Illiquidity in the Interbank Payment System following Wide-Scale Disruptions

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Abstract

We show how the financial system can become illiquid following wide-scale disruptions. Two forces are at play 1) operational problems and 2) changes in behavior by participants. We model the interbank payment system as an n-player game and utilize the concept of a potential function to describe the process by which one of multiple equilibria emerges after a wide-scale disruption. If the disruption is large enough, hits a key geographic area or hits a "Too Big to Fail" participant then coordination potentially breaks down and central bank intervention might be required to re-establish the socially efficient equilibrium. We also explore how the network topology of the underlying payment flow among banks affects the resiliency of coordination. The paper provides a theoretical framework to analyze the effects of events such as September 11, 2001.

1 Introduction

The events of September 11, 2001 and to a lesser extent the blackout of August 14, 2003 highlighted the fact that parts of the financial system are vulnerable to wide-scale disruptions. Moreover, the events underscored the fact that the financial system consists of a complex network of interrelated

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markets, infrastructures, and participants, and the inability of any of these entities to operate normally can have wide-ranging effects even beyond their immediate counter-parties. Not surprisingly, the financial industry and regulators are devoting considerable resources to business continuity measures and planning in order to strengthen the resiliency of the U.S. financial system. The primary objective is to minimize the immediate systemic effects on the financial system of large scale shocks.

At the apex of the U.S. financial system is a number of critical financial markets that provide the means for both domestic and international financial institutions to allocate capital and manage their exposures to liquidity, market, credit and other types of risks. These markets include Federal funds, foreign exchange, commercial paper, government and agency securities, corporate debt, equity securities and derivatives. Critical to the smooth functioning of these markets are a set of wholesale payments systems and financial infrastructures that facilitate clearing and settlement.[1] Operational difficulties of these entities or their participants can create difficulties for other systems, infrastructures and participants. Such spill overs might cause liquidity shortages or credit problems and hence potentially impair the functioning and stability of the entire financial system.

Wide-scale disruptions may not only present operational challenges for participants of the core payment and securities settlement system, but may also induce participants to change behavior in terms of how they conduct their business. The actions of participants have both the potential to mitigate, but also to augment the adverse effects.[2] Hence, understanding how participants react when faced with operational adversity by their counter-parties is crucial for both operators and regulators in terms of designing counter measures, devising policy, and providing emergency assistance if called for.

The backbone of the payment and securities settlement system in the U.S. is the Federal Reserve's Fedwire Funds Service (Fedwire). Fedwire is a Real Time Gross Settlement (RTGS) system where payments are settled individually and with instant finality in real time. Over 9,500 participants use Fedwire to send or receive time critical and/or large value payments on behalf of corporate and individual clients, settle positions with other financial institutions stemming from other payment systems, clearing arrangements or securities settlement, submit federal tax payments and buy and sell Federal Reserve funds. The average daily number of payments is approximately

[1] e.g. Depository Trust & Clearing Cooperation (DTCC), the Clearing House Inter-bank Payment System (CHIPS), Continuous Linked Settlement (CLS), and the Federal Reserve's Federal Funds and Securities Services (Fedwire).

[2] For a description of the extraordinary cooperative efforts to overcome the problems for the payment and securities settlement system caused by the events of September 11, 2001 see Federal Reserve Bank of New York [4].

[450,000] and the average value transferred is around [$1.6] trillion per day. Fedwire continued to operate during the events of September 11, 2001 and August 14, 2003, but in both cases the Federal Reserve had to intervene by extending the operating hours and by providing emergency liquidity.

On September 11, 2001, the massive damage to property and communications systems in lower Manhattan made it more difficult, and in some cases impossible, for many banks to execute payments to one another. The failure of some banks to make payments also disrupted the payments coordination by which banks use incoming payments to fund their own transfers to other banks. Once a number of banks began to be short of incoming payments, some became more reluctant to send out payments themselves. In effect, banks were collectively growing short of liquidity. The Federal Reserve recognized this trend toward illiquidity and provided liquidity through the discount window and open market operations in unprecedented amounts in the following week. Federal Reserve opening account balances peaked at more than $120 billion compared to approximately $15 billion prior. Moreover, the Federal Reserve waived the overdraft fees it normally charges. On September 14, daylight overdrafts peaked at $150 billion, more than 60 percent higher than usual. (see Ferguson [5]). Following the blackout in the Midwest and Northeast on August 14, 2003 commercial banks borrowed $785 million from the discount window compared to the normal range of $100 - $200 million.

Several recent papers (cf. McAndrews and Rajan [8], Bech and Garratt [2] and McAndrews and Potter [9]) advocate interpreting the payment decisions of banks participating in Fedwire as a game. McAndrews and Rajan argue that the timing of payments resembles a coordination game. This idea emerges in the formal analysis of the intraday liquidity management game of Bech and Garratt where it is shown that the game played by banks is special kind of coordination game known as the stag hunt game. In this game there are two Nash equilibria, one which involves early settlement of payments and the other late settlement. Evidence that banks select between these equilibria is found by McAndrews and Potter who document the breakdown and reemergence of coordinated payments following the events of September 11, 2001.

In what follows we shed some light on why coordination on early settlement occur in normal times and how operational difficulties for participants in Fedwire are likely to effect equilibrium selection. Our argument involves constructing a *potential function* for the game played by banks. The potential function is used to summarize welfare changes that result from movements by individual banks. Changes by individual banks that improve individual welfare necessarily increase potential. Hence, we look to adjustment processes that converge to potential maximizing strategy profiles as indicators of how the system will operate. We are able to characterize circumstances under which the system will converge to an early (versus late) payment equilibrium. At first, we assume that all banks are identical. Subsequently, we introduce heterogeneity across banks and are able to add new insight into the question, when a bank might be considered "too big to fail?". In addition, we investigate how different degrees of interconnectedness between banks, i.e., network topologies may effect resiliency vis-à-vis wide-scale disruptions.

2 The n-player, deterministic intraday liquidity management game

Envision an economy with n identical banks using a RTGS system operated by the central bank to settle interbank claims. Banks seek to minimize their settlement costs. The business day consists of two periods: morning and afternoon. Banks start the business day with a zero balance on their settlement accounts at the central bank. At dawn each bank receives $(n-1)$ requests from customers to pay a customer of each of the other $(n-1)$ banks \$1 as soon as possible. For simplicity, assume that banks either process all the requests in the morning period or postpone them all until the afternoon period. In order to process a payment request a bank must have sufficient funds in their account at the central bank. Banks without sufficient funds can borrow funds from the central bank. Unlike most other central banks, the Federal Reserve does not require intraday overdrafts to be backed by collateral for banks in good standing. Instead banks are charged a fee for such overdrafts. Here, it is assumed that banks are charged a fee $F > 0$ per dollar if their settlement account is overdrawn at the end of a period. Banks can avoid fees by delaying payments. However, delaying is costly as customers might demand compensation for late settlement or take their business elsewhere in the future. The cost of delaying faced by banks is $D > 0$ per dollar.[3] Hence, banks have to trade-off the potential cost of mobilizing liquidity against the cost of customer dissatisfaction.

