n_i$ and $\overline{x}_{i,k} = x_{i,k} - 1/2n_i$ at the symmetric equilibrium, we have

$$V_{i,k} = \alpha - \frac{5\beta}{4n_i} + r_i^* \equiv V_i. \tag{9}$$

In this expression, the second term represents the effect of improving the quality of matching. When the number of local firms rises, the average mismatch decreases, implying that the average wage increases. However, as shown by (7), an increase in the number of firms also leads to a lower capital price. Thus, the total impact of the number of firms on workers' welfare is a priori ambiguous. From (3), (8) and (9), we have

$$\begin{aligned} V_1^a - V_2^a &= \frac{5\beta}{4}\left(\frac{1}{n_2^a} - \frac{1}{n_1^a}\right) + \frac{\beta}{f}\left[\frac{l_1}{(n_1^a)^2} - \frac{l_2}{(n_2^a)^2}\right] \\ &= \frac{\beta}{4}\left(\frac{1}{n_2^a} - \frac{1}{n_1^a}\right) = \frac{\beta f}{4L}\left(\frac{1}{1-\theta} - \frac{1}{\theta}\right) \geq 0. \end{aligned}$$

Hence, increasing the share of country 1 in the global economy makes the residents of this country relatively better off than those of the small country's residents. The welfare gap between the two countries thus rises.

Summarizing the foregoing discussion, we have:

Proposition 1 *Consider two countries that have different sizes but the same relative endowment of capital and labor. When capital is immobile, consumers reach a higher utility level in the large country. Furthermore, the larger the difference in size, the larger the gap in individual welfare levels between the two countries.*

3 Capital mobility

In this section, we allow for capital mobility. As the number of firms in a country rises, the capital price effect has the nature of a pulling force, whereas the labor crowding effect acts as a pushing force. The international allocation of capital is thus the outcome of a process involving opposite forces. Furthermore, when the size of the local market increases, the labor-market pooling effect also has the nature of an attraction force. Throughout the rest of the paper, we assume that the number of firms is sufficiently large to avoid the integer problem and thus treat n_i as a real number.

3.1 Free market outcome

When capital is mobile between countries, capital flows to the country with higher capital price. Hence, arbitrage induces the capital prices in both countries to be the same:

$$r_1 = r_2.$$

Using (8), this equilibrium condition can be rewritten as follows:

$$\frac{l_1}{n_1^2} = \frac{l_2}{n_2^2} \tag{10}$$

so that $n_1^* > n_2^*$ if and only if $l_1 > l_2$, whereas $l_1 = l_2$ implies that $n_1^* = n_2^*$.

Since the amount of capital in the global economy is fixed, the total number of firms is still given by N. Because $n_1 + n_2 = N$, (10) allows us to determine the equilibrium number of firms in country i:

$$n_i^m = \frac{\sqrt{l_i}}{\sqrt{l_1} + \sqrt{l_2}} N \tag{11}$$

where the superscript m stands for the case of mobile capital. The corresponding value of the price of capital is then obtained by substituting (11) into (8):

$$r^m = \frac{\beta}{N} \left(\sqrt{\theta} + \sqrt{1-\theta} \right)^2$$

which is a decreasing and concave function of θ over the interval $(1/2, 1)$.

As long as the two countries have a different sizes, the mobility of capital generates a distribution of firms that differs from the one arising under autarky. Comparing (3) and (11), it is readily verified that

$$n_1^m < n_1^a \qquad n_2^m > n_2^a.$$

Thus, capital is exported from the large country to the small country. This should not come as a surprise as, under autarky, the price of capital is higher in the latter than in the former. Yet, the large country still retains a larger number of firms than the small one: $n_1^m > n_2^m$. Indeed, as there is no capital price effect in equilibrium, (7) and the equalization of profits between countries implies that the labor-market pooling effect generated by the large country must be exactly offset by a stronger labor crowding effect. This, in turn, means that the large country hosts a larger number of firms.

This is not the end of the story, however. Indeed, it is easy to see that the investment level per capita is smaller in the large country than in the small country:

$$\frac{n_1^m}{l_1} < \frac{n_2^m}{l_2}. \tag{12}$$

Using (4) and (12), we may then conclude that

$$\frac{n_1^m}{l_1} < \frac{n_1^a}{l_1} = \frac{n_2^a}{l_2} < \frac{n_2^m}{l_2}.$$

Hence, when capital is mobile, the large economy accommodates more firms than the small one, but it does so less than proportionally in terms of investment per capita. This is because country 1 has a larger labor pool that allows it to attract more firms, which then have to pay higher wages. In the process of international capital allocation, the negative labor crowding effect partly offsets the positive labor-market pooling effect and leads to a more dispersed distribution of capital.

