A Baseline New-Keynesian Open Economy Model

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EABCN Course

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- Baseline references

\- Gali and Monacelli (Restud 2005)

\- Faia-Monacelli (JMCB 2008)

1. Lay out basic NK open economy framework

2. Analysis of **optimal** monetary policy $\longrightarrow$ Deduce optimal **exchange rate** behavior

3. Analysis of simple interest rate rules

- **Baseline SOE framework**

1. Limit case of two-country setup: countries **asymmetric** in size

2. Complete international financial markets

3. Only **tradable** goods

4. Monopolistic competition and sticky prices in domestic (traded goods) sector

5. **Law of one price** in traded goods

6. Home bias in consumption $\longrightarrow$ Endogenous **real** exchange rate fluctuations

- World economy: two economic entities, size n and $(1-n)$ respectively

- Domestic Households: consume domestic and imported goods

$$C_t \equiv [(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} \; + \; \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{1}$$

Notice:

$\eta > 0 \equiv$ elasticity of substitution between domestic and foreign goods

$\gamma \equiv (1-n)\alpha \equiv$ weight of imported goods in consumption basket

- **Foreign** country consumption bundle

$$C_t^* \equiv [(1-\gamma^*)^{\frac{1}{\eta}} C_{F,t}^{*\ \frac{\eta-1}{\eta}} \ + \ (\gamma^*)^{\frac{1}{\eta}} \, C_{H,t}^{*\ \frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $\gamma^* \equiv n \ \alpha^*$.

- **Home bias in consumption**

$$(1-\gamma) = (1-(1-n)\alpha) \;\; > \;\; \gamma^* = n\alpha^* \tag{3}$$

$\longrightarrow$Notice

(i) symmetric case of $\alpha = \alpha^*$

(ii) limiting case $n \rightarrow 0$

$\longrightarrow$Home bias requires $\alpha < 1$.

- Each consumption bundle $C_{H,t}$ and $C_{F,t}$ composed of imperfectly substitutable varieties

- Optimal allocation of expenditure **within** each variety of goods:

$$C_{H,t}(i) = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \quad ; \quad C_{F,t}(i) = \frac{1}{1-n}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{4}$$

$$C_{H,t} \equiv \left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n [C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_{F,t} \equiv \left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 [C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di]^{\frac{\varepsilon}{\varepsilon-1}}$$

- **EXERCISE 1:** *derive demand functions for each individual variety*

- **Optimal** allocation of expenditure between **domestic** and **foreign** bundles yields:

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \qquad C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{5}$$

where

$$P_t \equiv [(1-\gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{6}$$

is the **CPI index**.

- **EXERCISE 2:** *derive demand functions for each bundle (H,F)*

- Households' problem

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \right\} \quad (7)$$

$$\underbrace{P_t C_t} + \sum_{h^{t+1}} \nu_{t+1,t} B_{t+1} \leq W_t N_t + \tau_t + B_t + \int_0^1 \Gamma_t(i) \quad (8)$$

→Note: aggregate C in budget constraint

Financial assets

- $\nu_{t+1,t} \equiv \nu(h_{t+1}|h_t)$ is the period-t price of a claim to one unit of **domestic unit of account** currency in state h_t divided by the probability of occurrence of that state.

- Each asset in the portfolio B_{t+1} pays one unit of domestic currency at time $t+1$ and in state h_{t+1}

Efficiency conditions

$$U_{c,t}\frac{W_t}{P_t} = -U_{n,t} \tag{9}$$

$$\beta\frac{P_t}{P_{t+1}}\frac{U_{c,t+1}}{U_{c,t}} = \nu_{t+1,t} \tag{10}$$

$$\lim_{j\to\infty} E_t\left\{\nu_{t+j,t}\ B_{t+j}\right\} = 0 \tag{11}$$

Note: each condition holds **for each state** h_{t+1}

- Structure of assets allows to price **one-period riskless** bonds $\longrightarrow$ **Arbitrage** condition

$$R_t = E_t \left\{ \nu_{t+1,t} \right\}^{-1}$$

$\longrightarrow$Obtain Euler condition:

$$R_t = \left[\beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{U_{c,t+1}}{U_{c,t}} \right\} \right]^{-1}$$

- **Law of One Price**

$$P_{F,t}(i) = \mathcal{E}_t \; P^*_{F,t}(i) \qquad \forall i \in [0,1]$$

where $\mathcal{E}_t$ is the *nominal exchange rate*, i.e., the price of foreign currency in terms of home currency, and $P^*_{F,t}(i)$ is the price of foreign good i denominated in foreign currency.

Foreign

- Budget constraint (in domestic units of account):

$$P_t^* C_t^* + \sum_{h^{t+1}} \nu_{t+1,t} \frac{B_{t+1}^F}{\mathcal{E}_t} \leq W_t^* N_t^* + \tau_t^* + \frac{B_t^F}{\mathcal{E}_t} + \int_0^1 \Gamma_t^*(i) \qquad (12)$$

- Efficiency condition for bonds' holdings

$$\beta \frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}} \frac{U_{c,t+1}^*}{U_{c,t}^*} = \nu_{t+1,t} \qquad (13)$$

- Take conditional expectations of (13)

- Impose arbitrage condition

$$R_t^* = \left(E_t \left\{ \nu_{t+1,t} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \right)^{-1}$$

- Obtain Euler condition:

$$R_t^* = \left[\beta \ E_t \left\{ \frac{P_t^*}{P_{t+1}^*} \frac{U_{c,t+1}^*}{U_{c,t}^*} \right\} \right]^{-1} \tag{14}$$

- **Notice**: UIP condition holds only up to **certainty equivalence** after log-linearizing the two conditions:

$$R_t^* = \left(E_t \left\{ \nu_{t+1,t} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \right)^{-1}$$

$$R_t = E_t \left\{ \nu_{t+1,t} \right\}^{-1}$$

- **Notice**:

(i) UIP not an independent equilibrium condition

(ii) Relevant equilibrium conditions are Euler equations in H and F

Budget Constraints and Risk Sharing

-Iterating budget constraint forward + TVC obtain **present value constraint** (in domestic units of account):

$$B_0 + \sum_{t=0}^{\infty} \left(\sum_{h^t} \nu_{t,0} \right) \left[W_t N_t + \int_0^1 \Gamma_t(i) \right] = \sum_{t=0}^{\infty} \left(\sum_{h^t} \nu_{t,0} \right) P_t C_t \qquad (15)$$

- Price system $\nu_{0,t}$ is obtained after iteration of Euler equation:

$$\nu_{t,0} = \beta^t \frac{U_{c,t}}{P_t} \frac{P_0}{U_{c,0}} \qquad (16)$$

- Similarly for **Foreign**

From Euler:

$$\nu^F_{t,0} = \left(\beta^t \frac{U^*_{c,t}}{P^*_t} \frac{P^*_0}{U^*_{c,0}}\right) \frac{\mathcal{E}_0}{\mathcal{E}_t} \equiv \nu^*_{t,0} \frac{\mathcal{E}_0}{\mathcal{E}_t} = \nu_{t,0} \tag{17}$$

- Iterating budget constraint + TVC (in domestic units of account)

$$B^F_0 + \sum_{t=0}^{\infty} \left(\sum_{h^t} \nu_{t,0} \right) \; \mathcal{E}_t \; \left[W^*_t N^*_t + \Gamma^*_t \right] = \sum_{t=0}^{\infty} \left(\sum_{h^t} \nu_{t,0} \right) \; P^*_t C^*_t \mathcal{E}_t \tag{18}$$

- Equating expressions for price systems $\nu^F_{t,0}$ and $\nu_{t,0}$ yields

$$\kappa \frac{U^*_{c,t}}{U_{c,t}} = \frac{\mathcal{E}_t P^*_t}{P_t} \equiv \underbrace{Q_t}_{\text{CPI rer}} \tag{19}$$

where $\kappa \equiv \frac{\mathcal{E}_0 \; P^*_0 \; U_{c,0}}{P_0 U^*_{c,0}}$.