Formally, the player set is $\mathcal{N} = \{1, 2, \ldots n\}$. The set of strategies for each bank i is $\mathcal{S}_i = \{m, a\}$ where m denotes "all requests processed in the morning" and a denotes "all requests postponed until the afternoon." Let $\mathcal{N}_{-i} = \mathcal{N}\backslash\{i\}$. The settlement cost of bank i depends on the strategies played by all banks and is given by $c(s_i, \mathbf{s}_{-i})$ where $s_i \in \mathcal{S}_i$ is the strategy played by bank i and where $\mathbf{s}_{-i} \in \mathcal{S}_{-i} = \times_{j\in\mathcal{N}_{-i}}\mathcal{S}_j$ is the $(n-1)$ dimensional vector of opponents strategies. The price that banks charge for processing is assumed to be fixed and for simplicity it is set to zero. The

[3] See Coleman [3] for a discussion of the evolution of the Federal Reserve's intraday credit policies.

payoff function for bank i is thus equal to the negative of the settlement cost function, that is

$$\pi_i(s_i, \mathbf{s}_{-i}) = -c(s_i, \mathbf{s}_{-i}) \tag{1}$$

The settlement cost a bank incurs by playing morning is simply the overdraft at noon times the overdraft fee per dollar. The overdraft at noon is given by the number of other banks playing afternoon, i.e., the number of banks from which the bank does not receive an offsetting payment (dollar) in the morning. On the other hand, if a bank plays afternoon then the settlement cost is the amount of payments delayed times the delay cost per dollar. Formally, the payoff function of bank i, for each strategy choice in $\mathcal{S}_i$, is given by

$$\pi_i(m, \mathbf{s}_{-i}) = -(n - 1 - |\mathbf{s}_{-i}|_m)F \tag{2}$$

$$\pi_i(a, \mathbf{s}_{-i}) = -(n - 1)D \tag{3}$$

where $|\mathbf{s}_{-i}|_a$ is the number of banks playing the afternoon strategy in the strategy profile $\mathbf{s}_{-i}$. In the following, we restrict ourselves to equilibria in pure strategies and measure efficiency in terms of aggregate payoff.

Example 1 *In the* 2*-player case the game is given by*

		Bank 2	
		m	a
Bank 1	m	$0, 0$	$-F, -D$
	a	$-D, -F$	$-D, -D$

If $D > F$ then (m, m) is the unique Nash equilibrium. If $D < F$, then the game is a stag hunt game. (m, m) is the efficient equilibrium.

Due to the structure of the payoff function we can transform the game into an "aggregation game" (see Rausser et al. [11]) where it suffices to keep track of the number of opponents playing a particular strategy. When bank i plays the morning strategy its payoff depends only on the number of opponents that play the strategy m. When bank i plays the strategy a, its payoff is independent of the strategies played by the opponents. Hence, the payoff functions π_i can be replaced with the functions $\tilde{\pi}_i(\cdot) : S_i \times \mathbb{Z}_+ \to \mathbb{R}$ that are specified as follows: $\tilde{\pi}_i(m, |\mathbf{s}_{-i}|_a) \equiv \pi_i(m, \mathbf{s}_{-i})$ and $\tilde{\pi}_i(a, |\mathbf{s}_{-i}|_a) \equiv -(n - 1)D$. The equilibria of the game are given by the following lemma.

Proposition 1

a) If intraday liquidity is relatively cheap, i.e., $F < D$ then the unique Nash equilibrium is $(m, \ldots, m)$.

b) If intraday liquidity is relatively expensive, i.e., $F \geq D$ then $(m, \ldots, m)$ and $(a, \ldots, a)$ are the only Nash equilibria.

Proof. From Eqs. (2) and (3) we have $\tilde{\pi}_i(m, 0) = 0 \geq \tilde{\pi}_i(a, 0) = -(n-1)D < 0$ for every $i \in \mathcal{N}$. Hence, $(m, \ldots, m)$ is a Nash equilibrium regardless of the relative magnitudes of F and D. Suppose $F \geq D$. Then, we have $\tilde{\pi}_i(a, n-1) = -(n-1)D \geq \tilde{\pi}_i(m, n-1) = -(n-1)F$ for every $i \in \mathcal{N}$. Hence, $(a, \ldots, a)$ is a Nash equilibrium if $F \geq D$. If $0 < |\mathbf{s}_{-i}|_a < n$ then at least one bank $i \in \mathcal{N}$ plays the strategy a and at least one bank $j \in \mathcal{N}_{-i}$ plays the strategy m. For a Nash equilibrium to exist it has to be true that $\tilde{\pi}_i(m, |\mathbf{s}_{-i}|_a) < \tilde{\pi}_i(a, |\mathbf{s}_{-i}|_a)$ and $\tilde{\pi}_j(a, |\mathbf{s}_{-i}|_a + 1) < \tilde{\pi}_j(m, |\mathbf{s}_{-i}|_a + 1)$. From Eq. (3) we have $\tilde{\pi}_j(a, |\mathbf{s}_{-i}|_a) = \tilde{\pi}_j(a, |\mathbf{s}_{-i}|_a + 1)$, and hence from the preceding strict inequalities, $\tilde{\pi}_i(m, |\mathbf{s}_{-i}|_a) < \tilde{\pi}_i(m, |\mathbf{s}_{-i}|_a + 1)$, which by Eq. (2) is a contradiction. ■

Note that whenever $F \geq D$ then our game is a coordination game and the efficient equilibrium in terms of minimizing both individual and aggregate settlement costs is $(m, \ldots, m)$. Furthermore, the set of equilibria is independent of the number of banks.

3 Convergence following wide-scale disruptions

The Interagency Paper on Sound Practices To Strengthen The Resilience Of The U.S. Financial System issued by the Federal Reserve Board, the Securities and Exchange commission and the Office of the Comptroller of the Currency defines a wide-scale disruption as an event that causes a severe disruption or destruction of transportation, telecommunication, power, or other critical infrastructure components across a metropolitan or other geographical area and the adjacent communities that are economically integrated with it; or that results in wide-scale evacuation or inaccessibility of the population within normal commuting range of the disruption's origin.

In the context of our model, we assume that the system is initially at the efficient equilibrium and we take a wide-scale disruption to mean an event that prevents a subset of banks from making payments as normal. Specifically, some banks are forced to play to the afternoon strategy, i.e., moving the system out of equilibrium. The size of the disruption can be measured by the share of banks that are disrupted. After the disruption we assume that the disrupted banks again become operational and that they as well as the banks not affected are free to choose either the morning or afternoon strategy.

The question we ask is: Will the disruption trigger a move to the inefficient afternoon equilibrium or not? Not surprisingly, the answer depends on the size of the disruption but is also depends on the incentive to coordinate via the relative cost delay and liquidity. Moreover, as we shall see in section 5 the answer is dependent on the structure of the banking system as well as the topology of the underlying interbank relationships.

At a first glance, our one shot n-player game does not appear to be well suited to analysis of off-equilibrium system dynamics following a shock. However, it turns out that the game played by banks is a potential game (à la Monderer and Shapley, [10]), and as such the dynamics of the system are conveniently and transparently understood by examining the potential function. Adjustments that increase the payoff of individual banks raise the value of the potential and adjustments that hurt banks lower it. Hence, we can visualize which direction the system will move following a disruption simply by looking at the "slope" of the potential function at the starting point produced by the disruption.[4]

3.1 Potential function

We now turn to a formal characterization of the potential function we utilize to analyze the n-player deterministic intraday liquidity management game.

Definition 1 (Potential Function) *A function P satisfying for every $s_i, s_i' \in \mathcal{S}_i$, $\mathbf{s}_{-i} \in \mathcal{S}_{-i}$ and $i \in \mathcal{N}$,*

$$\pi_i(s_i, \mathbf{s}_{-i}) - \pi_i(s_i', \mathbf{s}_{-i}) = P(s_i, \mathbf{s}_{-i}) - P(s_i', \mathbf{s}_{-i})$$

is a potential function for the n-player deterministic intraday liquidity management game.

The function P is unique up to an additive constant. In order to pin down a particular function we choose the normalization $P(m, ..., m) = 0$. The following example illustrates the construction of the potential function for the 2-player intraday liquidity management game

[4] A game with a potential function is called a potential game. A potential function contains all the information needed to compute the Nash equilibria of a potential game. Local maxima of a potential function correspond to Nash equilibria. At a local maximum no unilateral deviation increases the potential of the game. At a global maximum there is no alternative strategy profile that has higher potential.