Having said that, we now want to know whether our setting exhibits a "home market effect". According to Krugman (1980) and others, when the industry is characterized by increasing returns to scale, the large country would attract a 'more than proportionate' share of firms. Though standard, it will appear to be convenient to refer to such a property as the "static" home market effect. Define the share of firms in country 1 as $\lambda = n_1/N$. It then follows immediately from (11) that

$$\lambda^m = \frac{\sqrt{\theta}}{\sqrt{\theta} + \sqrt{1-\theta}}.$$

Accordingly, we have

$$\lambda^m = \frac{\sqrt{\theta}}{\sqrt{\theta}+\sqrt{1-\theta}} < \frac{\theta}{\theta+\sqrt{1-\theta}\sqrt{1-\theta}} = \theta$$

because $\theta > 1-\theta$, where the equality holds if and only if $\theta = 1/2$. Thus, unless the two countries have the same size, the large country hosts a 'less than proportional' share of the industry, implying the existence of a *reverse home market effect*. As shown by (10), this is because the labor-market pooling effect is proportional to the number of firms whereas the labor crowding effect is proportional to the square of the number of firms, thus making the large country "relatively" less attractive to firms. Consequently, even though more firms locate in country 1 than in country 2, country 1's share of firms is smaller than its consumption share.

Another way to look at the home market effect, which is very popular in empirical works (Head and Mayer, 2004), is to evaluate the elasticity of λ^m with respect to θ. Then, there is a "dynamic" home market effect once this elasticity is larger than one. In standard models of monopolistic competition with two countries and two sectors, the two effects are formally equivalent (Head *et al.*, 2002). However, as shown by Behrens *et al.* (2004), this is need not be the case in a multi-country setting. We show below that making such a distinction is critical too in our context. Indeed, we have

$$\frac{d\lambda^m}{d\theta}\frac{\theta}{\lambda^m} = \frac{1}{2\sqrt{1-\theta}\left(\sqrt{\theta}+\sqrt{1-\theta}\right)} > 0 \qquad \text{because} \qquad \frac{1}{2} < \theta < 1.$$

Furthermore, if

$$\varepsilon_\lambda \equiv \frac{d\lambda^m}{d\theta}\frac{\theta}{\lambda^m}$$

we have

$$\begin{aligned}
\varepsilon_\lambda &< 1 \qquad \text{when } \theta \in (1/2, \theta^m) \\
\varepsilon_\lambda &= 1 \qquad \text{when } \theta = \theta^m \equiv 1/2 + \sqrt{2}/4 \\
\varepsilon_\lambda &> 1 \qquad \text{when } \theta \in (\theta^m, 1)\,.
\end{aligned}$$

Hence, moving consumers from the small to the large country exacerbates the difference in net outputs. However, given that

$$\frac{d^2\lambda^m}{d\theta^2} = \frac{\left(4\sqrt{\theta}\sqrt{1-\theta}+1\right)(2\theta-1)}{4\left[\theta(1-\theta)\right]^{3/2}(\sqrt{\theta}+\sqrt{1-\theta})^4} > 0 \qquad \text{because } \theta > \frac{1}{2}$$

the elasticity ε_λ increases with θ. In other words, unless the two countries are very different, raising the size of the large country leads to a growing increase of its share of firms, which is less than proportional when $\theta < \theta^m$ and more than proportional when θ exceeds θ^m. Hence, there is no 'dynamic' home market effect when the size of country 1 is not very large, but such an effect arises for sufficiently large asymmetries in size. This may be explained as follows. At the equilibrium wage, the profit of a firm located in country 1 increases with the size of the labor market but decreases with the equilibrium capital price. As the latter falls when θ rises, both effects pull into the same direction, thus explaining why the elasticity ε_λ increases with the size of the large market. Furthermore, as the marginal impact of the market-labor pooling effect is constant whereas the marginal impact of the capital price effect is increasing in absolute value, the marginal aggregate effect increases; it becomes more than proportional once the large market is big enough.

The above arguments can be summarized as follows.

Proposition 2 *Consider two countries that have different sizes but the same relative endowment of capital and labor. When capital is mobile, the large country attracts more capital than the small one, but the capital-labor ratio is higher in the small country. Furthermore, capital mobility gives rise to a reverse static home market effect, but exhibits a dynamic home market effect when the size of the two countries is sufficiently different.*

3.2 The welfare implications of capital mobility

The equilibrium distribution of firms is first shown to minimize training costs in the global economy. Total training costs are given by

$$\begin{aligned} T(n_1, n_2) &= 2n_1 \int_0^{1/(2n_1)} l_1 \beta \sigma d\sigma + 2n_2 \int_0^{1/(2n_2)} l_2 \beta \sigma d\sigma \\ &= \frac{\beta l_1}{4n_1} + \frac{\beta l_2}{4n_2}. \end{aligned}$$

The first order condition for the minimization of T with respect to n_1 and n_2, taking $n_1 + n_2 = N$ into account, yields (10). Note that the net output of the global economy is $\alpha L - T(n_1, n_2)$. Hence, the equilibrium distribution of firms maximizes the net output of the global economy. In other words, allowing for *the free mobility of capital is globally efficient.* However, when capital is mobile, we may have redistributive effects.