Risk-sharing parameter κ.

- Representative agents in the two countries enter initial period 0 with predetermined (at *time -1*) wealth distribution given by a pair $\left\{B_0,\ B_0^F\right\}$.

Definition. *International risk-sharing requires a t=-1 trading of assets such that households in both countries face the same PV budget constraint at t=0.*

Lemma 1. *If the risk sharing arrangement is consistent with Definition 1, then* $\kappa = 1$.

Proof. Efficiency in Home requires:

$$\beta^t U_{c,t} = \Omega \ v_{0,t} P_t \tag{20}$$

where Ω is the Lagrange multiplier on PV constraint.

Similarly in Foreign

$$\beta^t U^*_{c,t} = \Omega^* \ v_{0,t} \ \mathcal{E}_t P^*_t \tag{21}$$

Combining

$$\kappa = \frac{\Omega}{\Omega^*} \tag{22}$$

If PV constraints need to be equalized, then $\Omega = \Omega^* \rightarrow \kappa = 1$

- Foreign demand for domestic variety i :

$$
\begin{aligned}
C^*_{H,t}(i) &= \frac{1}{n}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} \underbrace{C^*_{H,t}} \\
&= \frac{1}{n}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} \underbrace{\gamma^*\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t}
\end{aligned}
\tag{23}
$$

- **Terms of trade**

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{24}$$

- **Real exchange rate**

$$Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}.$$

- **CPI-PPI ratio**

$$\frac{P_t}{P_{H,t}} = [(1-\gamma) + \gamma {S_t}^{1-\eta}]^{\frac{1}{1-\eta}} \equiv g(S_t) \tag{25}$$

with $g_{s,t} \equiv \frac{\partial g(S_t)}{\partial S_t} > 0$.

- Terms of trade and real exchange

$$Q_t = S_t \frac{P_t^*}{P_{F,t}^*} \left(\frac{P_t}{P_{H,t}} \right)^{-1} \tag{26}$$
$$= S_t \, \frac{g^*(S_t)}{g(S_t)} \equiv q(S_t)$$

where

$$\frac{P_t^*}{P_{F,t}^*} = [(1-\gamma^*) + \gamma^* {S_t}^{\eta-1}]^{\frac{1}{1-\eta}} \equiv g^*(S_t) \tag{27}$$

with $q_{s,t} \equiv \frac{\partial q(S_t)}{\partial S_t} > 0$ and $g_{s,t}^* \equiv \frac{\partial g^*(S_t)}{\partial S_t} < 0$.

- **Production**

\- Each monopolistic firm i in Home produces a homogenous good according to the production function:

$$Y_t(i) = A_t F(N_t(i)) \tag{28}$$

where A_t is a labor productivity shifter (common across firms) and $F(\bullet)$ is a homogeneous function with $F_{n,t} \equiv \frac{\partial F(\cdot)}{\partial N_t} > 0$

- The cost minimizing choice of labor input :

$$\frac{W_t}{P_{H,t}(i)} = \frac{MC_t}{P_{H,t}(i)} \, A_t F_{n,t} \tag{29}$$

where MC denotes the nominal marginal cost (common across firms)

- **Price setting**: two scenarios

1. Set **one period** in advance

2. Under quadratic costs (isomorphic to Calvo)

- One-period price setting

- No international price discrimination.

- Each producer i chooses the price $P_{H,t}(i)$ to satisfy **local and foreign demand** and to maximize expected discounted nominal profits

Maximization problem:

$$\text{Max } E_{t-1}\left\{\nu_{t-1,t}\ \left[P_{H,t}(i)Y_t(i) - W_t N_t(i)\right]\right\}$$

subject to

$$Y_t(i) \leq \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}\ Y_t \tag{30}$$

$$Y_t(i) = A_t F(N_t(i)) \tag{31}$$

By using (30) and (28) we can rewrite the profit function:

$$\Gamma_t(i) = \left\{ \nu_{t-1,t} \left[\left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{1-\varepsilon} P_{H,t} Y_t - W_t \, h \left(\left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \frac{Y_t}{A_t} \right) \right] \right\}$$

where $h(\bullet) \equiv F^{-1}\left(\frac{Y_t(i)}{A_t}\right) = F^{-1}\left(P_{H,t}(i), P_{H,t}, A_t, Y_t\right) = N_t(i)$.

First order condition with respect to $P_{H,t}(i)$:

$$E_{t-1}\left\{\nu_{t-1,t}\ \left[(1-\varepsilon)\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t\ +\ \varepsilon\frac{W_t}{P_{H,t}}h_{p,t}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon-1}\frac{Y_t}{A_t}\right]\right\}=0 \quad (32)$$

where $h_{p,t} \equiv \frac{\partial\ h(\cdot)}{\partial\ P_{H,t}(i)}$

- **EXERCISE 4:** *derive first order condition for price setting*

Notice

(i) If linear technology $Y_t(i) = A_t N_t(i) \rightarrow h_{p,t} = 1$ for all t.

(ii) Recall that $\frac{\partial h(\cdot)}{\partial P_{H,t}(i)} = \left(\frac{\partial F}{\partial h}\right)^{-1} = \left(\frac{\partial F}{\partial N_t(i)}\right)^{-1}$.

Dividing through by $\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}$, writing the product wage as $\frac{W_t}{P_{H,t}} = \frac{W_t}{P_t} g(S_t)$ and using (16) we obtain:

$$\frac{\beta P_{t-1}}{U_{c,t-1}} E_{t-1} \left\{ \frac{U_{c,t}\ Y_t}{P_t} \left[\frac{P_{H,t}(i)}{P_{H,t}} - \frac{\frac{W_t}{P_t} g(S_t)}{A_t F_{n,t}(i)} \left(\frac{\varepsilon}{\varepsilon - 1} \right) \right] \right\} = 0 \qquad (33)$$

→**Optimal pricing condition**

- **Market clearing** for domestic variety i must satisfy:

$$
\begin{aligned}
Y_t(i) &= n\ C_{H,t}(i) + (1-n)\ C^*_{H,t}(i) \qquad (34) \\
&= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[\begin{array}{c} (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \\ + \frac{(1-n)}{n}\gamma^* \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t \end{array}\right] \\
&= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[\begin{array}{c} (1-(1-n)\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \\ + (1-n)\alpha^* \left(\frac{P_{H,t}}{\mathcal{E}_t P^*_t}\right)^{-\eta} C^*_t \end{array}\right]
\end{aligned}
$$

Notice: PPP does not necessarily hold $\rightarrow \mathcal{E}_t P^*_t \neq P_t$

- **Symmetric equilibrium**: $P_{H,t}(i) = P_{H,t}$, $N_t(i) = N_t$ and $Y_t(i) = Y_t$ for all i and t.

- **Small open economy**

- Relative size of Home is negligible relative to the rest of the world, i.e., $n \longrightarrow 0$.

$\longrightarrow$Implications : $P^*_{F,t} = P^*_t \;\longrightarrow\; g^*(S_t) = 1$

- **Real exchange** rate becomes:

$$Q_t = \frac{S_t}{g(S_t)} = q(S_t) \tag{35}$$

- **Symmetric degree of home bias**

$$\alpha = \alpha^*$$

- Rewrite **market clearing** condition

$$
\begin{aligned}
Y_t &= g(S_t)^\eta \; [(1-\alpha)C_t + \alpha q(S_t)^\eta C_t^*] && (36) \\
&= (1-\alpha)g(S_t)^\eta C_t \; + \; \alpha S_t^\eta C_t^*
\end{aligned}
$$

- Price setting condition can be written

$$E_{t-1}\left\{\left[\frac{A_t F(N_t)\ U_{c,t}}{g(S_t)} + \mu U_{n,t}\ \omega(N_t)\right]\right\} = 0 \qquad (37)$$

where $\omega(N_t) \equiv \frac{F(N_t)}{F_{n,t}}$ and $\mu \equiv \frac{\varepsilon}{\varepsilon - 1}$.