Example 2 *In the 2-player case, changes in Bank i's payoff that result from unilateral deviations are given by*

$$\pi_i(m,m) - \pi_i(a,m) = D$$
$$\pi_i(m,a) - \pi_i(a,a) = D - F$$

Hence, by Definition 1 the potential function has to satisfy

$$\begin{array}{ll} P(m,m) - P(a,m) = D & P(m,a) - P(a,a) = D - F \\ P(m,m) - P(m,a) = D & P(a,m) - P(a,a) = D - F \end{array} \tag{4}$$

This implies that $P(m,a) = P(a,m)$. Letting $P(m,m) = D$ we have

$$P(s) = \begin{cases} D & \text{if} \quad s = (m,m) \\ 0 & \text{-} \quad s = (m,a) \\ 0 & \text{-} \quad s = (a,m) \\ F - D & \text{-} \quad s = (a,a) \end{cases}$$

If $F < 2D$ then (m,m) globally maximizes P, otherwise (a,a) is the global potential maximizer. If $D < F < 2D$ then (a,a) is a local maximum. If $F > 2D$ then (m,m) is a local maximum.

The function P maps strategy profiles into real numbers and hence for large numbers of players it is difficult to visualize. However, in this application the value of the potential function depends only on the total number of players playing the morning strategy. Hence, we can summarize P with a new function $\tilde{P} : \mathbb{Z}_+ \to \mathbb{R}$, defined in the following way. For any $x \in \{0, 1, ..., n\}$, $\tilde{P}(x)$ is equal to the value of the function $P(\cdot)$ evaluated at any strategy profile in the set $\{\mathbf{s} \in \mathcal{S} : |\mathbf{s}|_m = x\}$.

From Eqs. (2) and (3) we have that

$$\tilde{P}(x) - \tilde{P}(x-1) = (x-n)F + (n-1)D \tag{5}$$

for all $x \in \{1, \ldots, n\}$. Solving Eq. (5) by induction and assuming that $\tilde{P}(n) = 0$ yields (see appendix A)

$$\tilde{P}(x) = \frac{1}{2}(n-1-x)(n-x)F - (n-x)(n-1)D \tag{6}$$

which is a second order polynomial in x. For large n there is no discernable difference between n and $n-1$ and thus the potential function is approximately equal to

$$\tilde{P}(x) \simeq \breve{P}(\theta) = n^2(\frac{1}{2}\theta^2 F - \theta D) \tag{7}$$

where $\theta \equiv \frac{n-x}{n}$ is the share of banks playing the afternoon strategy. For the ease of exposition, we shall use this version of the potential function below and treat it as a differentiable function defined over $\theta \in [0,1]$. The global and local maxima of $\breve{P}(\theta)$ and, by virtue of construction, the Nash equilibria of the n-player deterministic intraday liquidity management game are summarized in the following proposition (without proof).

Proposition 2

a) If $F < D$ then $\theta = 0$, i.e., $s = (m, \ldots, m)$, globally maximizes $\breve{P}$ and there are no other local maxima.

b) If $D \leq F < 2D$ then $\theta = 0$ globally maximizes $\breve{P}$ and $\theta = 1$, i.e., $s = (a, \ldots, a)$, is a local maximum.

c) If $F = 2D$ then both $\theta = 0$ and $\theta = 1$ globally maximize $\breve{P}$.

d) If $F > 2D$ then $\theta = 1$ globally maximizes $\breve{P}$ and $\theta = 0$ is a local maximum.

4 Simple adjustment process

Monderer and Shapley [10] specify a simple adjustment process that converges to a Nash equilibrium of a potential game in a finite number of steps. In their process it is assumed that, whenever the strategy profile is not a Nash equilibrium, one player deviates to a strategy that makes him better off. Unilateral deviations that increase the payoff of the deviator raise the value of the potential while unilateral deviations that increase the payoff of the deviator lower it. Hence, once a Nash equilibrium is reached (there are no more self-improving, unilateral deviations) the process terminates and the potential function will be at a maximum in the sense that its value cannot be increased by varying *any single player's strategy.* Endpoints of the simple adjustment process are local maxima of the potential function, i.e., Nash equilibria.

From Eq. (7), we have that the potential function is a second order polynomial in θ with a minimum at $\theta^* = \frac{D}{F}$. If the slope of $\breve{P}(\theta)$ is *negative* then unilateral deviations to *morning* strategy are *profitable* for banks while unilateral deviations to the *afternoon* strategy are *profitable* when the slope is *positive*. Hence, the simple adjustment process has the following properties:[5]

[5] The convergence results for the simple adjustment process are, in fact, quite more general. In fact, the conditions presented for convergence of the simple adjustment process apply to any best-response adjustment process based on arbitrary orderings of sequential unilateral deviations or simultaneous deviations

Proposition 3

a) Suppose $F < D$. The simple adjustment process leads to the equilibrium $\theta = 0$, i.e., $s = (m, \ldots, m)$ from any starting point.

b) Suppose $F = D$. The simple adjustment process leads to the equilibrium $\theta = 0$ from any starting point except $\theta = 1$, i.e., $s = (a, \ldots, a)$. At $\theta = 1$ all players are indifferent between playing a or making a unilateral deviation to m.

c) Suppose $D < F$. The simple adjustment process leads to the equilibrium $\theta = 0$ from starting points on the left of θ^* and to the equilibrium $\theta = 1$ from starting points to the right of left θ^*.

Proof. See Appendix B ■

The different cases outlined above are illustrated in Fig 1. The figure displays the potential function, $\breve{P}(\theta)$ normalized by n^2 for different values of D and F. The case of $F < D$ is illustrated by the lowest (dark blue) curve. As indicated by the first property all deviations to m are profitable after a disruption of any size in this case. The implication is that the system will self reverse to the (efficient) morning equilibrium regardless of the size of the disruption. In other words, the system is resilient to a disruption of any size in terms of maintaining coordination on early processing. The same is true for the case where $F = D$ (illustrated by the red curve) save the situation where the disruption hits all n banks, cf. property 3.

Instances of the case $F > D$ are illustrated by the top three curves in Fig. 1, i.e., the green, the purple and light blue curves, respcetively. In each instance disruptions that are sufficiently large will move the system to a point to the left of the minimum $\theta^* = \frac{D}{F}$ indcuing the banks to converge to the (inefficient) afternoon equilibrium ($\theta = 1$). The higher the cost of liquidity from the central bank, F, is relative to the cost of delay, D, the lower is the minimum and thus the larger is the "basin of attraction" for the afternoon equilibrium. For example, if $F = 2D$ then the disruption needs to hit at least half the banks in order for the system to converge to the inefficient equilibrium.