To see it, we compare the welfare levels reached in each country at the market outcome with and without capital mobility. From $r_1 = r_2$ and (11), the utility difference across countries in the tax case is given by

$$V_1^m - V_2^m = \frac{5\beta}{4}\left(\frac{1}{n_2^m} - \frac{1}{n_1^m}\right) = \frac{5\beta f\,(2\theta - 1)}{4L\sqrt{\theta}\sqrt{1-\theta}} > 0$$

because $\theta > 1/2$ (and $n_1^m > n_2^m$). Furthermore, the welfare gap rises as the size discrepancy increases.

Turning to comparisons of welfare under mobility and autarky, standard calculations show that

$$\begin{aligned} V_i^m - V_i^a &= \left(\frac{1}{n_i^{im}} - \frac{1}{n_i^m}\right)\left[\frac{5\beta}{4} - \frac{\beta l_i}{f}\left(\frac{1}{n_i^m} + \frac{1}{n_i^a}\right)\right] \\ &= \frac{\beta f\,(l_j - \sqrt{l_i}\sqrt{l_j})\,(l_j - 3l_i - 4\sqrt{l_i}\sqrt{l_j})}{4(l_1 + l_2)^2 l_i}. \end{aligned}$$

As $\theta > 1/2$ and, hence, $l_1 > l_2$, this implies that

$$V_1^m - V_1^a > 0.$$

Hence, *the large country always gains from capital mobility.* Though intuitive, this result is not immediate. Indeed, country 1's residents get higher capital incomes because its price rises when it can be invested abroad, but they earn lower wages on their local labor market because the number of local firms is lower.

The implications of capital mobility for the small country are even less straightforward. Because $l_1 - \sqrt{l_1}\sqrt{l_2} > 0$, it turns out that $V_2^m - V_2^a > 0$ holds if and only if

$$l_1 - 3l_2 - 4\sqrt{l_1}\sqrt{l_2} = (4\theta - 3 - 4\sqrt{\theta}\sqrt{1-\theta})L > 0.$$

This is a second degree inequality that is satisfied on the unit interval if and only if $\theta > \theta_c \equiv (5+\sqrt{7})/8 > 1/2$. Thus, we have:

Proposition 3 *Compared to the autarky case, capital mobility always raises the utility level in the large country. However, the utility level in the small country decreases if and only if countries have very different sizes.*

When capital is mobile, the global output net of training costs increases and reaches its maximum at the equilibrium distribution of firms. However, these gains need not benefit each country. Indeed, when capital is mobile, we know that some firms move to the small country. Hence, the large country's capital income rises but its labor income falls, whereas these two effects go in the opposite direction in the small country. On the one hand, in the large country, the gains resulting from the higher price of capital for country 1's residents always more than compensate their wage decrease. This is because country 1 hosts more firms than country 2, thus making the marginal and negative impact of the labor crowding effect weak enough, whereas the marginal and positive impact of the capital price effect remains strong enough. On the other, consumers in country 2 earn higher wages under capital mobility than under autarky (see Figure 1). Whether these gains are large enough to compensate for the lower price of capital now depends on the relative size of the two countries. As the large country gets bigger, the wage level in the small country goes down, but its decrease is sharper under autarky than under capital mobility (see Figure 2). Consequently, when θ is sufficiently large, the gains in wage income may compensate the loss in capital income. By contrast, when country sizes are similar, such a compensation is not possible. This shows that *country size matters for the welfare implications of capital mobility.*

4 Capital taxation

4.1 The tax game

This section considers two local governments that maximize the utility level of their residents by imposing taxes on firms and by distributing the proceeds to consumers as lump-sum transfers. Let s_i and t_i denote the lump-sum transfer to consumers and the lump-sum tax on firms in country i.[2] Note that the former may be positive ($s_i > 0$) and the latter negative ($t_i < 0$), thus meaning that government i may decide to subsidy firms and, therefore, to tax its residents instead of taxing capital.

In this case, the profit of a firm (2) and the utility level of a worker (9) become:

$$\pi^*_{i,k} = \frac{\beta l_i}{n_i^2} - r_i f - t_i \tag{13}$$

$$V_i = \alpha - \frac{5\beta}{4n_i} + r_i + s_i. \tag{14}$$

In what follows, we consider a standard two stage game in which local governments, first, determine s_i and t_i simultaneously and, then, firms enter the market, decide where to locate and pay the corresponding wage. From now on, we will refer to the fist stage game as the tax game. The equilibrium concept we adopt is a subgame perfect Nash equilibrium. As usual, the model is solved by backward induction.