- **Flexible Price Equilibrium**

$\longrightarrow$Price setting condition simplifies to:

$$\Phi_t \equiv -\frac{U_{n,t}\ g(S_t)}{A_t F_{n,t} U_{c,t}} = \mu^{-1} \equiv \Phi_t^f \tag{38}$$

$\longrightarrow$Each firm would optimally choose to replicate a *constant* markup

$\longrightarrow$**Open-economy** dimension: presence of the relative price $g(S_t)$

- **Gradual Price Adjustment**

- Rotemberg (1982) $\longrightarrow$ quadratic cost of adjusting prices

$$\frac{\vartheta}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}-1\right)^2 \tag{39}$$

If $\vartheta = 0$, prices are flexible.

Each producer chooses the price $P_{H,t}(i)$ of variety i to maximize expected nominal discounted profits:

$$E_t \left\{ \sum_{t=0}^{\infty} \nu_{0,t} \left[P_{H,t}(i) Y_t(i) - W_t N_t(i) - \frac{\vartheta}{2} \left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right)^2 P_{H,t} \right] \right\} \tag{40}$$

subject to

$$Y_t(i) \leq \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t \tag{41}$$

$$Y_t(i) = A_t F(N_t(i)) \tag{42}$$

→**First order condition**:

$$\nu_{0,t}\left\{(1-\varepsilon)\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}Y_t\ +\ \varepsilon\frac{W_t}{P_{H,t}}h_{p,t}\ \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon-1}\frac{Y_t}{A_t}\right\} \tag{43}$$

$$\begin{aligned} =\ & \nu_{0,t}\ P_{H,t}\ \vartheta\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}-1\right)\frac{1}{P_{H,t-1}(i)} \\ & -\ E_t\left\{\nu_{0,t+1}\ P_{H,t+1}\ \vartheta\left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)}-1\right)\frac{P_{H,t+1}(i)}{P_{H,t}(i)^2}\right\} \end{aligned} \tag{44}$$

where, again, $h_{p,t}\equiv\frac{\partial\ h(\cdot)}{\partial\ P_{H,t}(i)}$.

- In a **symmetric equilibrium** (which implies $P_{H,t}(i) = P_{H,t}$ for all i and t):

$$\pi_{H,t}(\pi_{H,t} - 1) = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{g(S_t)}{g(S_{t+1})} \pi_{H,t+1}(\pi_{H,t+1} - 1) \right\} + \frac{\varepsilon Y_t}{\vartheta} \left(\frac{\frac{W_t}{P_t} g(S_t)}{A_t F_{n,t}} - \frac{\varepsilon - 1}{\varepsilon} \right) \tag{45}$$

$\longrightarrow$Non-linear **NKPC**

- **EXERCISE 5**: *derive first order condition for price setting under quadratic costs*

- Real marginal cost

$$MC_t^r \equiv \frac{W_t \; g(S_t)}{P_t A_t F_{n,t}}$$

$\longrightarrow$Notice: increasing in the terms of trade

- NKPC under quadratic cost of adjustment

$$\pi_{H,t} = \beta \ E_t\{\pi_{H,t+1}\} + \lambda_{ROT} \ mc_t$$

where

$$\lambda_{ROT} \equiv \frac{\varepsilon}{\vartheta}$$

- Resource constraint will comprise a price adjustment cost factor

$$A_t N_t = (1-\alpha) g (S_t)^{\eta} C_t \; + \; \alpha S_t^{\eta} C_t^{*} + \frac{\vartheta}{2} \left(\pi_{H,t} - 1 \right)^2 \tag{46}$$

- **Equilibrium** under gradual price adjustment

- Allocations for $\left\{C_t, N_t, \pi_{H,t}, S_t, R_t\right\}$ given the **exogenous** processes $\{C_t^*,\ A_t\}$

$$\pi_{H,t}(\pi_{H,t}-1) = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{g(S_t)}{g(S_{t+1})} \pi_{H,t+1}(\pi_{H,t+1}-1) \right\} + \frac{\varepsilon Y_t}{\vartheta} \left(\frac{-\frac{U_{n,t}}{U_{c,t}} g(S_t)}{A_t F_{n,t}} - \frac{\varepsilon - 1}{\varepsilon} \right) \qquad \textbf{NKPC}$$

$$A_t N_t = (1-\alpha) g(S_t)^{\eta} C_t + \alpha S_t^{\eta} C_t^* + \frac{\vartheta}{2}\left(\pi_{H,t}-1\right)^2 \qquad \textbf{Market clearing}$$

$$\frac{U^*_{c,t}}{U_{c,t}} = q(S_t) \qquad \textbf{Risk-sharing}$$

$$R_t = \left[\beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{U_{c,t+1}}{U_{c,t}} \right\}\right]^{-1} \qquad \textbf{Euler}$$

$+$ specification of **monetary policy** rule

- Alternative: **Calvo pricing**

$$\max_{\overline{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \left[Y_{t+k} \left(\overline{P}_{H,t} - MC^n_{t+k} \right) \right] \right\}$$

s.t.

$$Y_t(i) \leq \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \; Y_t \tag{47}$$

$$Y_t(i) = A_t F(N_t(i)) \tag{48}$$

$\overline{P}_{H,t}$ must satisfy the first order condition

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \; Y_{t+k} \; (\overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} \; MC_{t+k}^n)] \right\} = 0 \tag{49}$$

- Log-linear pricing rule under Calvo

$$\overline{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k}^n\}$$

$$P_{H,t} \equiv \left[\theta \; P_{H,t-1}{}^{1-\varepsilon} + (1-\theta) \; (\overline{P}_{H,t})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{50}$$

- Combining →NKPC

$$\pi_{H,t} = \beta \; E_t\{\pi_{H,t+1}\} + \lambda \; \widehat{mc}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

- Notice: λ differs from λ_{ROT}, but **not** because of openness

- **Alternative monetary policy rules**

1. Simple Taylor-type rule: **CPI based**

$$R_t = \overline{R}\Pi_t^{\phi_\pi}$$

2. Taylor rule **GDP-inflation** based

$$R_t = \overline{R}\Pi_{H,t}^{\phi_\pi}$$

3. Managed **exchange rate**

$$R_t = \overline{R}\Pi_{H,t}^{\phi_\pi}(\mathcal{E}_t/\mathcal{E}_{t-1})^{\phi_e}$$

4. Targeting rules

$$\Pi_t = 1 \quad \forall t$$

$$\Pi_{H,t} = 1 \ \forall t$$

$$\mathcal{E}_t = \overline{\mathcal{E}} \ \forall t \qquad \text{(Peg)}$$

- **Log-linear dynamic framework**

- Market clearing condition: first order approx. around zero inflation steady state with zero net exports ($Y_t = C_t$)

- Assume rest of the world closed economy: $c_t^* = y_t^*$

$$y_t = (1 - \alpha)c_t + (1 - \alpha)\eta\alpha s_t + \eta\alpha s_t + \alpha y_t^*$$

