

# Rationalizing Trading Frequency and Returns: Maybe Trading **IS** Good for You

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## Motivation

Barber and Odean (2000)

“

Our most dramatic empirical evidence supports the view that **overconfidence leads to excessive trading** ...households that trade frequently earn a net annualized geometric mean return of 11.4 percent, and those that trade infrequently earn 18.5 percent. These results are consistent with models where trading emanates from **investor overconfidence**, but are **inconsistent** with models where trading results from **rational expectations**.

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- "Trading Is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors,"The Journal of Finance, 2000, **3229 cites** .
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Fact

BO Interpretation

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## What We Do

- **Ask:** Does a model of rational and irrational traders support these findings?

## What We Do

- ❏ **Ask:** Does a model of rational and irrational traders support these findings?
  
- ❏ **Approach**
  - ❏ Dynamic household optimization with portfolio adjustment costs
  - ❏ Beliefs
    - ▶ Impose rational expectations for baseline
    - ▶ Irrationality modeled as: belief miscalibration
  - ❏ Simulated Method of Moments (SMM) to match turnover rate and return moments from BO data

## Data

- ❏ from a large discount brokerage firm
- ❏ all accounts, 78,000 households, Jan 91 - Dec 96
- ❏ focus on common stocks only (66,465 hh)
  
- ❏ median position: 3 stocks, \$15k
- ❏ trade (monthly): median sales/purchase: \$ 7k/6k
- ❏ median commission (monthly): \$ 106

Sampled households have open accounts before Jan 1991. Household level medians.

# Moments

Table: Data Moments: Our Calculation v.s. BO

case	Mean					DR
	t1	t2	t3	t4	t5	
Time Series	0.00	0.0039	0.0134	0.0291	0.1155	<b>-0.0010</b>
Cross Section	0.00	0.00	0.00	0.00	0.2721	<b>-0.0009</b>
BO	0.0010	0.0124	0.0289	0.0598	0.2149	<b>-0.0046</b>

This table reports data moments. Here "ti" is the turnover rate for quintile "i" and "DR" is the difference in the net return between the highest and lowest turnover rate quintiles. The row labeled BO is from Table V of BO.

- ❖ **Time Series:** Balanced Panel  
 the quintiles of the time series average of the turnover rates by household.
- ❖ **Cross Section:** Balanced Panel  
 the *time series average* of the quintiles computed from the cross section for each month.
- ❖ **BO:** directly taken from Barber and Odean (2000)

## Findings

- ❖ Our model with **rational** households can **essentially** match **turnover rate** and **net return differential** moments
  - ❖ turnover rates driven by fixed costs, idiosyncratic income shocks and return shocks
  - ❖ net return reduced by trading costs
- ❖ model fit with irrational beliefs
  - ❖ **no** improvement when matching model-consistent moments.
  - ❖ **slight** improvement **only** matching moments directly taken from BO(2000)



# Model

## Environment

- ❖ Infinite horizon, monthly
- ❖ Rational households: beliefs about stochastic processes are consistent with data
- ❖ 2 assets
  - ❖ stocks: stochastic return, costly to trade
  - ❖ bonds: non-stochastic return, no costs of adjustment
- ❖ All are stock market participants
  - ❖ extensive margin: adjust stock holding or not
  - ❖ intensive margin: size of adjustment
- ❖ Introduce irrational beliefs (misperceptions, advice) later

### Formalities

Exogenous income and stock return processes, as well as the stock trading costs are estimated from data directly.

Return

Income

## Costs

Consumption if adjustment:

$$c = \psi y + R^b b + R^s s - b' - s' - C(s, s') - F \quad (1)$$

### Internal Costs:

- ❖  $\psi \leq 1$  is an opportunity cost
- ❖  $F$  is a fixed cost

### Trading Costs: $C(s, s')$

- ❖ estimated from BO monthly (1991-96) household account data
- ❖ fixed, linear and non-linear costs [Details](#)
- ❖ impacts net return dependence on trading volume

## Income Process

- 1, annual income process estimated using PSID:1968-97: AR1 (0.84,0.29)
- 2, estimate monthly income process to match annual moments