[2] Because firms would not operate under negative profits, the nonegativity of profits is here a natural constraint to satisfy. It yields an upper bound on t_i, which is itself bounded above by the highest possible output of country i, that is, αl_i.

Consider the second stage subgame induced by s_i and t_i $(i = 1, 2)$. The capital price in each country is then given by

$$r_i = \frac{1}{f}\left(\frac{\beta l_i}{n_i^2} - t_i\right). \tag{15}$$

Free entry and the equalization of capital prices leads to the condition:

$$\frac{\beta l_1}{n_1^2} - t_1 = \frac{\beta l_2}{n_2^2} - t_2. \tag{16}$$

Hence, firms' decisions in the second stage imposes a constraint, given by (16), on the tax game between countries 1 and 2.[3]

We now consider the tax game. Country i's government, which fully anticipates the influence of its decision on r_i and on the resulting distribution of firms determined by (16), maximizes (14) with respect to s_i and t_i under the budget constraint

$$s_i l_i = t_i n_i.$$

Substituting (15) and $s_i l_i = t_i n_i$ into (14), we obtain

$$V_i = \alpha - \frac{5\beta}{4n_i} + \frac{1}{f}\left(\frac{\beta l_i}{n_i^2} - t_i\right) + \frac{n_i t_i}{l_i}. \tag{17}$$

Substituting $n_2 = N - n_1$ into (16), we get

$$\frac{\beta l_i}{n_i^2} - t_i = \frac{\beta l_j}{(N - n_i)^2} - t_j. \tag{18}$$

Thus, the welfare problem of government i amounts to maximizing (17) with respect to t_i, subject to the constraint (18).

We have just seen that, at any equilibrium of the tax game, the following condition must be satisfied:

$$g(t_i, n_i; t_j) \equiv \frac{\beta l_i}{n_i^2} - t_i - \frac{\beta l_j}{(N - n_i)^2} + t_j = 0.$$

[3] We disregard the possibility that both governments tax away the whole surplus made by firms in a country. This is because the price of capital would be zero in each country, thus making the distribution of firms undetermined.

Totally differentiating $g(t_i, n_i; t_j)$ for a given t_j yields $dn_i/dt_i(t_i)$. Taking the total differential of V_i and using dn_i/dt_i then leads to

$$t_i^* = \frac{3\beta l_i}{4n_i^2} + \frac{2\beta l_i l_j}{n_j^3}\left(\frac{n_i}{l_i} - \frac{1}{f}\right). \tag{19}$$

A similar argument holds for country $j \neq i$. From $n_1 + n_2 = N = (l_1 + l_2)/f$, it follows that

$$\frac{l_i}{f} - n_i = n_j - \frac{l_j}{f}. \tag{20}$$

Substituting (19) into (18), rearranging terms, and using (20), we see that (18) may be rewritten as follows:

$$\frac{1}{l_1}\left(\frac{l_1}{n_1}\right)^2\left(\frac{7}{8} - \frac{l_1}{fn_1}\right) = \frac{1}{l_2}\left(\frac{l_2}{n_2}\right)^2\left(\frac{7}{8} - \frac{l_2}{fn_2}\right). \tag{21}$$

Thus, if a Nash equilibrium of the tax game exists, there is a pair (t_1^*, t_2^*) such that (i) t_i^* maximizes $V_i(t_i, t_j^*)$, $i, j = 1, 2$ and $j \neq i$ and (ii) at the resulting distribution of firms, both the conditions $n_1 + n_2 = N$ and (21) are satisfied.

The existence of a (pure-strategy) Nash equilibrium in tax games is known to be a very problematic issue. Even in the case of simple games such as those by Wilson (1986) and Zodrow and Mieszkowski (1986), the existence of a Nash equilibrium has so far been proven only under special circumstances. For example, Laussel and Le Breton (1998) as well as Rothstein (2004) simplify the tax game by ignoring consumers' capital income, whereas Bucovetsky (2003) shows the existence of a Nash equilibrium in a tax game with a continuum of countries. This state of affairs has recently led Bayindir-Upmann and Ziad (2005) to focus on a more general concept, namely a *local Nash equilibrium*, in which t_i^* is the best reply of government i against t_j^* $(j \neq i)$ provided that t_i is restricted to small variations around t_i^* (formally, t_i must belong to some neighborhood of t_i^*).[4] Clearly, when the tax game has a standard Nash equilibrium, this equilibrium is local; however, the tax game may have a local equilibrium while a standard equilibrium fails to exist. However, the existence results by Bayindir-Upmann and Ziad do not apply here because they

[4] A similar approach has been proposed by Bonanno (1988) in oligopoly theory.

focus on symmetric games, whereas ours involves asymmetric countries. Yet, the following result, which is proven in Appendix A, shows that a local equilibrium always exists in our setting.