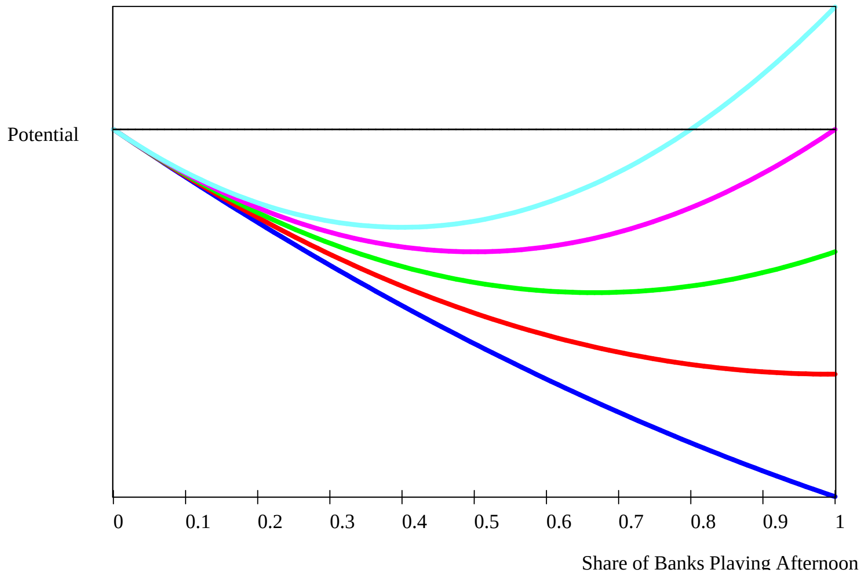

Figure 1: $P(\theta)/n^2$ - ($F < D$, $F = D$, $D < F < 2D$, $F = 2D$, $F > 2D$)

In the situation, where a wide-scale disruption hits and the system is either likely to, is in the process of, or already has converged to the inefficient equilibrium, the central bank might intervene in order to ensure the smooth operation of the payment system. Lacker [7] provides a detailed overview of the Federal Reserve's response to the terrorist attacks on September 11, 2001. Shortly after the attacks, the Federal Reserve issued several statements to the effect that the Fedwire was open and operating and that the discount window was available to meet liquidity needs. Moreover, the Federal Reserve began to make liquidity widely available and for the period from September 11 through September 21, it waived the daylight overdraft fees for all account holders and eliminated the penalty normally charged in excess of the effective Federal Funds rate on overnight overdrafts. Both policy responces are consistent with this model. The statements can been seen as an attempt to encourage the banks to keep coordinating on the efficient equilibrium. Moreover, the infusion of liquidity and elimination of fees served to lower the cost of liquidity F to virtually zero so that deviations to m became profitable. Once the system has returned to the morning equilibrium (or had made sufficient progress) the original F could be restored.

5 Banking Structure and Network Topology

So far, we have considered an economy with a homogenous banking structure consisting of n identical banks. Obviously, this is - in many ways - an unsatisfactory representation. For example, in Fedwire the top 100 participants account for [95]% of the value and [60]% of the volume. Moreover, it is often argued that the financial sectors resiliency to different types of shocks depends critically on the interconnectedness of the interbank market see e.g. Allen and Gale [1]. In the following, we provide a simple extension of our model that allows us to investigate how both heterogeneity across banks and different degrees of completeness of interbank relationships might impact the resiliency of the interbank payment systems vis-à-vis (wide-scale) disruptions.

5.1 Mergers and Payment Topology

Envision a new economy with one large bank and a number of smaller identical banks. Assume that the large bank is created by merging $k > 1$ out of the n identical banks in our previous economy. For notational convenience, assume that banks 1 through $k < n$ are the ones that are merged. Let the merged bank be denoted by k. The new player set is $\widetilde{\mathcal{N}} = \{k, k+1, \dots n\}$. A key distinction is that transfers between customers of the merged bank are handled internally within that bank. Consequently following a merger, the total volume of payments transferred via the RTGS system decreases. The larger the merger the larger the decrease. Specifically, the total volume (and value) of payments transferred ex post is $n(n-1) - 2(k-1)$ compared to $n(n-1)$ ex ante.

We assume that a merger can lead to a shift in the payment flow. For example, the merger might lead to an increase in the volume of transactions involving the merged bank. This would occur if there were positive externalities associated with banking transactions or if larger banks were able to provide better payments services due to economies of scale or scope. We do not formally model the determinants of the underlying payment flows. Rather we consider different exogenous payment flows by including the parameter $\alpha \geq 0$, which represents the fraction of each dollar that small banks continue to transact with each other following a merger. The remaining $1 - \alpha$ of each dollar small banks transacted with each other before the merger now goes to the merged bank. For example, if the merger is size k, each small bank sends α dollars to each of the other small banks and sends $k + (1-\alpha)(n-k-1)$ dollars to the merged bank. We keep the total payment flow fixed, and hence the large bank sends the same amount to each of the small banks.

If $\alpha = 1$, then there is no shift in payment volume between the large and the smaller banks, but the analysis of the banking system is still effected by the merger because there are no longer any coordination issues associated with transfers within the merged bank. Values $\alpha < 1$ correspond to the scenario discussed above where the small banks are losing market share vis-à-vis the merged bank. If $\alpha > 1$, then the small banks are gaining market share vis-à-vis the merged bank. In order to keep all payment flows positive we require $\alpha \leq \frac{n-1}{n-k-1} = \bar{\alpha}$.

By varying α, we create different degrees of interconnectedness or network topologies. If $0 < \alpha < \bar{\alpha}$, we have a complete graph, where all banks exchange payments with each other. In the two special cases where $\alpha = 0$ or $\alpha = \bar{\alpha}$, we get topologies where not all banks are exchanging payments with each other. If $\alpha = 0$, then we have a *star* graph where the small banks do not interact with each other, but instead, only interact with the "money center" bank. If $\alpha = \bar{\alpha}$, then we have a *disconnected* graph where the small banks are interacting with each other but not with the merged bank which only process internal transactions. Examples of the payment flows in an economy with originally five banks following a merger of size $k = 2$ are shown in Fig. 2.

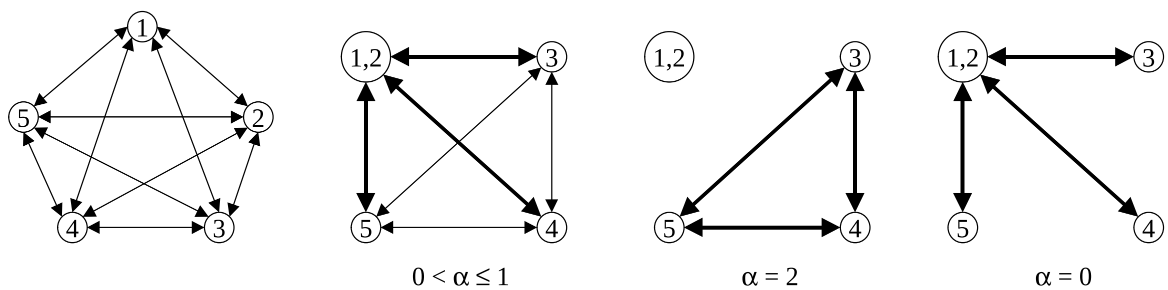

Figure 2: Payment flows before and after merger ($n = 5$ and $k = 2$)

5.2 Potential Function for the Heterogenous Economy

The payoff function of the merged bank k is given by

$$\pi_k(m, \mathbf{s}_{-k}) = -(k + (1-\alpha)(n-k-1))(n-k-|\mathbf{s}_{-k}|_m)F \tag{8}$$

$$\pi_k(a, \mathbf{s}_{-k}) = -(k + (1-\alpha)(n-k-1))(n-k)D \tag{9}$$

The payoff functions of the $i = k+1, ..., n$ small banks are given by

$$\pi_i(m, s_k, \mathbf{s}_{-i,k}) = -(n-1-(k+(1-\alpha)(n-k-1))1_{s_k=m} - \alpha\,|\mathbf{s}_{-i,k}|_m)F \tag{10}$$

$$\pi_i(a, s_k, \mathbf{s}_{-i,k}) = -(n-1)D \tag{11}$$

where $1_{s_k=m}$ is the indicator function.