- ❖ without unemployment shock
  - ❖ serial correlation:  $\rho = 0.978$
  - ❖ std of innovation:  $\sigma = 0.116$
- ❖ with unemployment Risk
  - ❖ monthly spells calculated from Bureau of Labor Statistics data: job loss: 0.014, job finding: 0.27
  - ❖ replacement rate = 0.40
  - ❖ **estimated** to match annual moments given monthly job flows
  - ❖  $\rho = 0.996$ ,  $\sigma = 0.084$ , conditional on employment

no income data for BO households. [back](#)

## Return Process

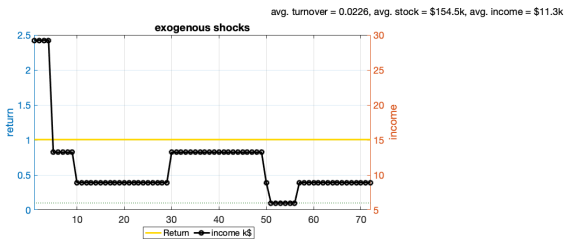
- ❖ bonds: 1.0017 monthly (1.02 annual)
- ❖ stocks (monthly):
  - ❖ measured as S&P index monthly 1967-94, Center for Research in Security Prices (CRSP) data
  - ❖ mean return: 1.0061
  - ❖ std: 0.0439
  - ❖ no serial correlation

No correlation between return and income process

[back](#)

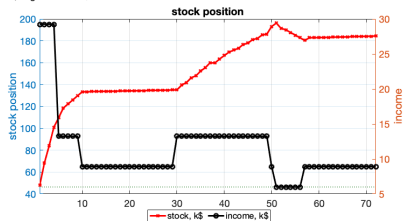
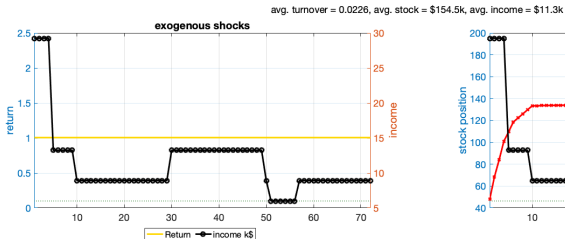
## An example: Response to Income Shocks

## Response to Income shocks



Avg. hh. income = \$6k. (de) accumulation after (low) high income realization.  
Turnover rate depends on stock position.