Proposition 4 *The tax game always has a local Nash equilibrium. Furthermore, this equilibrium is unique when we restrict ourselves to interior solutions.*

Regarding the equilibrium distribution of firms, the following result is shown to hold in Appendix B.

Proposition 5 *Assume that the tax game has an equilibrium. Then, the equilibrium number of firms per capita is larger in the small country than in the large country. However, the large (small) country has more firms per capita at the tax-game outcome than at the no-tax outcome.*

This proposition implies that

$$\frac{n_1^m}{l_1} < \frac{n_1^g}{l_1} < \frac{n_2^g}{l_2} < \frac{n_2^m}{l_2} \tag{22}$$

where the superscript g represents the capital mobile case with active local governments. Since $n_1 + n_2 = N$ can be rewritten as

$$l_1 \left(\frac{n_1}{l_1} - \frac{1}{f} \right) = l_2 \left(\frac{1}{f} - \frac{n_2}{l_2} \right)$$

(22) implies that

$$\frac{n_1^g}{l_1} < \frac{1}{f} < \frac{n_2^g}{l_2} \tag{23}$$

the inequality being strict if and only if countries have different sizes. Combining (16) with (19), we obtain

$$t_2^* > 0$$

because $\theta > 1/2$. Moreover, from (22), it follows that $7/8 - l_1/(fn_1^g) > 7/8 - l_2/(fn_2^g)$. This and (21) thus yields

$$\frac{l_2}{n_2^{g2}} > \frac{l_1}{n_1^{g2}} \tag{24}$$

whereas the equality holds when $\theta = 1/2$. Thus, (16) implies that

$$t_1^* - t_2^* = \beta \left[\frac{l_1}{(n_1^g)^2} - \frac{l_2}{(n_2^g)^2} \right] < 0.$$

We may summarize our results as follows.

Proposition 6 *At the tax-game outcome, the government of the small country always taxes firms. However, the government of the large country either subsidies or taxes firms. When it taxes firms, its tax level is always lower than the one chosen by the government of the small country.*

Propositions 5 and 6 imply that the subsidies and taxes set by local governments distorts the economy. Without competing local governments, the number of firms per capita is larger in the small country than the large country ($n_2^m/l_2 > n_1^m/l_1$). Consider now a situation in which each firm is given the same subsidy in each country. The tax burden on each consumer is then heavier in the small country than in the large one. This difference in the tax burden means that local governments have different incentive in maximizing the workers' utility according to the size of their country: the government in the small country always levies taxes on firms and transfers the tax revenue to its residents, whereas the government of the large country may or may not do so. As a result, more firms locate in the large country in the case of competing local governments than in the no-tax case ($n_1^g > n_1^m$ and $n_2^m > n_2^g$). Since (n_1^m, n_2^m) minimizes the total training costs in the economy, (n_1^g, n_2^g) typically leads to a lower output than does (n_1^m, n_2^m). This in turn implies that tax competition reduces the total output in the economy.

If capital is immobile, governments do not use taxes nor subsidies (see (3) and (17)). Hence, from the global viewpoint, Propositions 5 and 6 imply that *tax competition induces wasteful redistribution from the small to the large country.*

4.2 Tax competition vs. tax coordination

It remains to compare the welfare level reached in each country at the tax-competition and no-tax (efficient) outcomes. First, we have

$$\begin{aligned} V_1^g - V_2^g &= \frac{\beta}{4}\left(\frac{1}{n_2^g} - \frac{1}{n_1^m}\right) + \frac{n_2^g}{l_2}\left[\frac{\beta l_2}{(n_2^g)^2} - t_2\right] - \frac{n_1^g}{l_1}\left[\frac{\beta l_1}{(n_1^g)^2} - t_1\right] \\ &= \frac{\beta}{4}\left(\frac{1}{n_2^g} - \frac{1}{n_1^m}\right) + \frac{n_2^g}{l_2}\left[\frac{\beta l_1}{(n_1^g)^2} - t_1\right] - \frac{n_1^g}{l_1}\left[\frac{\beta l_1}{(n_1^g)^2} - t_1\right] \\ &= \frac{\beta}{4}\left(\frac{1}{n_2^g} - \frac{1}{n_1^m}\right) + \left[\frac{\beta l_1}{(n_1^g)^2} - t_1\right]\left(\frac{n_2^g}{l_2} - \frac{n_1^g}{l_1}\right), \end{aligned}$$

where the second equality follows from (16). Because $n_1^m > n_2^m$, (22) implies that $n_2^g < n_2^m < n_1^m < n_1^g$ and, hence, $1/n_2^g - 1/n_1^g > 0$. Moreover, (19) and (23) imply that $\beta l_1/\left(n_1^g\right)^2 - t_1 > 0$. Therefore, we have

$$V_1^g - V_2^g > 0.$$

Substituting (19) into (17), we also have

$$\begin{aligned} V_2^g - V_2^m &= \frac{5\beta}{4n_2^m} - \frac{\beta}{2n_2^g} - \frac{3\beta l_2}{4f\left(n_2^g\right)^2} + \frac{\beta l_2}{f}\left[\frac{1}{(n_2^g)^2} - \frac{1}{(n_2^m)^2}\right] \\ &\quad + \frac{2\beta l_1 l_2}{(n_1^g)^3}\left(\frac{n_2^g}{l_2} - \frac{1}{f}\right). \end{aligned}$$