Example 3 *Consider the 4-player case and assume that banks 1 and 2 merge (i.e., $k=2$). The payoffs of the merged bank are given by $\pi_2(m,a,a) = -2(3-\alpha)F$, $\pi_2(m,m,a) = \pi_2(m,a,m) = -(3-\alpha)F$, $\pi_2(m,m,m) = 0$, and $\pi_2(a,\cdot) = -2(3-\alpha)D$ where the first argument of $\pi_2(\cdot)$ is the strategy of the merged bank. The payoffs of the small banks (3 or 4) are given by $\pi_i(m,m,a) = -\alpha F$, $\pi_i(m,m,m) = 0$, $\pi_i(m,a,a) = -3F$, $\pi_i(m,a,m) = -(3-\alpha)F$, and $\pi_i(a,\cdot) = -3D$ where the first argument of $\pi_i(\cdot)$ is the strategy of bank i and the second argument is the strategy of the merged bank.*

As before, the payoff to a bank, that plays afternoon, is independent of the strategies played by the opponents and the structure of the payoff function allows us to transform the game into an "aggregation game". For the small banks it suffices to keep track of the bank's own strategy, the strategy of the large bank and the number of other small banks playing the morning strategy. Hence, the payoff functions π_i, $i \neq k$, can be replaced with the functions $\hat{\pi}_i(\cdot) : S_i \times S_k \times \mathbb{Z}_+ \to \mathbb{R}$ that are specified as $\hat{\pi}_i(s_i, s_k, |\mathbf{s}_{-i,k}|_m) \equiv \pi_i(s_i, s_k, \mathbf{s}_{-i,k})$. For example, in Ex. 3, $\hat{\pi}_i(m,m,0) = \pi_i(m,m,a) = -\alpha F$. In case of the large bank, it suffices to keep track of it's own strategy and the number of small banks playing the morning strategy. Thus we define $\tilde{\pi}_k(\cdot) : S_k \times \mathbb{Z}_+ \to \mathbb{R}$ as $\hat{\pi}_k(s_k, |\mathbf{s}_{-k}|_m) \equiv \pi_k(s_k, \mathbf{s}_{-k})$. Thus, in Ex. 3, we have $\hat{\pi}_2(m,0) = \pi_2(m,a,a) = -2(3-\alpha)F$. For each strategy choice of the big bank, the value of the potential function depends only on the total number of small banks playing the morning strategy. Hence, we can proceed using a potential function for the game, $\hat{P}(\cdot) : S_k \times \mathbb{Z}_+ \to \mathbb{R}$, defined to satisfy

$$\hat{P}(s_k, |(s_{k+1}...s_i...s_n)|_m) - \hat{P}(s_k, \left|(s_{k+1}...s_i'...s_n)\right|_m) = \hat{\pi}_i(s_i, s_k, |\mathbf{s}_{-i,k}|_m) - \hat{\pi}_i(s_i', s_k, |\mathbf{s}_{-i,k}|_m) \quad (12)$$

and

$$\hat{P}(s_k, |\mathbf{s}_{-k}|_m) - \hat{P}(s_k', |\mathbf{s}_{-k}|_m) = \hat{\pi}_k(s_k, |\mathbf{s}_{-k}|_m) - \hat{\pi}_i(s_k', |\mathbf{s}_{-k}|_m) \quad (13)$$

for all $i \in \{k+1, \dots n\}$ and $s_i, s_k \in \{m, a\}$. Using Eqs. (8) - (11), we have that

$$\hat{P}(m,x) - \hat{P}(m,x-1) = -\alpha(n-k-x)F + (n-1)D \quad (14)$$

$$\hat{P}(a,x) - \hat{P}(a,x-1) = -(n-1-\alpha(x-1))F + (n-1)D \quad (15)$$

$$\hat{P}(m,x) - \hat{P}(a,x) = -(\alpha k + (1-\alpha)(n-1))((n-k-x)F - (n-k)D) \quad (16)$$

for $x = 1, \dots, n-k$. Solving Eqs. (14) - (16) by assuming that $\hat{P}(m, n-k) = 0$ yields

$$\hat{P}(m,x) = \frac{\alpha}{2}(n-k-1-x)(n-k-x)F - (n-k-x)(n-1)D \quad (17)$$

and

$$\hat{P}(a,x) = (n-1-\frac{\alpha}{2}(n-k+x-1))(n-k-x)F \\ -((n-k-x)(n-1)+(\alpha k+(1-\alpha)(n-1))(n-k))D \tag{18}$$

for $x = 0, \ldots, n-k$. Both $\hat{P}(m,x)$ and $\hat{P}(a,x)$ are second order polynomials in x.

Example 4 *Continuing Ex. 3. Setting $\hat{P}(m,2) = 0$ yields the following potential function $\hat{P}(s_k, x)$*

$s_k \setminus x$	0	1	2
m	$\alpha F - 6D$	$-3D$	0
a	$(6-\alpha)F - (12-2\alpha)D$	$(3-\alpha)F - (9-2\alpha)D$	$-(6-2\alpha)D$

For large $n-k$ there is no discernable difference between $n-k-1$ and $n-k$ and thus the potential functions in Eqs. (17) and (18) are approximately equal to

$$\hat{P}(m,x) \simeq \bar{P}(m,\theta) = (n-k)^2 \left(\frac{\alpha F}{2}\theta^2 - \phi D\theta\right) \tag{19}$$

and

$$\hat{P}(m,x) \simeq \bar{P}(a,\theta) = (n-k)^2 \left(\frac{\alpha}{2}F\theta^2 + ((\phi-\alpha)F - \phi D)\theta - (\phi-\alpha)D\right), \tag{20}$$

respectively (see Appendix C). Now $\theta = \frac{n-k-x}{n-k}$ is the share of *small* banks playing the afternoon strategy and $\phi = \frac{n}{n-k}$ is a non-linear measure of the size of the merger, k. Again, for the ease of exposition, we shall use this version of the potential function below and treat it as a differentiable function defined over $\theta \in [0,1]$. Note that in the money center case where $\alpha = 0$ the potential functions become first order polynomials in θ and in the case where $\alpha = \phi$ (the disconnected graph) the two potential functions are the same.

As before, the maxima of the potential function correspond to the Nash equilibria of the game and we summarize them in the following proposition.

Proposition 4

1. *If $F < D$ then $s_k = m$ and $\theta = 0$ globally maximize $\bar{P}$ and there are no other local maxima.*

2. *If $D \leq F < 2D$ then $s_k = m$ and $\theta = 0$ globally maximizes $\bar{P}$ and $s_k = a$ and $\theta = 1$ is a local maximum.*

3. *If $F = 2D$ then $s_k = m$ and $\theta = 0$ as well as $s_k = a$ and $\theta = 1$ globally maximize $\bar{P}$.*

4. *If $F > 2D$ then $s_k = a$ and $\theta = 1$ globally maximizes $\bar{P}$ and $s_k = m$ and $\theta = 0$ is a local maximum.*

If liquidity is relatively cheap ($F < D$) then our game has a unique equilibrium and coordination is not needed to sustain the efficient equilibrium. Moreover, the adjustment process always leads to the efficient equilibrium. Hence, early processing of payments will not be affected by any wide-scale disruption. In contrast, if liquidity is relatively expensive ($F \geq D$) then our game has two equilibria, one where all banks play the morning strategy and one where they play the afternoon strategy. We shall focus on the latter case below.[INSERT A FOOTNOTE HERE ARGUING THAT THIS CASE IS RELEVANT AND SUPPORTED BY EMPIRICAL EVIDENCE. CITE McANDREWS PAPER.]