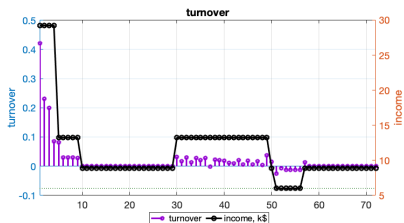
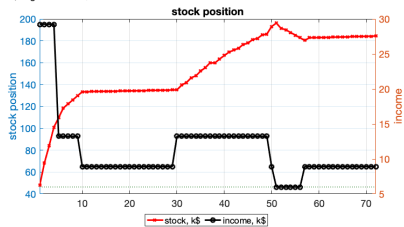
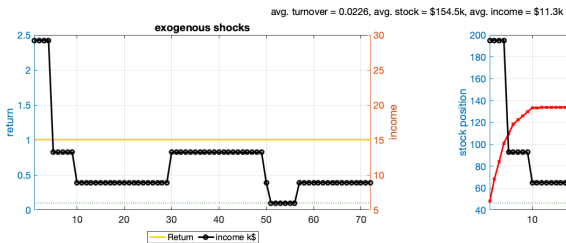
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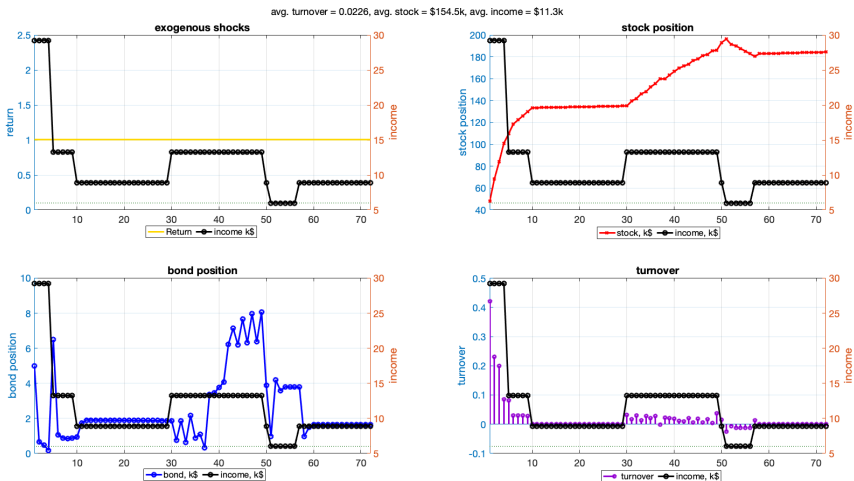
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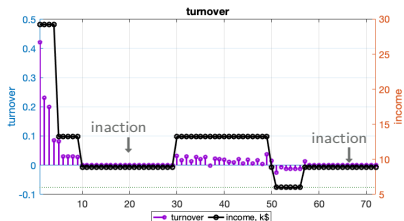
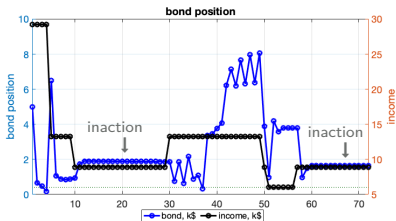
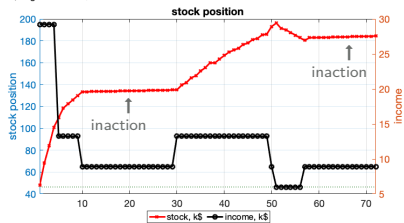
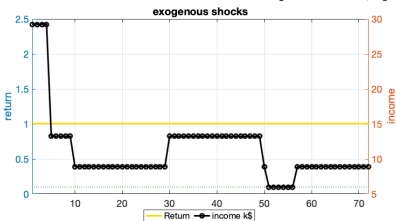
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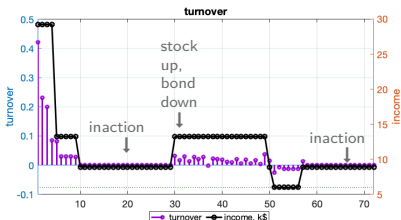
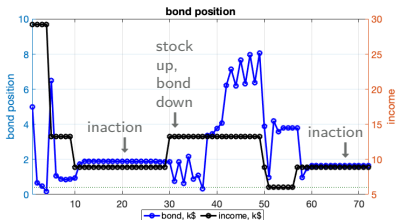
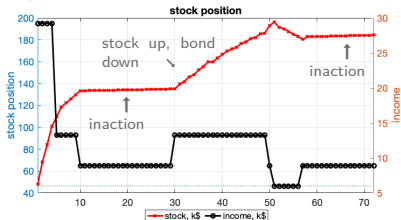
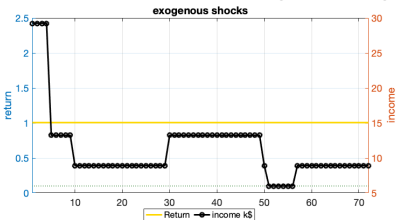
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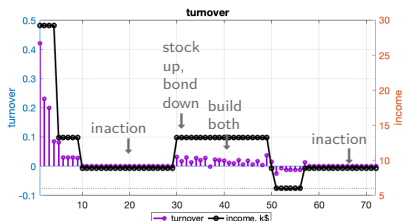
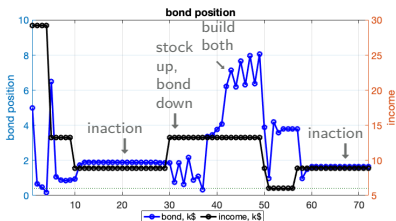
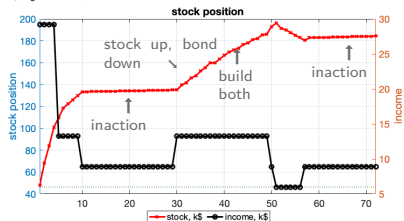
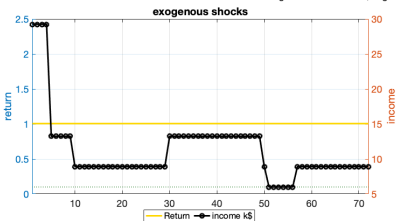
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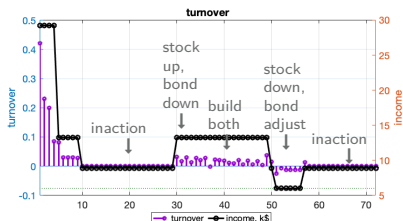
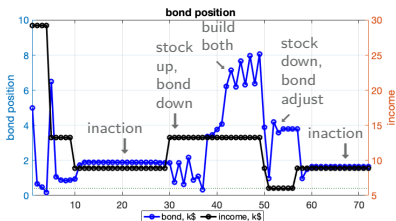
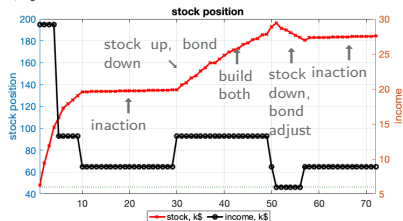
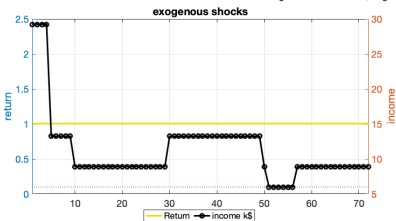
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## Response to Income shocks