Since (23) gives $l_2/(f n_2^g) < 1$, we have

$$\frac{5\beta}{4n_2^m} - \frac{\beta}{2n_2^g} - \frac{3\beta l_2}{4f\left(n_2^g\right)^2} > \frac{5\beta}{4}\left(\frac{1}{n_2^m} - \frac{1}{n_2^g}\right).$$

As a result, we obtain

$$V_2^g - V_2^m > \left(\frac{1}{n_2^m} - \frac{1}{n_2^g}\right)\left[\frac{5\beta}{4} - \frac{\beta l_2}{f}\left(\frac{1}{n_2^g} + \frac{1}{n_2^m}\right)\right] + \frac{2\beta l_1 l_2}{(n_1^g)^3}\left(\frac{n_2^g}{l_2} - \frac{1}{f}\right).$$

That $n_2^m > n_2^g$ and (23) imply that $V_2^g - V_2^m > 0$ if $5\beta/4 - (\beta l_2/f)\left(1/n_2^g + 1/n_2^m\right) < 0$. Since $n_2^m > n_2^g$ and (23) give that $1/f < n_2^g/l_2 < n_2^m/l_2$, it is readily verified that

$$\frac{5\beta}{4} - \frac{\beta l_2}{f}\left(\frac{1}{n_2^g} + \frac{1}{n_2^m}\right) < \frac{5\beta}{4} - \frac{\beta l_2}{f}\frac{2}{n_2^m} < -\frac{3\beta}{4} < 0.$$

Hence, we have

$$V_2^g - V_2^m > 0.$$

Finally, as total output is lower under tax competition, it must be that

$$V_1^g - V_1^m < 0.$$

Consequently, we have:

Proposition 7 *The large country's residents are better off at the no-tax outcome, whereas the small country's residents prefer the tax-game outcome.*

Finally, consider the case of cooperation between governments. Using (17) and (18), it is straightforward that the average utility level in the global economy is given by

$$\begin{aligned} \overline{V} &= \frac{1}{L}\left(l_1 V_1 + l_2 V_2\right) \\ &= \alpha L - T(n_1, n_2). \end{aligned}$$

As seen in the previous section, T is minimized in the no-tax case. Hence, *cooperation leads to the same outcome as zero tax rate in both countries.*

5 Concluding remarks

We have developed a new tax competition model that bears some resemblance with Ottaviano and Ypersele (2005). Table 1 gives the number of firms per capita and the tax (or subsidy) levels in the two settings, where the superscript o stands for the socially optimal number of firms.

	Here	Ottaviano and van Ypersele (no clustering case)
Optimal (Tax cooperation)	$\frac{n_1^o}{l_1} < \frac{n_2^o}{l_2}$	$\frac{n_2^o}{l_2} < \frac{n_1^o}{l_1}$
Capital mobile case (no government)	$\frac{n_1^m}{l_1} = \frac{n_1^o}{l_1} < \frac{n_2^o}{l_2} = \frac{n_2^m}{l_2}$	$\frac{n_2^m}{l_2} < \frac{n_2^o}{l_2} < \frac{n_1^o}{l_1} < \frac{n_1^m}{l_1}$
Tax competition	$\frac{n_1^o}{l_1} < \frac{n_1^g}{l_1} < \frac{n_2^g}{l_2} < \frac{n_2^o}{l_2}$ $t_2^* > t_1^*, t_2^* > 0$	$\frac{n_2^o}{l_2} < \frac{n_2^g}{l_2} < \frac{n_1^g}{l_1} < \frac{n_1^o}{l_1}$ $t_2^* < t_1^* < 0$

Table 1: Comparison of results with those in Ottaviano and Ypersele (2005)

In Ottaviano and Ypersele (2005), the large country has more firms per capita than does the small country under both cooperation and competition. By contrast, we have seen that the large country has less firms per capita in both situations. This is because we have a reverse home market effect, whereas the home market effect holds in Ottaviano and Ypersele. It is hard to believe that such a difference in results is due to the sole existence of strategic interactions in our setting. Indeed, as shown by Head *et al.* (2002), strategic competition on the product market does not suffice to invalidate the home market effect. Consequently, predictions may be very different. For example, whereas firms are always taxed in Ottaviano and Ypersele (2005), firms may receive subsidies in our setting. This suggests that product-market and input-market analyses need not be symmetric.