5.3 Convergence Following Disruptions

Once more, we assume that the system initially is in the efficient morning equilibrium but that it is hit by a disruption which forces a set of banks - that may or may not include the merged bank - to play the afternoon strategy. In other words, the share of small banks playing the afternoon strategy changes from 0 to $\theta' > 0$. The adjustment process following the disruption of both the small banks and the merged bank is governed by the potential functions $\bar{P}(s, \theta)$, $s \in \{m, a\}$ in Eqs. (19) and (20). A unilateral deviation of a small bank to the morning strategy lowers the share of small banks playing the afternoon strategy, θ. Hence, if the slope $\frac{\partial \bar{P}(s,\theta)}{\partial \theta}$ is negative then such a deviation increases the potential and it is profitable for the small bank which consequently will choose to deviate. For the merged bank the interesting quantity is the difference between the two equations, i.e, $\bar{P}(m, \theta) - \bar{P}(a, \theta)$ which gives the change in potential associated with a unilateral deviation. Again, if the change in potential is positive, then the merged bank will choose to deviate.

The potential functions are second order polynomials in θ and an illustrative example is shown in Fig. 3.

Clearly, the minima as well as the intersection point of $\bar{P}(m, \theta)$ and $\bar{P}(a, \theta)$ are important in terms of the dynamics of the adjustment process. We have the following lemma

Lemma 1 *Let θ^*_m be the minimum of $\bar{P}(m, \theta)$, θ^*_a be the minimum of $\bar{P}(a, \theta)$ and θ^c be the point at which $\bar{P}(m, \theta)$ and $\bar{P}(a, \theta)$ intersects. We have that $\theta^*_a < \theta^c < \theta^*_m$ for $0 < \alpha < \phi$.*

Proof. In Appendix C, we show that $\theta^*_a = \frac{\phi}{\alpha}\frac{D}{F} - \frac{\phi - \alpha}{\alpha}$, $\theta^c = \frac{D}{F}$ and $\theta^*_m = \frac{\phi}{\alpha}\frac{D}{F}$ for $0 < \alpha < \phi$. ■

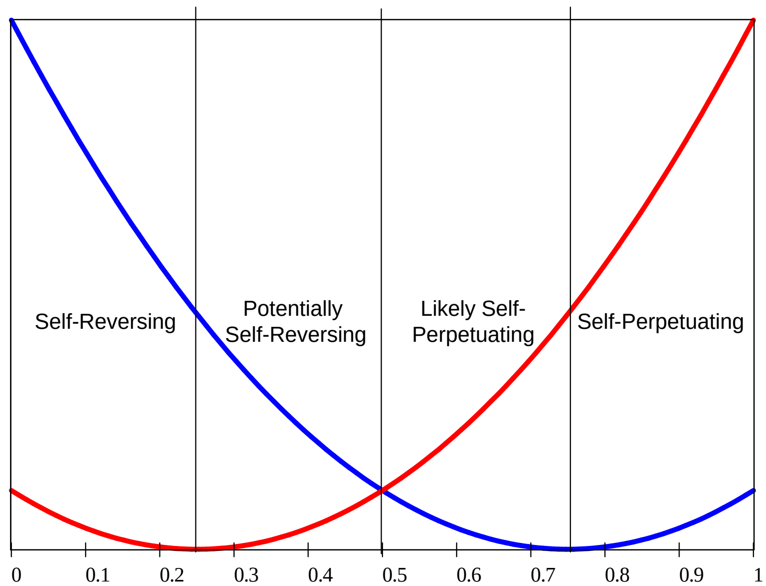

Figure 3: $\hat{P}(m,\theta)/(n-k)^2$ and $\hat{P}(a,\theta)/(n-k)^2$

Lemma 1 implies that there are potentially four qualitavely different cases to consider in terms of the adjustment process following a wide-scale disruption to the economy with heterogenous banking structure.

1. **Self-reversing**. In this case the share of small banks affected is relatively small, i.e., $\theta' < \theta_a^*$ and thus we have that $\frac{\partial \bar{P}(m,\theta)}{\partial \theta} < 0$, $\frac{\partial \bar{P}(a,\theta)}{\partial \theta} < 0$ and $\bar{P}(m,\theta) - \bar{P}(a,\theta) > 0$. Hence unilateral deviations to back to the morning strategy is profitable from the perspetive of both the small banks as well as the large bank is protitable. In other words, the system will revert back to the morning equilibrium even if the merged bank is hit by the disruption.

2. **Potentially self-reversing**. In this case, $\theta_m^* < \theta' < \theta^c$ and thus we have that $\frac{\partial \bar{P}(m,\theta)}{\partial \theta} < 0$, $\frac{\partial \bar{P}(a,\theta)}{\partial \theta} > 0$ and $\bar{P}(m,\theta) - \bar{P}(a,\theta) > 0$. Hence unilateral deviations to back to the morning strategy is profitable from the perspective of the small banks if and only if the the merged bank is not affected by the disruption. Hence, tf the merged bank is not affected by the disruption, then the system will revert back to the morning equilibrium on its own. On the other hand, if the merged bank is, then it is profitable for the smalls banks not affected by the disruption to change their strategy to afternoon and small banks affected will stick with the afternoon strategy. So while it immediately following the disruptions is profitable for

a merged bank to revert back to the morning strategy, this will not continue to be true as the small banks adjust to the afternoon strategy. If θ increases above θ^c then the merged will not revert back to the morning strategy. Basically, the adjustment process following the disruption is a horse race between the merged bank becoming operational again and the number of small banks deciding to cease coordinating on early processing.

3. **Likely self-perpetuating**. In this case, $\theta^c < \theta' < \theta_m^*$ and thus we have that $\frac{\partial \bar{P}(m,\theta)}{\partial \theta} < 0$, $\frac{\partial \bar{P}(a,\theta)}{\partial \theta} > 0$ and $\bar{P}(m,\theta) - \bar{P}(a,\theta) < 0$. Here, the disruption affected so many small banks that the merged bank does not have an incentive to maintain or restore coordination on the the morning equilibrium. Hence, the system will perpetuate towards the inefficient afternoon equilibrium if the merged bank is affected by the disruption and it will only move towards the efficient morning equilibrium if the merged bank is slow to respond after a disruption that does not affect it. In this case small banks becoming operational again might push θ below θ^c ensuring that the merged bank does not deviate to the afternoon.

4. **Self-perpetuating**. In this case, $\theta_m^* < \theta'$ and thus we have that $\frac{\partial \bar{P}(m,\theta)}{\partial \theta} > 0$, $\frac{\partial \bar{P}(a,\theta)}{\partial \theta} > 0$ and $\bar{P}(m,\theta) - \bar{P}(a,\theta) < 0$. Hence, unilateral deviations to back to the morning strategy is not profitable from the perspetive of the small banks as well as the merged bank. In other words, the system will perpetuate towards the inefficient afternoon equilibrium even if the merged bank is not hit by the disruption.

In the self-perpetuating cases the system will not revert back to the efficient morning equilibrium if the merged bank is hit by the disruption. We have

Proposition 5 *A merged bank is too big to fail in terms of maintaining coordination on early processing if* $\theta' > \frac{D}{F}$.

The width of regions over which the adjustment process belong to the four cases, depends on the particular set of parameters. The potential functions intersects at an interior point ($\theta^c \in]0,1[$) whenever $F > D$ but depending on the parameter values θ_a^* might be negative and θ_m^* might be greater than one. This implies that the self reversing or/and self-pepetuating cases are not relevant for a certain set of parameters. Moreover, a change in either the cost of delay, cost of liquidity, a shift in the topology of the payment flow and the size of the merger widens or narrows the different regions as shown in Tab. 1

			Too Big to Fail	
	Self Reversing	Potentially Self Reversing	Likely Self Perpetuating	Self Perpetuating
$D \uparrow$	Widens	Widens	*Narrows*	*Narrows*
$F \uparrow$	Narrows	Narrows	**Widens**	**Widens**
$\alpha \uparrow$	Widens	Narrows	Narrows	Widens
$\phi \uparrow$	Narrows	Widens	Widens	Narrows

Note: Italic indicates that the too big to fail region narrows and bold that it widens

Table 1: Regions and Changes in Parameters

6 Conclusion

The containment of systemic risk within the U.S. financial system in the event of a wide-scale disruption rest on the rapid recovery and resumption of the core clearing and settlement activities that support the financial markets. Fedwire is a one of the most critical components of clearing and settlement process and hence the resilience of the system itself and its participants is of paramount importance. As shown by McAndrews and Rajan (2000) and McAndrews and Potter (2002), coordination in the transmission of payments is a vital part of the day-to-day operations in Fedwire. The ability to maintain coordination, where banks tend to settle promptly and synchronize their payment activity, can potentially be instrumental in mitigating the impact of a wide-scale disruption to the financial system.