- Rearranging:

$$y_t = (1 - \alpha)c_t + (2 - \alpha)\eta\alpha s_t + \alpha y_t^*$$

- Using **risk-sharing** to eliminate y_t^*:

$$c_t = y_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t$$

- Substituting and rearranging:

$$y_t = c_t + \underbrace{\frac{\alpha\omega}{\sigma} s_t}_{\text{terms of trade effect}}$$

$$\omega \equiv \sigma\eta + (1-\alpha)(\sigma\eta - 1)$$

- Notice: ω crucial parameter$\longrightarrow$

$$\alpha = 0 \quad \rightarrow \quad y_t = c_t \qquad \text{(closed economy)}$$

$$\sigma\eta = 1 \;\text{ or }\; \sigma = \eta = 1 \quad \rightarrow \quad \omega = 1 \;\text{ (isomorphism)}$$

- From Euler condition obtain **open economy AD** equation

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(r_t - E_t\{\pi_{t+1}\} - \rho\right) - \frac{\alpha\omega}{\sigma} E_t\{\Delta s_{t+1}\} \qquad (51) \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(r_t - E_t\{\pi_{H,t+1}\} - \rho\right) - \frac{\alpha\Theta}{\sigma} E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\left(r_t - E_t\{\pi_{H,t+1}\} - \rho\right) + \alpha\Theta\, E_t\{\Delta y^*_{t+1}\} \end{aligned}$$

where

$$\Theta \equiv (\sigma\eta - 1)(2 - \alpha) = \omega - 1$$

$$\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha) + \alpha\omega}$$

Notice: role of **openness**

1. Affect (inverse) intertemporal elasticity of substitution in consumption.

2. Foreign output growth affects domestic output growth (if $\omega \neq 1$)

- Effect of openness on σ_α ambiguous:

$$\frac{\partial \sigma_\alpha}{\partial \alpha} = \frac{-2\sigma(\sigma\eta - 1)(1-\alpha)}{[1 + (\sigma\eta - 1)\alpha(2-\alpha)]^2}$$

$\rightarrow$Elasticity σ_α **falls** with openness ($1/\sigma_\alpha$ rises) if

$$\underbrace{\eta}_{\text{trade elasticity}} > \underbrace{\frac{1}{\sigma}}_{\text{intl. elast in consumption}}$$

$\rightarrow$Demand can be sensitive to real interest rates if net exports sensitive via **terms of trade** movements

- **Real Marginal Cost**

$$
\begin{aligned}
mc_t &= (w_t - p_{H,t}) - a_t \\
&= (w_t - p_t) + (p_t - p_{H,t}) - a_t \\
&= \sigma\ c_t + \varphi\ n_t + \alpha\ s_t - a_t \\
&= \sigma\ y_t^* + \varphi\ y_t + s_t - (1+\varphi)\ a_t
\end{aligned}
$$

- Substituting for s_t

$$
mc_t = \underbrace{(\sigma_\alpha + \varphi)}\ y_t - (1+\varphi)\ a_t + \underbrace{(\sigma - \sigma_\alpha)\ y_t^*}_{\text{open economy factor}}
$$

- **Labor supply channel** of openness $\longrightarrow$ openness affects also sensitivity of real mc to **domestic** output via σ_α

Intuition: from C/leisure condition

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} = \frac{W_t}{P_{H,t}} \frac{1}{g(S_t)}$$

Hence movements in the terms of trade affect the **product** wage (marginal cost) for any given real consumption wage

- Effect on **labor market** equilibrium

$$mc_t = w_{H,t} - a_t$$

$\longrightarrow$Express in the $(w_{H,t}, n_t)$ space

$$w_{H,t} = (\sigma_\alpha + \varphi)\, n_t + \sigma_\alpha a_t + (\sigma - \sigma_\alpha)\ \ y_t^*$$

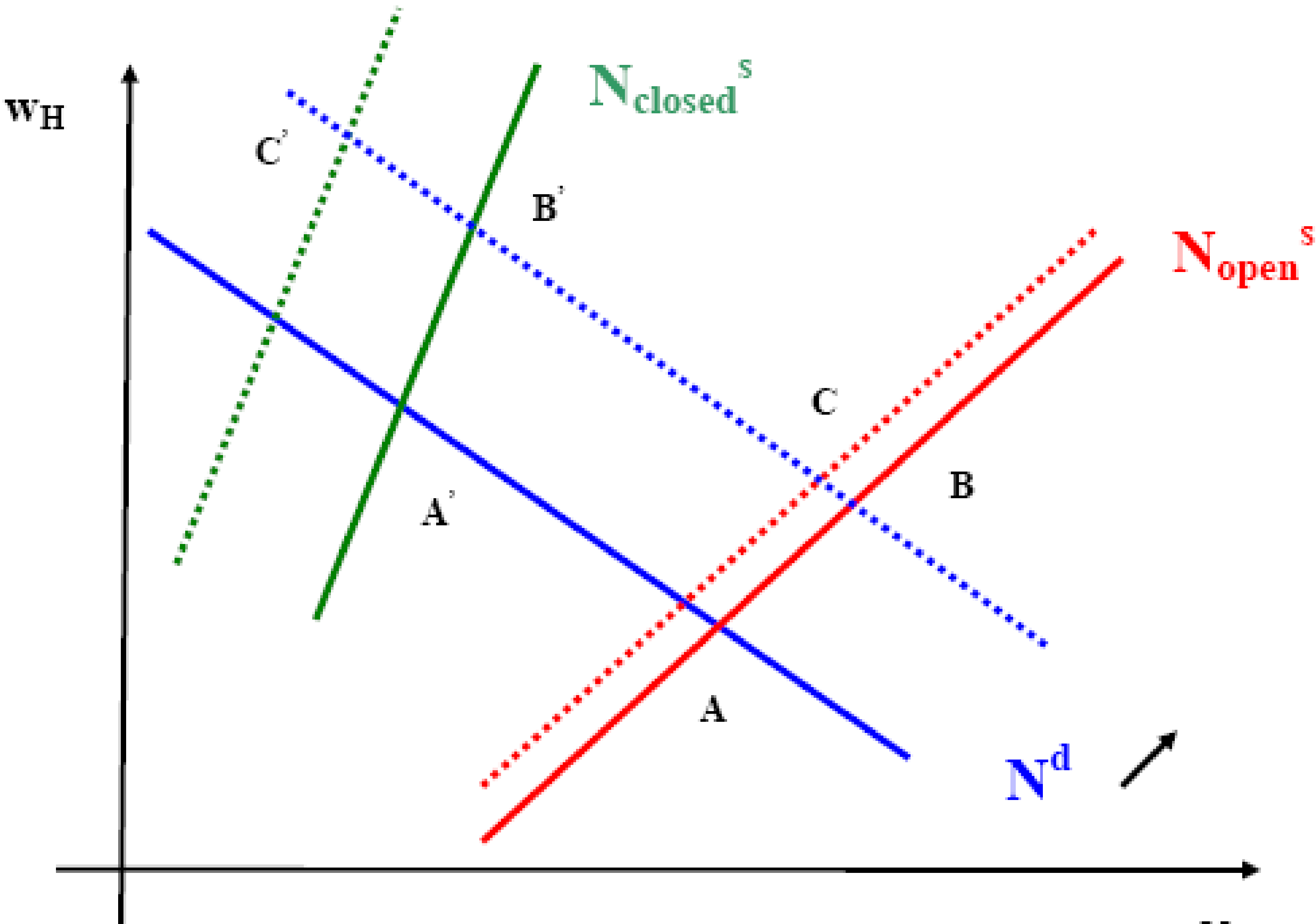
$\longrightarrow$Openness (via σ_α) affects (i) **slope** of labor supply schedule and (ii) sensitivity to **technology** shock

- Response to a **technology** shock$\rightarrow$

(i) Labor demand shifts out (same closed and open)

(ii) Closed economy: wealth effect on labor supply (depends on σ) $\rightarrow$ Labor supply shifts inward