There are similarities, however, between the both settings. First, the difference in the number of firms per capita is smaller under competition than under cooperation. Second, both papers stress "the importance of being large" in that the large country is better off under competition than under cooperation.

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Appendix A. Proof of Proposition 4.

1. Existence of a local Nash equilibrium in the tax game. From (18), it follows

$$\begin{aligned} \frac{dn_i}{dt_i} &= -\frac{1}{2\beta}\left(\frac{l_i}{n_i^3}+\frac{l_j}{n_j^3}\right)<0 \\ \frac{d^2n_i}{dt_i^2} &= 6\beta\left(\frac{dn_i}{dt_i}\right)^2\left(\frac{l_j}{n_j^4}-\frac{l_i}{n_i^4}\right). \end{aligned} \tag{A1}$$

The first-order condition for the maximization of V_i with respect to t_i is

$$\frac{dV_i}{dt_i}=\frac{n_i}{l_i}-\frac{1}{f}+\left(\frac{5\beta}{4n_i^2}-\frac{2\beta l_i}{fn_i^3}+\frac{t_i}{l_i}\right)\frac{dn_i}{dt_i}=0 \tag{A2}$$

whereas the second-order derivative of V_i with respect to t_i yields

$$\begin{aligned}\frac{d^2V_i}{dt_i^2} &= \frac{2}{l_i}\frac{dn_i}{dt_i} + \frac{\beta}{n_i^3}\left(\frac{6l_i}{fn_i} - \frac{5}{2}\right)\left(\frac{dn_i}{dt_i}\right)^2 \\ &\quad + \left(\frac{5\beta}{4n_i^2} - \frac{2\beta l_i}{fn_i^3} + \frac{t_i}{l_i}\right)\frac{d^2n_i}{dt_i^2}.\end{aligned}$$

Using (A1) and (A2), d^2V_i/dt_i^2 may then be rewritten as follows:

$$\begin{aligned}\frac{d^2V_i}{dt_i^2} &= \left(-\frac{4\beta l_j}{l_i n_j^3} - \frac{13\beta}{2n_i^3} + \frac{6\beta l_i}{fn_i^4}\right)\left(\frac{dn_i}{dt_i}\right)^2 \\ &\quad +6\beta\left(\frac{n_i}{l_i} - \frac{1}{f}\right)\left(\frac{l_i}{n_i^4} - \frac{l_j}{n_j^4}\right)\frac{dn_i}{dt_i} \\ &= \Phi_i\left(\frac{dn_i}{dt_i}\right)^2 + 6\beta\Psi_i\frac{dn_i}{dt_i}\end{aligned}$$

where

$$\begin{aligned}\Phi_i &\equiv -\frac{4\beta l_j}{l_i n_j^3} - \frac{13\beta}{2n_i^3} + \frac{6\beta l_i}{fn_i^4} \\ \Psi_i &\equiv \left(\frac{n_i}{l_i} - \frac{1}{f}\right)\left(\frac{l_i}{n_i^4} - \frac{l_j}{n_j^4}\right).\end{aligned}$$

Hereafter, we show that the second-order condition is always satisfied at any point for which both the first-order conditions of the two countries ($dV_1/dt_1 = 0$ and $dV_2/dt_2 = 0$) and the equalization of capital prices (18) hold. This shows that t_i^* is the local best reply of government i against t_j^*. As will be shown later, such a point exists and is unique, satisfying properties (22), (23) and (24).

When $i = 1$, (22) implies that

$$\frac{n_1^g}{l_1} > \frac{n_1^m}{l_1} = \frac{1}{\left(\theta + \sqrt{\theta}\sqrt{1-\theta}\right)f} > \frac{4}{5f}$$

implying that

$$\frac{5}{4} > \frac{l_1}{fn_1^g}$$

so that

$$\Phi_1 < -\frac{4\beta l_2}{l_1\left(n_2^g\right)^3} - \frac{13\beta}{2\left(n_1^g\right)^3} + \frac{30\beta}{4\left(n_1^g\right)^3}.$$

Furthermore, $n_1^m > n_2^m$ and (22) imply that $n_2^g < n_2^m < n_1^m < n_1^g$. It then follows from (24) that

$$\frac{l_2}{(n_2^g)^3} > \frac{l_1}{(n_1^g)^3}$$

which in turn implies that

$$\Phi_1 < -\frac{4\beta l_1}{l_1 (n_1^g)^3} - \frac{13\beta}{2 (n_1^g)^3} + \frac{30\beta}{4 (n_1^g)^3} = -\frac{3\beta}{(n_1^g)^3} < 0. \tag{A3}$$