avg. turnover = 0.0226, avg. stock = \$154.5k, avg. income = \$11.3k



Avg. hh. income = \$6k. (de) accumulation after (low) high income realization.  
Turnover rate depends on stock position.

# Intuition

- Trading patterns reflect both income shocks and adjustment costs, as well as return shocks
- large trades from tail shocks plus "pent up" adjustment
- turnover rates also reflect financial wealth
- net returns are lower when trading volume is high

# Irrational Agents Approach

- ❖ Households optimize but
  - ❖ allow alternative beliefs about exogenous processes
  - ❖ believe "signal" about future returns
  - ❖ "Overconfidence" is not clear in BO papers
- ❖ Procedure
  - ❖ specify where beliefs differ: (i) income, (ii) returns, (iii) advice
  - ❖ Estimate Parameters
  - ❖ Simulation based on actual not perceived processes
  - ❖ Compare to baseline estimates and fit

Details



## Approach: SMM

$$J = \min_{(\Theta)} \left( M^s(\Theta) - M^d \right)' W \left( M^s(\Theta) - M^d \right). \quad (2)$$

$W$  is the identity matrix.<sup>1</sup>

Parameters:

- discount factor:  $\beta$
- Preferences:  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
- Adjustment Costs:  $(\psi, F)$ , 2 cases

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<sup>1</sup>Optimal weighting matrix for robustness

## Main Results

## Results: Key Points

### Time Series and Cross Section

- ❖ match turnover and net return moments with rational agents
- ❖ fit better with Unemployment Risk (UR)
- ❖ irrational agents do not improve fit
- ❖ robustness exercise

### BO Moments

- ❖ expand turnover moments to include portfolio rebalancing
- ❖ rational agents match turnover and net return moments
- ❖ slight improvement with irrational beliefs about unemployment risk or volatility of the return process
- ❖ consumption loss from irrationality is small

Our hh v.s. BO hh

## Time Series: Moments

Data and Model Moments: Time Series

case	t1	t2	t3	t4	t5	DR
Data	0	0.0039	0.0134	0.0291	<b>0.1155</b>	<b>-0.0010</b>
	Rational					
No UR	<b>0.0065</b>	0.0113	0.0168	0.0247	<b>0.1151</b>	<b>-0.0017</b>
UR	<b>0.0065</b>	0.0104	0.0157	0.0242	<b>0.1159</b>	<b>-0.0018</b>
	Irrational					
$\delta$	0.0065	0.0113	0.0168	0.0247	0.1151	-0.0017
$\sigma_R$	0.0065	0.0115	0.0174	0.0262	0.1076	-0.0018
$\rho_R$	0.0065	0.0113	0.0168	0.0248	0.1152	-0.0017
$\mu_R$	0.0066	0.0113	0.0174	0.0266	0.1137	-0.0026
good news	0.0065	0.0113	0.0168	0.0247	0.1151	-0.0017
bad news	0.0063	0.0112	0.0171	0.0259	0.1160	-0.0017

This table reports data and simulated moments by household averages over time. The panel is balanced. "ti" is the turnover rate for quintile "i" and "DR" is the difference in the net return between the highest and lowest turnover rate quintiles.