(iii) **Open** economy (if $\sigma\eta > 1$): $\alpha \rightarrow \downarrow \sigma_\alpha \rightarrow$ N^s schedule shifts inward less $\rightarrow$ **More expansionary** on hours and less on real product wage.



w_H
$N_{closed}{}^s$
C'
B'
$N_{open}{}^s$
C
B
A'
A
N^d
N

Canonical Representation

- Output Gap

$$x_t \equiv y_t - \overline{y}_t$$

- Domestic **natural** level of output

$\longrightarrow mc_t = 0 \longrightarrow$

$$\overline{y}_t = \Omega + \Gamma \ a_t + \alpha\Psi \ y_t^* \tag{52}$$

where $\Omega \equiv \frac{v-\mu}{\sigma_\alpha+\varphi}$, $\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha+\varphi} > 0$, and $\Psi \equiv -\frac{\Theta\ \sigma_\alpha}{\sigma_\alpha+\varphi}$

- Domestic real marginal cost and output gap related according to:

$$mc_t = (\sigma_\alpha + \varphi) \; x_t$$

- **New Keynesian Phillips curv**e in terms of the output gap:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha \, x_t \tag{53}$$

where $\kappa_\alpha \equiv \lambda\,(\sigma_\alpha + \varphi)$.

- For $\alpha = 0$ (or $\sigma = \eta = 1$) the slope coefficient is given by $\lambda\,(\sigma + \varphi)$ as in the standard, closed economy NKPC.

- Effect of **openness** via slope coefficient κ_α

- If $\sigma\eta > 1 \longrightarrow$ elasticity σ_α falls with openness $\longrightarrow$ slope κ_α falls

An increase in openness **lowers** the size of the adjustment in the terms of trade necessary to absorb a change in domestic output (relative to world output)$\longrightarrow$ dampens impact of that adjustment on marginal cost and inflation.

- Dynamic **IS equation** for the open economy in terms of the output gap:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\alpha}(r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t) \tag{54}$$

where the **natural real interest rate** reads

$$\overline{rr}_t \equiv \rho - \sigma_\alpha \Gamma(1 - \rho_a)\ a_t + \alpha\sigma_\alpha(\Theta + \Psi)\ E_t\{\Delta y^*_{t+1}\}$$

- Openness affects natural real rate via $E_t\{\Delta y^*_{t+1}\}$ but also via σ_α

- Canonical representation

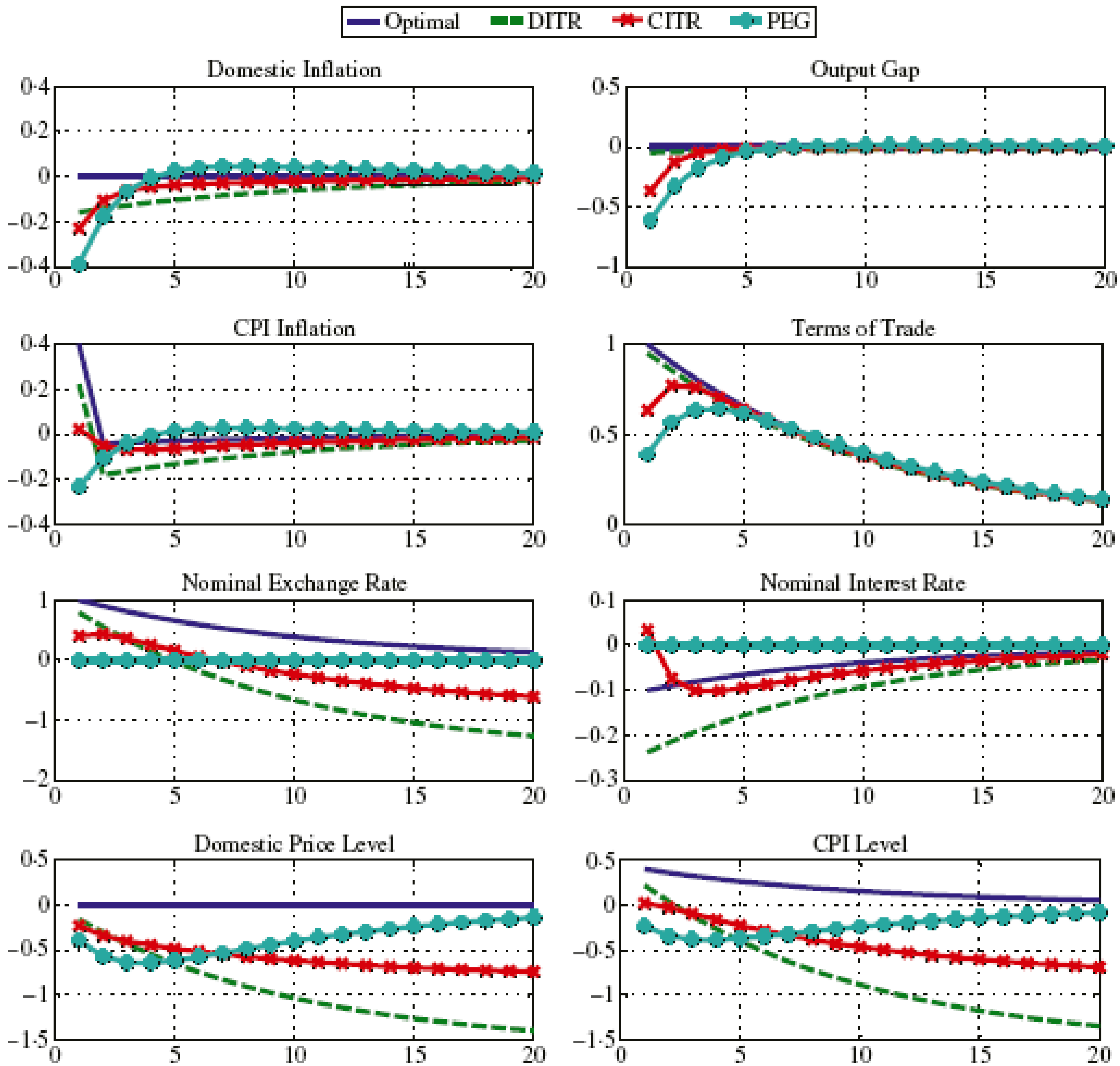
$$\pi_{H,t} = \beta \; E_t\{\pi_{H,t+1}\} + \kappa_\alpha \; x_t$$

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\alpha}\,(r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t)$$