Similarly, (24) yields

$$\frac{l_2}{(n_2^g)^4} > \frac{l_1}{(n_1^g)^4} \tag{A4}$$

which, combined with (22), implies that

$$\Psi_1 > 0. \tag{A5}$$

For $i = 2$, (23) implies that

$$1 > \frac{l_1}{f n_2^g}.$$

Hence, we obtain

$$\Phi_2 < -\frac{4\beta l_1}{l_2 (n_1^g)^3} - \frac{13\beta}{2 (n_2^g)^3} + \frac{6\beta}{(n_2^g)^3} = -\frac{4\beta l_1}{l_2 (n_1^g)^3} - \frac{\beta}{2 (n_2^g)^3} < 0. \tag{A6}$$

Similarly, (A4) and (22) lead to

$$\Psi_2 > 0. \tag{A7}$$

From (A1), (A3) and (A5) to (A7), it then follows that

$$\frac{d^2 V_i}{dt_i^2} < 0$$

so that the second-order consition is satisfied for each $i = 1, 2$.

2. Uniqueness of the local Nash equilibrium in the tax game. Substituting $n_2 = N - n_1$ into (21), we see that both the left hand side (LHS) and the right hand side (RHS) of (21) depend on n_1. The LHS is negative and increases in n_1 in the interval $(0, 8l_1/7f)$, and is positive in $(8l_1/7f, N]$. Moreover, it is readily verified that $\lim_{n_1 \to 0} LHS = -\infty$ and $LHS|_{n_1 = 8l_1/(f} = 0$.

The RHS is positive in $[0, N - 8l_2/7f)$, and is negative and decreasing in n_1 in $(N - 8l_2/7f, N)$. We can also see that $RHS|_{n_1=N-8l_2/7f} = 0$ and $\lim_{n_1 \to N} RHS = -\infty$ (see Figure 3 for an illustration). Because $8l_1/7f - [N - 8l_2/7f] = N/7 > 0$, from Figure 3, we can easily see that the following proposition holds. Q.E.D.

Appendix B. Proof of Proposition 5

When $\theta = 1/2$ (i.e., $l_1 = l_2$), it is obvious that $n_1^g = n_2^g = N/2$. Since $n_1^m = n_2^m = N/2$, we obtain

$$\frac{n_1^m}{l_1} = \frac{n_1^g}{l_1} = \frac{n_2^g}{l_2} = \frac{n_2^m}{l_2}.$$

When $\theta > 1 - \theta$ (i.e., $l_1 > l_2$), from (21), we have

$$\left(\frac{n_2^g/l_2}{n_1^g/l_1}\right)^2 = \frac{7/8 - l_2/fn_2^g}{7/8 - l_1/fn_1^g}\frac{l_1}{l_2}. \tag{B1}$$

From (11), we also have

$$\left(\frac{n_2^m/l_2}{n_1^m/l_1}\right)^2 = \frac{l_1}{l_2}. \tag{B2}$$

Let us define γ as follows:

$$\gamma = \frac{7/8 - l_2/(fn_2^g)}{7/8 - l_1/(fn_1^g)}.$$

From Figure 1, we can see that $N - 8l_2/(7f) < n_1^g < N < 8l_1/(7f)$. This and $n_2 = N - n_1$ imply that $7/8 - l_i/(fn_i^g) < 0$ for $i = 1, 2$, and that $\gamma > 0$. (B1) and (B2) yield

$$\frac{n_2^g/l_2}{n_1^g/l_1} = \gamma^{1/2}\frac{n_2^m/l_2}{n_1^m/l_1} = \left(\frac{\gamma l_1}{l_2}\right)^{1/2}. \tag{B3}$$

Now assume that $\gamma \geq 1$. This implies that $7/8 - l_1/fn_1^g \geq 7/8 - l_2/fn_2^g$, which is reduced to $n_1^g/l_1 \geq n_2^g/l_2$. However, since $\gamma \geq 1$ and $l_1 > l_2$, (B3) yields $n_2^g/l_2 > n_1^g/l_1$, which is a contradiction. Therefore, it must be true that $\gamma < 1$. This means $7/8 - l_2/fn_2^g > 7/8 - l_1/fn_1^g$, which gives

$$\frac{n_2^g}{l_2} > \frac{n_1^g}{l_1}. \tag{B4}$$

Furthermore, from (B3), we obtain

$$\frac{n_2^m/l_2}{n_1^m/l_1} > \frac{n_2^g/l_2}{n_1^g/l_1}. \tag{B5}$$

Note that $n_1 + n_2 = N$ implies

$$\frac{n_2/l_2}{n_1/l_1} = \frac{Nl_1}{l_2 n_1} - \frac{l_1}{l_2}.$$

Substituting this into (B5) gives

$$n_1^g > n_1^m. \tag{B6}$$

Similarly, we have

$$\frac{n_2/l_2}{n_1/l_1} = \frac{1}{Nl_2/l_1 n_2 - l_2/l_1}$$

which and (B5) yield

$$n_2^m > n_2^g. \tag{B7}$$

The proposition then follows from (B4), (B6) and (B7). Q.E.D.