## Time Series: Parameter Estimates

Table: Parameter Estimates: Time Series

case	$\beta$	$\gamma$	$\psi$	IR Parm	DP	$J$
				Rational		
No UR	0.9876	2.1659	0.9868			<b>1.3e-04</b>
UR	0.9892	2.2552	0.9803			<b>1.2e-04</b>
				Irrational		
$\delta$	0.9876	2.1659	0.9867	<b>0.0149</b>	<b>0.0140</b>	1.3e-04
$\sigma_R$	0.9876	2.1658	0.9868	0.0439	0.0439	1.3e-04
$\rho_R$	0.9876	2.1659	0.9868	0.0004	0	1.3e-04
$\mu_R$	0.9883	2.2407	0.9854	1.0057	1.0061	1.3e-04
good news	0.9876	2.1659	0.9868	0.0000	0	1.3e-04
bad news	0.9877	2.1670	0.9864	0.0000	0	1.2e-04

This table reports estimated parameters for the various cases:  $\beta$  is the discount factor,  $\gamma$  is relative risk aversion and  $\psi$  is the fraction of income remaining after portfolio adjustment. For the irrational cases, "IR Parm" is the estimated beliefs and "DP" is the parameter from the data.  $J$  is the difference between model moments and data moments as described in equation (2).

Identification

Robustness

Cross Section Moments

Cross Section Params

## BO: Moments

Table: Data and Model Moments: BO

case	t1	t2	t3	t4	t5	DR
Data	<b>0.0019</b>	0.0124	0.0289	0.0598	<b>0.2149</b>	<b>-0.0046</b>
Rational: Time Series, balanced						
No UR	<b>0.0129</b>	0.0248	0.0364	0.0496	<b>0.2148</b>	<b>-0.0032</b>
UR	<b>0.0047</b>	0.0184	0.0311	0.0454	<b>0.2128</b>	<b>-0.0028</b>
Irrational: Time Series						
$\tilde{\delta}$	<b>0.0085</b>	0.0211	0.0328	0.0568	<b>0.2160</b>	<b>-0.0042</b>
$\tilde{\sigma}_R$	<b>0.0079</b>	0.0225	0.0334	0.0557	<b>0.2136</b>	<b>-0.0028</b>

This table reports data and simulated moments. "ti" is the turnover rate for quintile "i" and "DR" is the difference in the net return between the highest and lowest turnover rate quintiles.

Full Table

Our hh v.s. BO hh

## BO: Parameter Estimates

Table: Parameter Estimates: BO

case	$\beta$	$\gamma$	$\psi$	$\Gamma$	IR Parm	DP	$J$
Rational: Time Series, Balanced							
No UR	0.9830	1.7351	0.9947	na	na	na	4.36e-04
UR	0.9767	1.6029	0.9917	na	na	na	2.62e-04
Irrational: Time Series							
$\tilde{\delta}$	0.9828	1.4122	0.9951	na	0.0368	0.0140	1.44e-04
$\tilde{\sigma}_R$	0.9832	1.5123	0.9898	na	0.0958	0.0439	1.80e-04

This table reports estimated parameters for the various cases:  $\beta$  is the discount factor,  $\gamma$  is relative risk aversion and  $\psi$  is the fraction of income remaining after portfolio adjustment.  $J$  is the difference between model moments and data moments as described in equation (2).