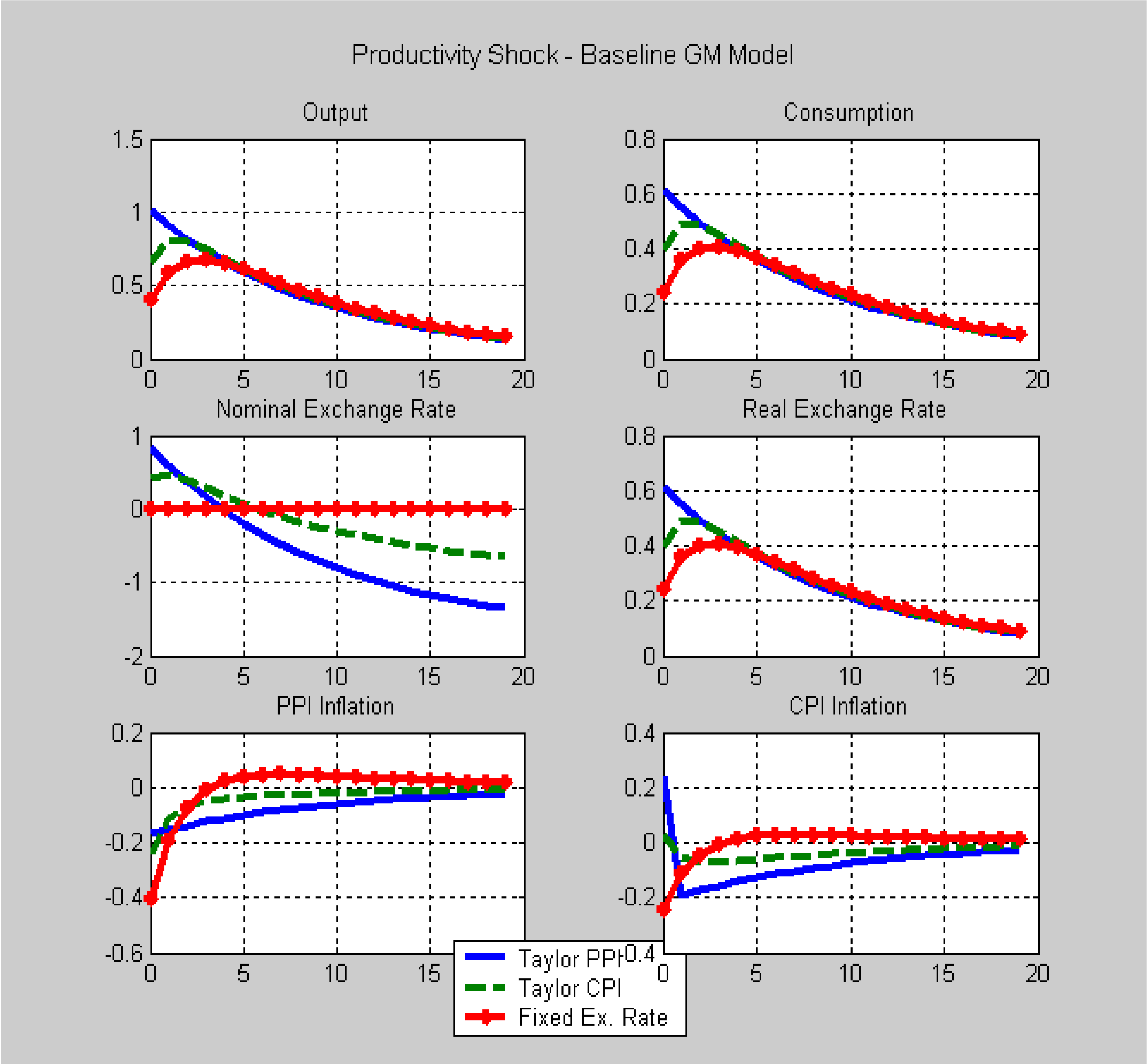
+ monetary policy rule

- **EXERCISE 6:** *solve canonical form of the model using method of undetermined coefficients*

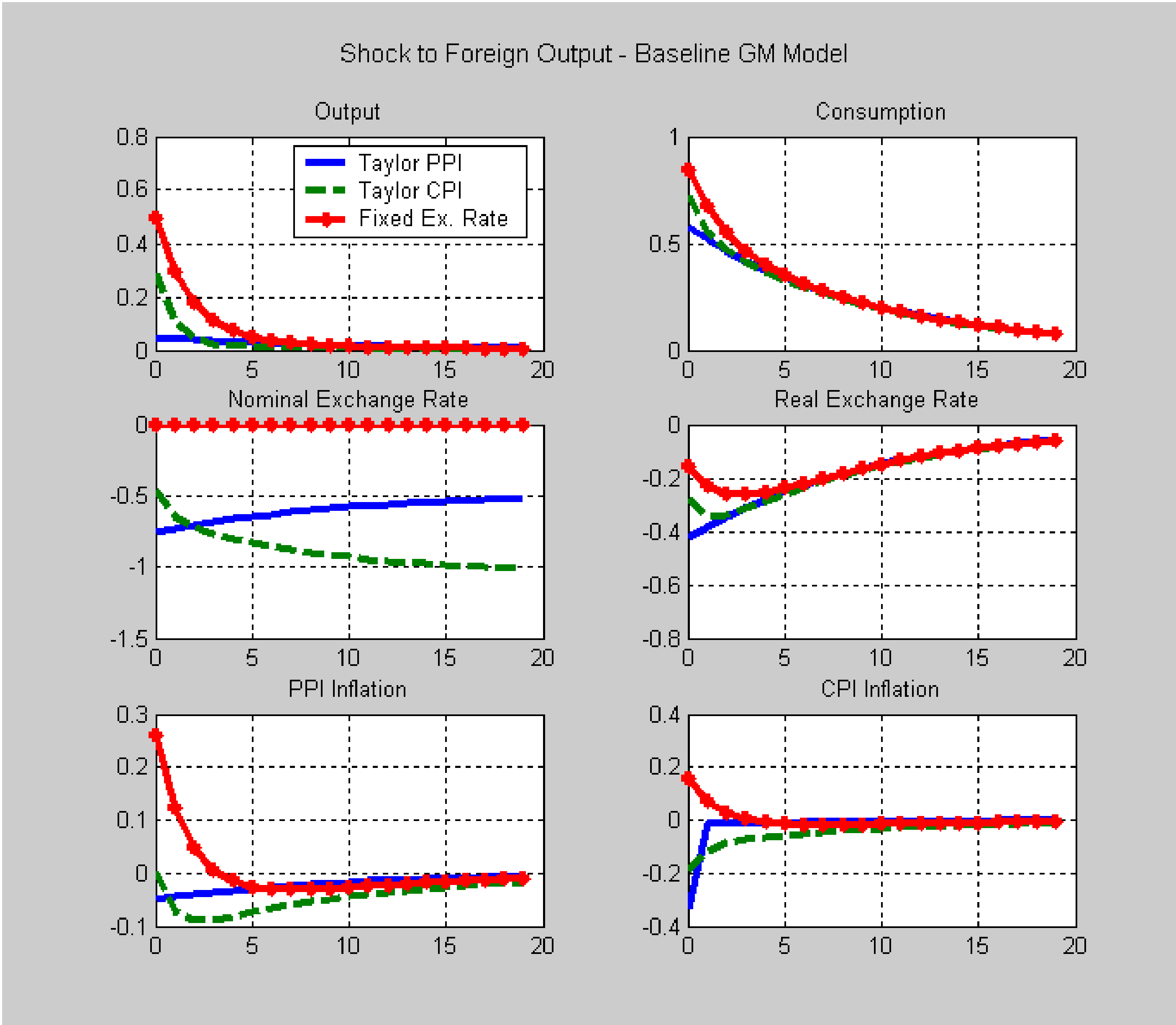
- Impulse responses to a **Home productivity** shock