In this paper we have investigated how a wide-scale disruptions is likely to impair the smooth functioning of the interbank payment system. Such a disruption will almost by definition - create operational difficulties for the system and its participants in some way or another. However, perhaps less obvious the operational difficulties of some participants may induce other participants to change behavior in terms of how they process payments. In particular, this may lead to a break down of coordination. This paper argues that the ability of banks in Fedwire to maintain payment coordination following a wide-scale disruption depends critically on a number of different factors. First of all, and in retrospect not too surprising, the size of the disruption. A disruption that affects a large part of the nation or a disruption that hits a key geographical area is more likely to result in the break down of payment coordination as more banks presumably will experience operational difficulties.

Secondly, the ability also depends on the relative cost of liquidity and the cost of postponing payments. The cheaper the liquidity, the more likely banks will be to maintain coordination by them selves. The Fed's response to the tragic events of September 11, 2001 in terms of providing unprecedented amount of liquidity to the system at virtually zero cost was in part aimed at minimizing the risk of banks holding back processing payments.

Thirdly, we argue that the banking structure can influence the smooth functioning of the payment system after a wide-scale disruption and that a bank can be considered too big to fail in a new interesting way. We showed that the resiliency of a large bank could be important not only in terms of its share of the payment flow but also in terms of this bank being pivotal in maintaining the coordination. Obviously, this is not comprehensive list of factors determining whether or not payment coordination can be maintained. Another important factor not addressed specifically in this paper is the timing of the disruption. The clearing and settlement cycle over the course of the day consists of a range of critical times where different type processes take place. For example, settlement of foreign exchange transactions through Continuous Linked Settlement (CLS) bank occur in a narrow window very early in the morning whereas the bulk of the activity from Treasury market occur in the afternoon. The effect of the disruption is going to depend on what time of the day it occurs and how long it persists. On one hand, an early time of impact give Fedwire and its participants more time to recover but on the other hand less of the days business has already settled. The overall effect is ambiguous.

7 Appendix

A Potential function, homogenous case

From Eq. (5) we have

$$
\begin{aligned}
\tilde{P}(n) - \tilde{P}(n-1) &= (n-1)D \\
\tilde{P}(n-1) - \tilde{P}(n-2) &= -F + (n-1)D \\
\tilde{P}(n-2) - \tilde{P}(n-3) &= -2F + (n-1)D \\
\tilde{P}(n-3) - \tilde{P}(n-4) &= -3F + (n-1)D \\
&\vdots \\
\tilde{P}(2) - \tilde{P}(1) &= -(n-2)F - (n-1)D \\
\tilde{P}(1) - \tilde{P}(0) &= -(n-1)F + (n-1)D
\end{aligned}
$$

Let $\tilde{P}(n) = 0$. Then $\tilde{P}(n-1) = -(n-1)D$ and so forth

$$\begin{aligned}
\tilde{P}(n-2) &= \tilde{P}(n-1) + F - (n-1)D = F - 2(n-1)D \\
\tilde{P}(n-3) &= \tilde{P}(n-2) + 2F - (n-1)D = 3F - 3(n-1)D \\
\tilde{P}(n-4) &= \tilde{P}(n-3) + 3F + (n-1)D = 6F - 4(n-1)D \\
\tilde{P}(n-5) &= \tilde{P}(n-4) + 4F + (n-1)D = 10F - 5(n-1)D \\
&\vdots
\end{aligned}$$

Hence, we have

$$\tilde{P}(x) = \frac{1}{2}(n-1-x)(n-x)F - (n-x)(n-1)D$$

For large n the potential function is approximately equal to

$$\breve{P}(\theta) = n^2(\frac{1}{2}\theta^2 F - \theta D) \simeq \frac{1}{2}\frac{n-1-x}{n}\frac{n-x}{n}F - \frac{n-x}{n}\frac{n-1}{n}D$$

where $\theta \equiv \frac{n-x}{n} \in [0,1]$.

B Convergence of the simple adjustment process

Proof. The derivative of the potential function $\breve{P}(\theta)$ is $n^2(F\theta - D)$ and the minimum is at $\theta^* = \frac{D}{F}$. If $F < D$ then $\frac{d\breve{P}(\theta)}{d\theta} < 0$ for all $\theta \in [0;1]$. Hence all unilateral deviations from a to m are profitable, and the simple adjustment process converges to $(m,\ldots,m)$. If $F = D$ then $\frac{d\breve{P}(\theta)}{d\theta} = n^2 D(\theta - 1) < 0$ for all $\theta \in [0;1[$ and $\frac{d\breve{P}(\theta)}{d\theta} = 0$ for $\theta = \theta^* = 1$. If $D < F$ then $\frac{d\breve{P}(\theta)}{d\theta} < 0$ if $\theta \in [0;\theta^*[$. Hence starting from points to the below θ^* the simple adjustment process leads to the equilibrium $(m,\ldots,m)$ and starting from points to the above θ^* the simple adjustment process leads to the equilibrium $(a,\ldots,a)$. ■

C Potential function, Heterogeneous Case

Let $\hat{P}(m, n-k) = 0$. From Eqs. (14) - (16) we have that $\hat{P}(m, n-k-1) = -(n-1)D$. Moreover, we have

$$\begin{aligned}
\hat{P}(m, n-k-2) &= \hat{P}(m, n-k-1) + \alpha\,(n-k-(n-k-1))F - (n-1)D \\
&= \alpha F - 2(n-1)D, \\
\hat{P}(m, n-k-3) &= \hat{P}(m, n-k-2) + \alpha\,(n-k-(n-k-2))\,F - (n-1)D \\
& 3\alpha F - 3(n-1)D, \\
\hat{P}(m, n-k-4) &= \hat{P}(m, n-k-3) + \alpha\,(n-k-(n-k-3))\,F - (n-1)D \\
&= 6\alpha F - 4(n-1)D
\end{aligned}$$

and so forth. Hence, we have

$$\hat{P}(m, x) = \frac{\alpha}{2}(n-k-1-x)(n-k-x)F - (n-k-x)(n-1)D \tag{21}$$

for $x = 1, ..., n-k$. Furthermore, from Eq. (16) we know that

$$\hat{P}(a, x) = \hat{P}(m, x) + \{\alpha k + (1-\alpha)(n-1)\}[(n-k-x)F - (n-k)D]$$