Full Table

Economic Significance of Irrationality

## Conclusions

- ❖ BO attributes the data pattern to overconfidence: we **don't concur**
- ❖ take challenges to models of rational agents very seriously and to the data
- ❖ models with **rational agents can match** the turnover moments and net return differentials emphasized in BO
- ❖ introducing various forms of **irrationality does NOT improve the fit** of the model with model-consistent moments;
- ❖ some form of irrationality **slightly** improves the fit when matching expanded moments from BO, **small economic significance**



# Appendix

## Formalities

### Extensive Margin

$$v(\Omega) = \max\{v^a(\Omega), v^n(\Omega)\}$$

for all  $\Omega = (y, b, s, R^s)$ .

### Intensive Margin

#### Adjust:

$$v^a(\Omega) = \max_{b' \geq 0, s' \geq 0} u(c) + \beta E_{\Omega'|\Omega} v(\Omega')$$

*s.t.*

$$c = \psi y + R^b b + R^s s - b' - s' - C(s, s') - F.$$

#### No Adjust:

$$v^n(\Omega) = \max_{b' \geq 0} u(c) + \beta E_{\Omega'|\Omega} v(\Omega')$$

*s.t.*

$$c = y + R^b b - b'$$

$$s' = R^s s.$$

## Policy Functions

- Extensive:  $Z(\Omega) \in \{0, 1\}$ , indicates adjustment
- Intensive:  $(b(\Omega), s(\Omega))$
- Large Turnover due to:
  - adjustment costs
  - large income shocks
  - stock return shocks
    - wealth effects from return on stocks
    - return paid to stock account
- Decision rules depend on
  - trading costs,  $C(s, s')$ , impact measured net return
  - conditional beliefs,  $E_{\Omega'|\Omega}$
- Solved at monthly frequency

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## Types of Irrationality

### ❖ Beliefs about Income

- ❖ Variance:  $\sigma$
- ❖ Unemployment Risk:  $\delta$

### ❖ Beliefs about the Return Process

- ❖ Persistence of Stock Returns:  $\tilde{\rho}$
- ❖ Mean of Stock Return:  $\mu_R$
- ❖ Variance  $\sigma_R$

### ❖ Signals

- ❖ stock broker provides "inside information": optimism or pessimism
- ❖ assumed to be believed
- ❖ large response in portfolio composition and savings

## Noisy Advice

- ❏ common story about brokers inducing excessive turnover with large trading costs
- ❏ introduce signal  $z$  about future stock return
  - ❏ optimism: high return with probability one
  - ❏ pessimism: low return with probability one
  - ❏ has no predictive power but trusted by household
  - ❏ estimate probability of signal,  $p$

back

## Revised Model

$$v(\Omega, z) = \max\{v^a(\Omega, z), v^n(\Omega, z)\}$$

for all  $(\Omega, z)$ .

$$v^a(\Omega, z) = \max_{b' \geq 0, s' \geq 0} u(c) + \beta E_{\Omega' | \Omega, z} \int_z v(\Omega', z') dG(z)$$

s.t.

$$c = \psi y + R^b b + R^s s - b' - s' - C(s, s') - F.$$

$$v^n(\Omega, z) = \max_{b' \geq 0} u(c) + \beta E_{\Omega' | \Omega, z} \int_z v(\Omega', z') dG(z)$$

s.t.

$$\begin{aligned} c &= y + R^b b - b' \\ s' &= R^s s. \end{aligned}$$

## Trading Costs

Model: ( $s_{-1}$  is value of current stock holdings)

$$\text{Buy} : C^b(s_{-1}, s) = \nu_0^b + \nu_1^b(s - s_{-1}) + \nu_2^b(s - s_{-1})^2$$

$$\text{Sell} : C^s(s_{-1}, s) = \nu_0^s + \nu_1^s(s_{-1} - s) + \nu_2^s(s - s_{-1})^2.$$

Table: Estimated Trading Costs

Parameter	Buying	Selling
Constant $\nu_0^i$	56.10 (0.05)	61.44 (0.061)
Linear $\nu_1^i$	0.0012 (1.63e-06)	0.0014 (1.93e-06)
Quadratic $\nu_2^i$	$-1.01e^{-10}$ (2.88e-13)	$-1.28e^{-10}$ (9.26e-13)
Adj. $R^2$	0.251	0.359
Number of Observations	1,746,403	1,329,394