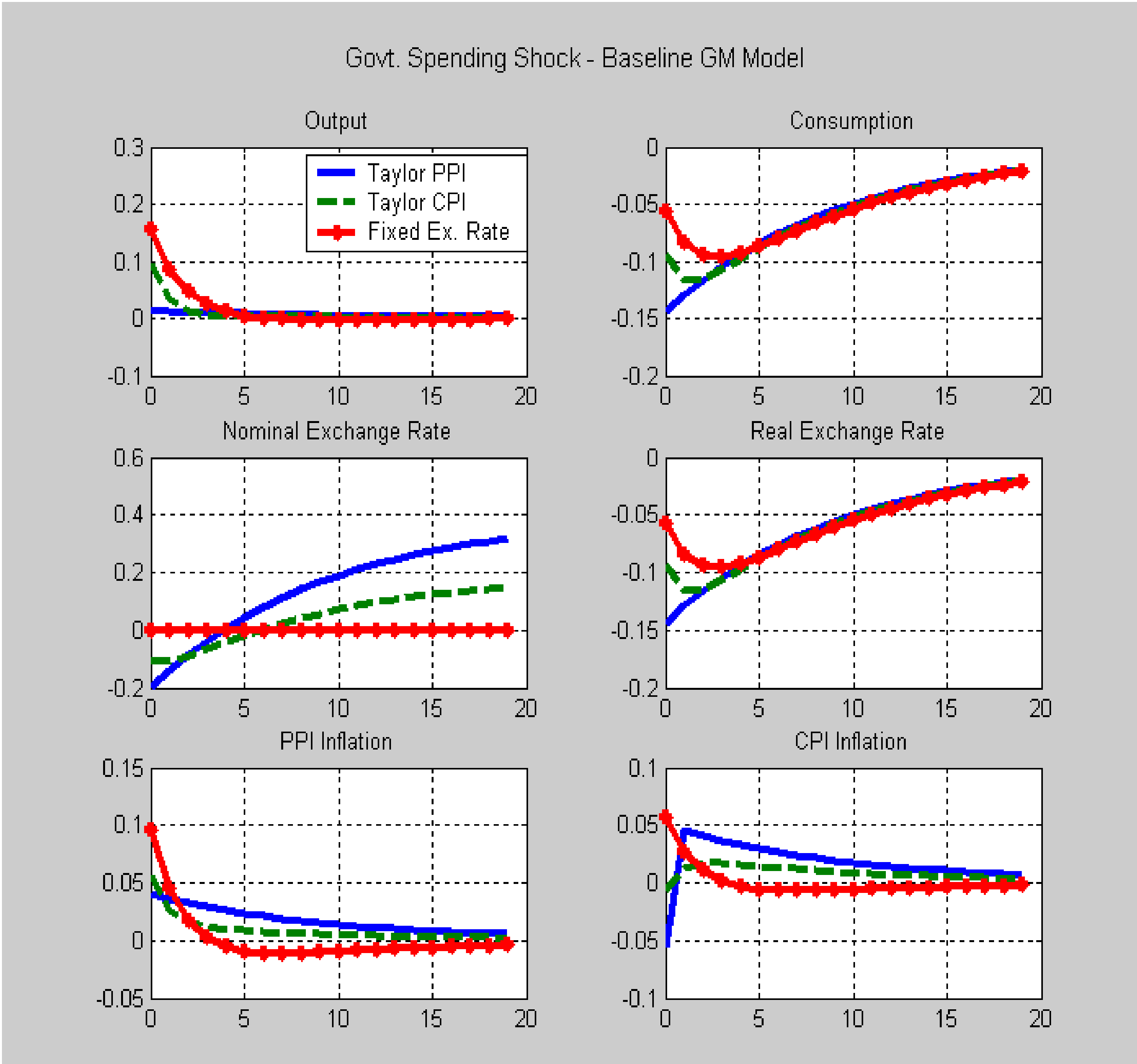

Optimal
DITR
CITR
PEG
Domestic Inflation
Output Gap
CPI Inflation
Terms of Trade
Nominal Exchange Rate
Nominal Interest Rate
Domestic Price Level
CPI Level

- Alternative Monetary Rules



Productivity Shock - Baseline GM Model
Output
Consumption
Nominal Exchange Rate
Real Exchange Rate
PPI Inflation
CPI Inflation
Taylor PPI
Taylor CPI
Fixed Ex. Rate


Shock to Foreign Output - Baseline GM Model
Output
Taylor PPI
Taylor CPI
Fixed Ex. Rate
Consumption
Nominal Exchange Rate
Real Exchange Rate
PPI Inflation
CPI Inflation



Govt. Spending Shock - Baseline GM Model
Output
Taylor PPI
Taylor CPI
Fixed Ex. Rate
Consumption
Nominal Exchange Rate
Real Exchange Rate
PPI Inflation
CPI Inflation

1. Model: **productivity** shock produces real ex. rate **depreciation**

2. Model: **government** spending shock produces real **appreciation**

VAR evidence seems to point in opposite direction

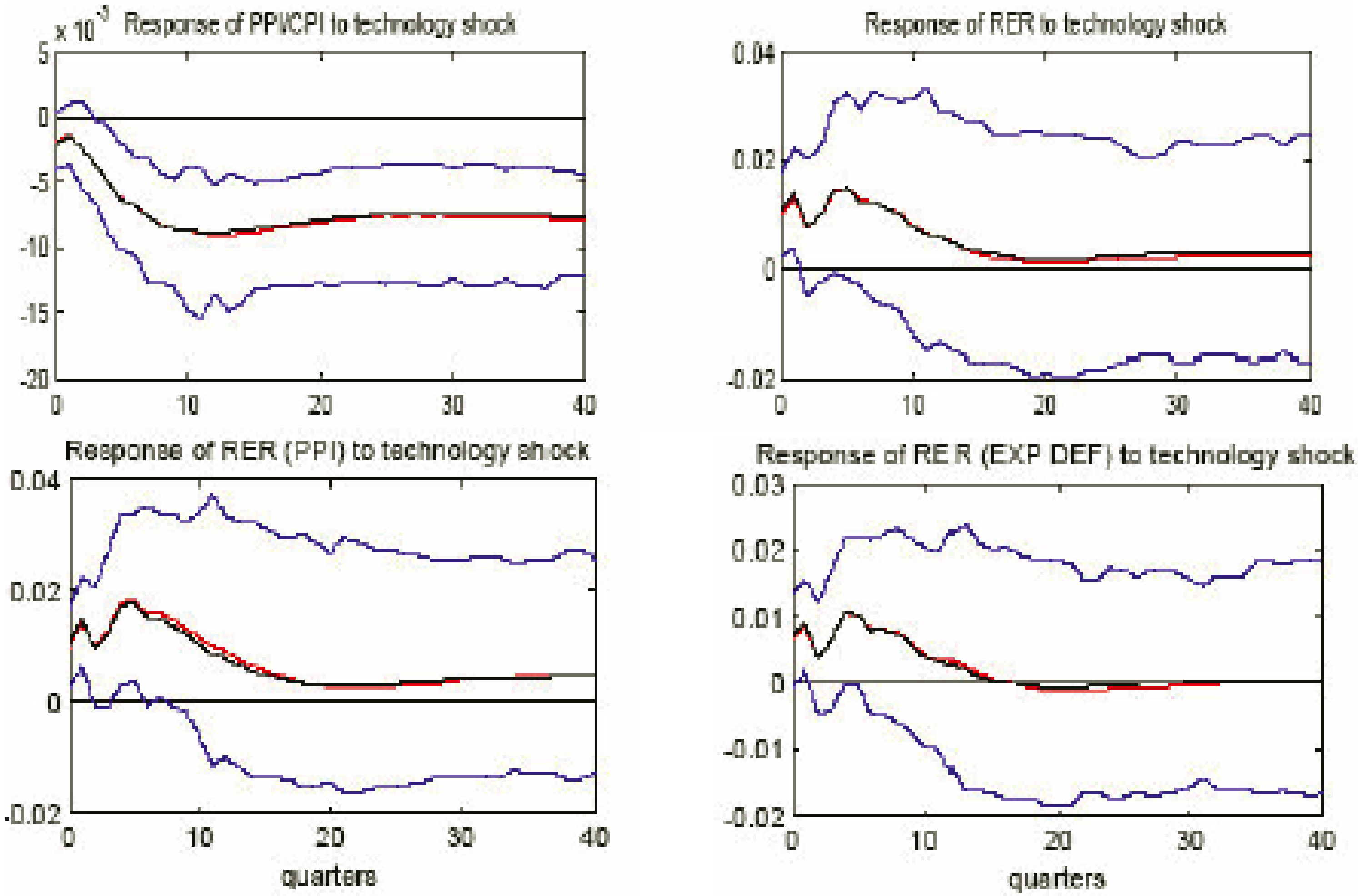
1. Corsetti et. al. 2007: productivity shock **may** produce real appreciation (depending on degree of openness?) both in RER and terms of trade