Substituting in Eq (**??**) yields

$$\begin{aligned}
\hat{P}(a, x) &= [\frac{\alpha}{2}(n-k-1-x) + \alpha k + (1-\alpha)(n-1)](n-k-x)F - [\circ]D \\
&= [n-1-\frac{\alpha}{2}(n-k+x-1)](n-k-x)F \\
&- [(n-k-x)(n-1) + \{\alpha k + (1-\alpha)(n-1)\}(n-k)]D
\end{aligned} \tag{22}$$

for $x = 1, ..., n-k$. Both $\hat{P}(m, x)$ and $\hat{P}(a, x)$ are second order polynomials in x. For large $n-k$ the potential functions are approximately equal to

$$\begin{aligned}
\bar{P}(m, \theta) &= (n-k)^2 \left(\frac{\alpha}{2}\theta^2 F - \phi\theta D\right) \\
&\simeq (n-k)^2 \left(\frac{\alpha}{2}\frac{n-k-1-x}{n-k}\frac{n-k-x}{n-k}F - \frac{n-k-x}{n-k}\frac{n-1}{n-k}D\right)
\end{aligned}$$

and

$$
\begin{aligned}
\bar{P}(a,\theta) &= (n-k)^2\left(\frac{\alpha}{2}F\theta^2 + ((\phi-\alpha)\,F - \phi D)\,\theta - (\phi-\alpha)\,D\right) \\
&= (n-k)^2\left(\frac{\alpha}{2}\theta^2 F + (\phi-\alpha)\,\theta F - (\theta\phi + \phi - \alpha)\,D\right) \\
&= (n-k)^2\left(\frac{\alpha}{2}\theta^2 F + (\alpha\,(\phi-1) + (1-\alpha)\phi)\,\theta F - (\theta\phi + \alpha\,(\phi-1) + (1-\alpha))\,D\right) \\
&\simeq (n-k)^2\left(\left(\frac{\alpha}{2}\frac{n-k-1-x}{n-k} + \alpha\frac{k}{n-k} + (1-\alpha)\frac{n-1}{n-k}\right)\frac{n-k-x}{n-k}F - [\circ]D\right) \\
&\simeq (n-k)^2(\left([\circ]F - (\frac{n-k-x}{n-k}\frac{n-1}{n-k} + \alpha\frac{k}{n-k} + (1-\alpha)\frac{n-1}{n-k}\right)D
\end{aligned}
$$

where $\theta = \frac{n-k-x}{n-k} \in [0,1]$ and $\phi = \frac{n}{n-k} \in [1,\infty[$. Both potential functions are second order polynomials in θ. We have that $\bar{P}(m,\theta) = \bar{P}(a,\theta)$ for $\phi = \alpha$. Otherwise, $\bar{P}(m,\theta)$ intersects $\bar{P}(a,\theta)$ at $\theta^c = \frac{D}{F}$. $\bar{P}(m,\theta)$ and $\bar{P}(a,\theta)$ attain their minima at

$$
\frac{\partial \bar{P}(m,\theta)}{\partial \theta} = \alpha F\theta - \phi D = 0 \Rightarrow \theta_m^* = \frac{\phi}{\alpha}\frac{D}{F}
$$

and

$$
\frac{\partial \bar{P}(a,\theta)}{\partial \theta} = \alpha F\theta + (\phi-\alpha)\,F - \phi D \Rightarrow \theta_a^* = \frac{\phi}{\alpha}\frac{D}{F} - \frac{\phi-\alpha}{\alpha},
$$

respectively. The partial derivatives of the minima and intersection point with respect to the different parametes are given in

$\frac{\partial\theta}{\partial z}$	θ_a^*	θ^c	θ_m^*
D	$\frac{\phi}{\alpha}\frac{1}{F} > 0$	$\frac{1}{F} > 0$	$\frac{\phi}{\alpha}\frac{1}{F} > 0$
F	$-\frac{\phi}{\alpha}\frac{D}{F^2} < 0$	$-\frac{D}{F^2} < 0$	$-\frac{\phi}{\alpha}\frac{D}{F^2} < 0$
α	$\frac{\phi}{\alpha^2}(1-\frac{D}{F}) > 0$	0	$-\frac{\phi}{\alpha^2}\frac{D}{F} < 0$
ϕ	$-\frac{1}{\alpha}(1-\frac{D}{F}) < 0$	0	$\frac{1}{\alpha}\frac{D}{F} > 0$

Table 2: Type 1 Game

D Stuff

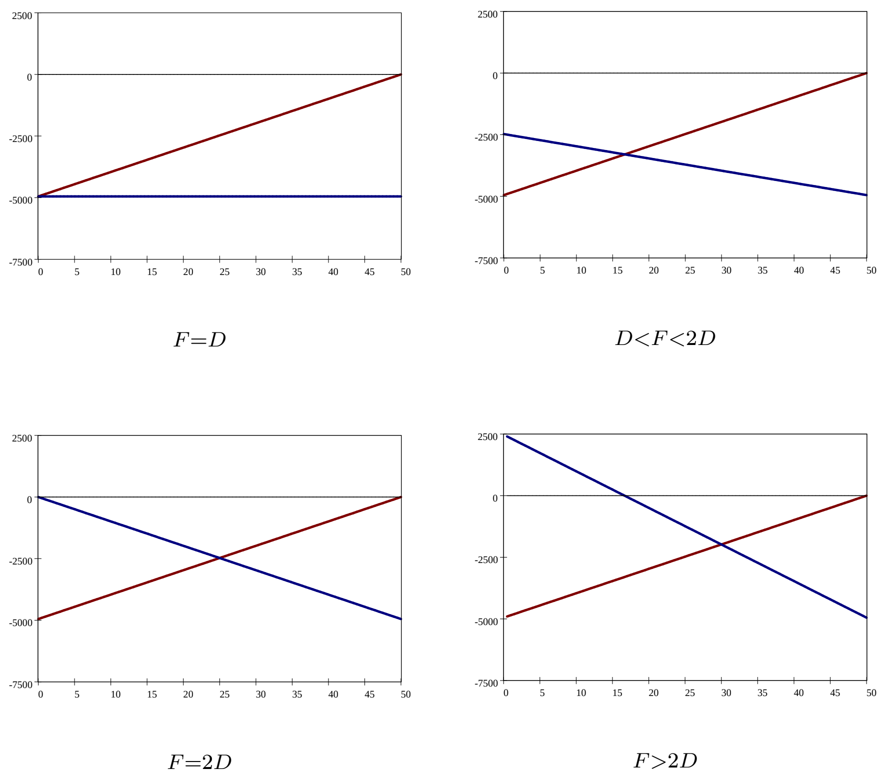

Figure 4: $\hat{P}(m,x)$ and $\hat{P}(a,x)$ - $n = 100$, $k = 50$, $\alpha = 0$

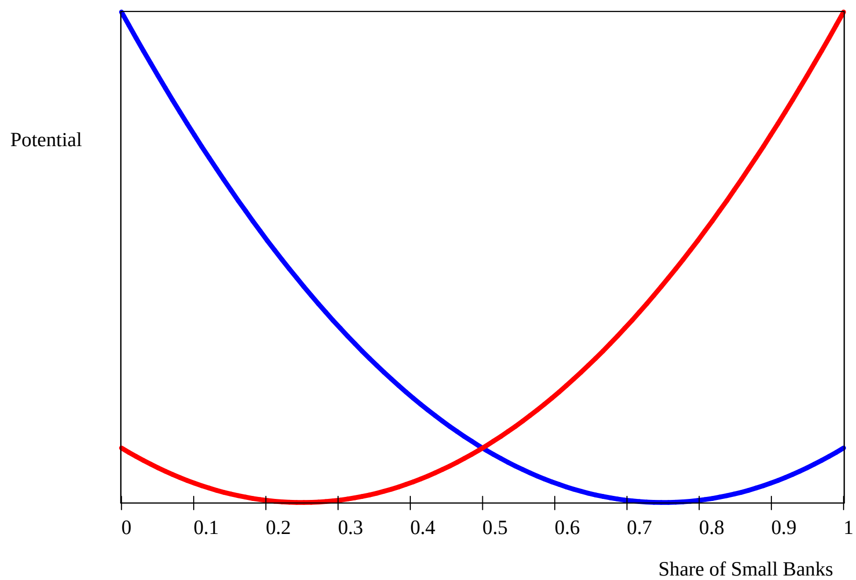

Figure 5: $\hat{P}(m, \theta)/(n-k)^2$ and $\hat{P}(a, \theta)/(n-k)^2$

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