# Moments

## Turnover Moments

### Turnover Rates

$$T_{i,t} \equiv \left| \left( \frac{s_{i,t} - s_{i,t-1}R_t^s}{s_{i,t-1}R_t^s} \right) \right|. \quad (3)$$

### Net Return for agent $i$

$$R_{i,t}^n = \frac{s_{i,t-1}R_t^s - C(s_{i,t} - s_{i,t-1}R_t^s)}{s_{i,t-1}}. \quad (4)$$

### Study difference in $R_{i,t}^n$ by quintile of turnover rate.

## time series vs cross sectional approaches

Moments Table



## Identification at Baseline Moments

Table: Elasticity of Moments to Parameter Values: Baseline

parm.	t1	t2	t3	t4	t5	DR
$\beta$	95.31	45.25	11.96	-14.43	-77.97	-95.39
$\gamma$	3.95	7.32	17.46	18.01	4.44	3.57
$\psi$	6.12	9.36	17.61	18.94	-20.27	21.11

This table reports the elasticity of moments with respect to parameters for the baseline model, based upon a 1 % increase.

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## Cross Section: Moments

Table: Data and Model Moments: Cross Section

case	t1	t2	t3	t4	t5	DR
Data	0.0000	0.0000	0.0000	0.0000	0.2721	-0.0009
	Rational					
No UR	0.0000	0.0000	0.0000	0.0000	0.2723	-0.0017
UR	0.0000	0.0000	0.0000	0.0015	0.2723	-0.0015
	Irrational					
$\rho_R$	0.0000	0.0000	0.0000	0.0000	0.2724	-0.0016

This table reports data and simulated moments. "ti" is the turnover rate for quintile "i" and "DR" is the difference in the net return between the highest and lowest turnover rate quintiles.

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## Cross Section: Parameter Estimates

Table: Parameter Estimates: Cross Section

case	$\beta$	$\gamma$	$\psi$	IR	DP	$J$
	Rational					
No UR	0.9795	1.0666	0.9729	na	na	6.30e-07
UR	0.9727	1.1050	0.9531	na	na	3.57e-07
	Irrational					
$\rho_R$	0.9790	1.0487	0.9626	0.0004	0	6.03e-07

This table reports estimated parameters for the various cases:  $\beta$  is the discount factor,  $\gamma$  is relative risk aversion and  $\psi$  is the fraction of income remaining after portfolio adjustment. For the irrational cases, "IR Parm" is the estimated beliefs and "DP" is the parameter from the data.  $J$  is the difference between model moments and data moments as described in equation (2).

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## Robustness

- ❏ Low Trading Costs: 10% of est. values
- ❏ No reinvestment if no adjustment: stock gains into bond account if no adjustment
- ❏ Fixed cost of adjustment instead of income loss

## Finding

The model with rational agents is able to reproduce both the return differential and high turnover rates

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# Robustness Results

Table: Parameter Estimates: Robustness

case	$\beta$	$\gamma$	$\psi$	$J$
Baseline	0.9876	2.1659	0.9868	1.3e-04
Low Cost	0.9876	2.1635	0.9868	2.0e-04
No Reinvestment	0.9881	2.3782	0.8716	3.0e-04
F	0.9854	2.8566	0.0469	1.2e-04

This table reports estimated parameters for the various cases:  $\beta$  is the discount factor,  $\gamma$  is relative risk aversion and  $\psi$  is the fraction of income remaining after portfolio adjustment.  $J$  is the difference between model moments and data moments as described in equation (2). The row labeled Baseline is the baseline estimation with no unemployment risk.

Table: Data and Model Moments: Robustness

case	t1	t2	t3	t4	t5	DR
Data	0	0.0039	0.0134	0.0291	0.1155	-0.0010
Baseline	0.0065	0.0113	0.0168	0.0247	0.1151	-0.0017
Low Cost	0.0073	0.0127	0.0201	0.0297	0.1101	-0.0070
No Reinvestment	0.0102	0.0156	0.0191	0.0250	0.1152	-0.0009
F	0.0064	0.0110	0.0168	0.0253	0.1145	-0.0013

This table reports data and simulated moments by household averages over time. The panel is balanced. "ti" is the turnover rate for quintile "i" and "DR" is the difference in the net return between the highest and lowest turnover rate quintiles. The row labeled Baseline is the baseline estimation with no unemployment risk.