2. Productivity shocks lower the PPI/CPI price

3. Monacelli-Perotti 2007: G shock produces real depreciation

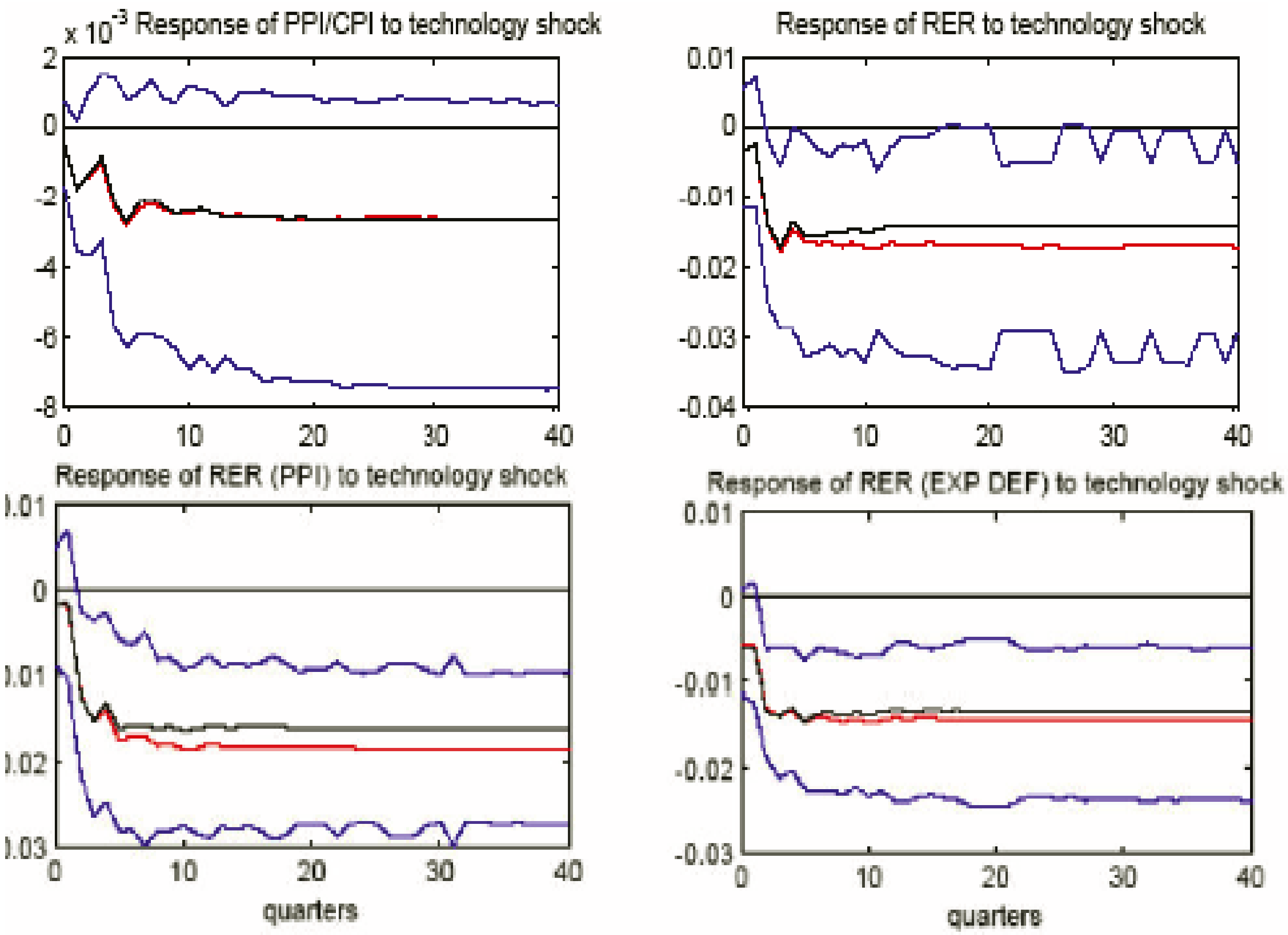
Response to productivity in T sector US: RER appreciation (RER rises)

(source Corsetti et al. 2007)

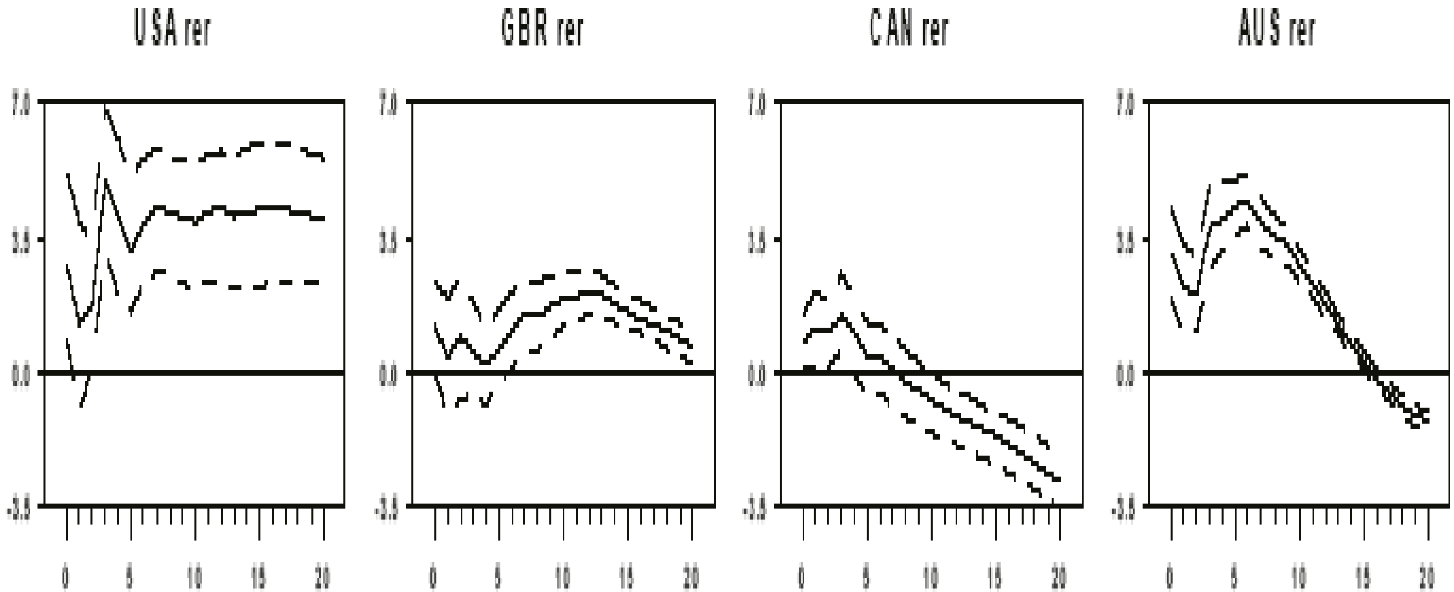


x 10^-3 Response of PPI/CPI to technology shock
Response of RER to technology shock
Response of RER (PPI) to technology shock
Response of RER (EXP DEF) to technology shock
quarters
quarters

United Kingdom: real depreciation (RER falls)


x 10^-3 Response of PPI/CPI to technology shock
Response of RER to technology shock
Response of RER (PPI) to technology shock
Response of RER (EXP DEF) to technology shock
quarters
quarters

Impulse response of **real effective exchange rate** to identified G shock (SVAR of Monacelli-Perotti)



Real exchange rate **depreciation** (G shock = 1% GDP)