## BO: Moments

Table: Data and Model Moments: BO

case	t1	t2	t3	t4	t5	DR	exit rate
Data	0.0019	0.0124	0.0289	0.0598	0.2149	-0.0046	0.0120
Rational: Time Series,balanced							
No UR	0.0129	0.0248	0.0364	0.0496	0.2148	-0.0032	na
UR	0.0047	0.0184	0.0311	0.0454	0.2128	-0.0028	na
Rational: Time Series,unbalanced							
No UR	0.0061	0.0105	0.0157	0.0249	0.2176	-0.0060	0.0010
UR	0.0048	0.0083	0.0143	0.0220	0.2133	-0.0079	0.0005
Rational: Cross Section							
No UR	0.0022	0.0112	0.0268	0.0609	0.2126	-0.0049	na
UR	0.0000	0.0022	0.0161	0.0401	0.2120	-0.0052	na
Irrational: Time Series							
$\tilde{\delta}$	0.0085	0.0211	0.0328	0.0568	0.2160	-0.0042	na
$\tilde{\sigma}_R$	0.0079	0.0225	0.0334	0.0557	0.2136	-0.0028	na

This table reports data and simulated moments. "ti" is the turnover rate for quintile "i" and "DR" is the difference in the net return between the highest and lowest turnover rate quintiles.

## BO: Parameter Estimates

Table: Parameter Estimates: BO

case	$\beta$	$\gamma$	$\psi$	$\Gamma$	IR Parm	Parm	$J$
Rational: Time Series, Balanced							
No UR	0.9830	1.7351	0.9947	na	na	na	4.36e-04
UR	0.9767	1.6029	0.9917	na	na	na	2.62e-04
Rational: Time Series, Unbalanced							
No UR	0.9886	2.5459	0.9683	0.0612	na	na	0.0015
UR	0.9927	1.3656	0.9987	0.0120	na	na	0.0018
Rational: Cross-Section							
No UR	0.9469	2.0056	0.9996	na	na	na	1.26e-05
UR	0.9797	1.5313	0.9986	na	na	na	6.67e-04
Irrational: Time Series							
$\tilde{\delta}$	0.9828	1.4122	0.9951	na	0.0368	0.0140	1.44e-04
$\tilde{\sigma}_R$	0.9832	1.5123	0.9898	na	0.0958	0.0439	1.80e-04

This table reports estimated parameters for the various cases:  $\beta$  is the discount factor,  $\gamma$  is relative risk aversion and  $\psi$  is the fraction of income remaining after portfolio adjustment.  $J$  is the difference between model moments and data moments as described in equation (2).

## Economic Significance of Irrationality

the compensating differential given to rational agents if they used the irrational decision rules: is calculated by solving for  $\chi$  that satisfies

$$\begin{aligned} & \sum_{t=1}^{60} \beta^{t-1} U(\chi CR_{i,t}) + \beta^{60} \mathbb{E}_{y',R'|y,R} V(S') \\ &= \sum_{t=1}^{60} \beta^{t-1} U(CIR_{i,t}) + \beta^{60} \mathbb{E}_{y',R'|y,R} V(S') \end{aligned}$$

where  $U(\cdot)$  : the utility function for the rational agent,  $\mathbb{E}$ : rational expectation,  $V(S')$ : the value function of the rational agent in state  $S'$ .

$\chi \approx 99.4\%$ : A rational agent needs **0.6%** consumption compensation to behave like irrational: this is about \$36 or **less than the fixed cost of a single trade.** [back](#)



## To Be Clear: Not Literally BO Household

- ❏ only bonds and stocks: no stock turnover
  - ❏ construct moments to measure portfolio adjustment
  - ❏ ignore stock rebalancing in baseline
  - ❏ add moments with rebalancing as robustness
  - ❏ trading costs from estimates
- ❏ no choice between direct and indirect holdings
- ❏ no data on income process for their households

### Moments from the BO panel data

- ❏ BO moments
- ❏ our moments: recalculate model consistent moments using time series (baseline) and cross section approaches from BO data with and without exit

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